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Modeling and Analysis of Quasi-2D Turbulence Dynamics in Shallow Waters

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Abstract

Introduction. The work is devoted to the study of the generation and development of turbulent structures in shallow-water flows. For optimal water resource management, it is necessary to know what the consequences will be if the flow system changes as a result of human intervention. Basically, all fluid flows that relate to the practice of civil engineering are turbulent in nature. These are, for example, river and channel flows, tidal currents in the oceans and coastal seas. Shallow currents in the environment often include a wide range of vortex scales, ranging from micro-scale vortices to large-scale coherent structures with horizontal length scales that far exceed the depth of water ($L \gg H$). The existence of such large structures is a typical characteristic of turbulence in shallow flow. This indicates the need for a systematic analysis of the problem, as well as modeling of such complex formalized systems. The purpose of this work is to model and analyze the dynamics of quasi-2D turbulence structures.

Materials and Methods. Large-scale quasi-2D coherent structures (2 DCS) are investigated depending on the source and localization in the liquid column. Turbulent flows in the channel satisfying incompressible Navier-Stokes equations are considered. The numerical experiment was carried out on the basis of the “large eddy simulation” (LES) approach.

The Results of the Study. Scenario of the dynamics of quasi-2D turbulence structures of the coastal zone is constructed, the formation of vortex structures is predicted.

Discussion and Conclusions. The development of two-dimensional turbulence in shallow flows illustrates the processes that control quasi-two-dimensional turbulence, including the merging of individual vortices. The main mechanism controlling the decay of 2DCS is the loss of energy due to friction on the bottom, while the larger the size of the vortex relative to the depth, the faster the direct dissipation of its kinetic energy occurs.

Keywords: turbulent structures, shallow water channels, large-scale quasi-2D coherent structures (2PCS), quasi-2D turbulence, vortex scale.

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Моделирование и анализ динамики квази-2D-турбулентности в мелководных водоемах

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Аннотация

Введение. Работа посвящена изучению генерации и развития турбулентных структур в мелководных потоках. Для оптимального управления водным ресурсом необходимо знать, какие будут последствия при изменении системы течения в результате вмешательства человека. В основном все потоки жидкости, которые относятся к практике гражданского строительства, имеют турбулентный характер. Это, например, речные и русловые потоки, приливные течения в океанах и прибрежных морях. Неглубокие течения в окружающей среде часто включают в себя широкий диапазон масштабов вихрей, начиная от микромасштабных вихрей и заканчивая крупномасштабными когерентными структурами с горизонтальными масштабами длины, которые намного превышают глубину воды ($L \gg H$). Существование таких крупных структур — типичная характеристика турбулентности при мелком течении. Это указывает на необходимость проведения системного анализа проблемы, а также моделирования подобных сложно формализуемых систем. Целью данной работы является моделирование и анализ динамики структур квази-2D-турбулентности.

Материалы и методы. Исследуются крупномасштабные квази-2D когерентные структуры (2DCS) в зависимости от источника и локализации в столбе жидкости. Рассматриваются турбулентные течения в канале, удовлетворяющие несжимаемым уравнениям Навье-Стокса. Численный эксперимент выполнен на основе подхода «моделирование крупных вихрей» (LES).

Результаты исследования. Построен сценарий динамики структур квази-2D-турбулентности береговой зоны. Предсказано формирование вихревых структур.

Обсуждение и заключения. Развитие двумерной турбулентности в неглубоких потоках служит иллюстрацией процессов, которые управляют квази-двумерной турбулентностью, включая слияние отдельных вихрей. Основным механизмом, управляющим распадом 2DCS, являются потери энергии из-за трения о дно. При этом, чем больше размер вихря относительно глубины, тем быстрее происходит прямое рассеивание его кинетической энергии.

Ключевые слова: турбулентные структуры, мелководные потоки, крупномасштабные квази-2D когерентные структуры (2DCS), квази-2D-турбулентность, масштаб вихрей.

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Introduction. In the context of fluid mechanics in the environment, it can be stated that almost all fluid flows have a turbulent character. Typically, a turbulent flow contains vortices, also called “coherent structures” or “turbulence structures”. These are dynamic recirculation structures resulting from the instability of the internal flow. Despite the fact that the size and boundaries of individual vortices cannot usually be identified unambiguously, it is often possible to determine the length and velocity scales to characterize the behavior of various types of vortices in the flow, especially dominant energy-containing large vortices. Shallow flows include a wide range of vortex scales, including large-scale coherent structures. Such vortices are often observed in areas with large horizontal velocity differences or near obstacles where the flow separates from the wall.

The turbulence of a shallow current, like any other, in the environment cannot be directly predicted in detail due to its chaotic nature. Careful experimental work (field or laboratory data) and detailed numerical modeling are required to predict the behavior of turbulence in real situations with some accuracy. This is of practical importance for improving the understanding and modeling of large-scale turbulence in shallow water. For optimal management of a water resource,

it is necessary to know with a high degree of probability what consequences will occur when the flow system changes (including turbulence) as a result of human intervention.

Turbulent flows are present everywhere in nature, shallow turbulent flows among them form an important subgroup. In fact, turbulence is a chaotic phenomenon. However, turbulence in shallow currents can be described as “organized chaos”. The organization in such flows is visible due to turbulent structures with length scales, usually exceeding the depth of water, which can have a relatively long lifetime.

A shallow flow is defined as three-dimensional, one dimension of which is significantly smaller than the other two dimensions. In the context of environmental hydromechanics, this smaller dimension is usually the depth of the water. G. H. Jirka [1] describes several mechanisms that cause large-scale turbulence structures, which are typical for shallow flows. Such large vortices are predominantly two-dimensional in nature. Their dynamics differ significantly from smaller-scale vortices ($L < H$), which have a completely three-dimensional character, i. e. relatively short lifetime ($T \approx L/U$), weaker cross-correlation and a continuous tendency to break up into smaller vortices. Although all turbulence in nature is essentially three-dimensional, shallow flow turbulence is often referred to as “quasi-2D” [2]. G. H. Jirka [1] introduced the abbreviation 2DCS to denote large-scale quasi-2D coherent structures. It is obvious that 2D objects in the stream are always accompanied by three-dimensional structures of a smaller scale. Figure 1 shows a color-synthesized image of the Taganrog Bay of the Azov Sea, obtained in March 2020 from the Landsat 8 remote sensing satellite, the figure shows the structures of turbulent currents in the Taganrog Bay, which have a quasi-two-dimensional character.

Turbulence is usually caused by a shift in the direction perpendicular to the local flow velocity, as a result of which the flow becomes unstable. The origin of such a lateral shift can always be traced either to wall friction (wall turbulence) or to a transverse velocity gradient within the region (free turbulence). An important mechanism responsible for the occurrence of internal velocity gradients is flow separation. Separation occurs when the boundary layer of the flow loses contact with the corresponding solid wall and breaks away from it. This may be due to geometric reasons (for example, the flow is not able to follow the complex shape of the boundary or smoothly bend around the corner) or for dynamic reasons (the pressure gradient in the flow disrupts the equilibrium of the local boundary layer). Separating flows include an area of strong transverse shear downstream from the separation site, which leads to a high intensity of turbulence and is often an area of flow recirculation [3–5].



Fig. 1. Color-synthesized image of the Taganrog Bay of the Azov Sea from the Landsat 8 remote sensing satellite, resolution 30 m

Shallow flows, jets and mixing layers are three common types of free, shallow, shear flows. In other cases, the presence of side walls leads to the fact that the separating flow creates a recirculation area.

Materials and methods

1. Shallow water and its effect on turbulence. Turbulent flows in the channel satisfy incompressible Navier-Stokes equations in a conservative form [6]:

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} + \frac{\partial P/\rho}{\partial x_i} - \frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = f_i, \quad (2)$$

where u is the velocity vector of the liquid (m/s); P is the hydrodynamic pressure (kg/ms^2); ρ is the constant density (fresh water, 1000 kg/m^3); ν is the constant kinematic molecular viscosity ($10^{-6} \text{ m}^2/\text{s}$) and f is the vector of body force per unit mass (m/s^2), which usually constitutes gravity, $i, j \in \{1, 2, 3\}$. The velocity field u is called divergence-free or solenoidal. The gravity of the body can be eliminated by including it in the pressure gradient. If we set $f_i = -\partial(g x_3)/\partial x_i$ (with acceleration of gravity $g = 9.81 \text{ m/s}^2$) and determine the so-called non-hydrostatic normalized pressure through $\rho = P/\rho + g x_3$, equation (2) can be rewritten as:

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} + \frac{\partial \rho}{\partial x_i} - \frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = 0. \quad (3)$$

Equations (1) and (3) must be supplemented with appropriate initial and boundary conditions. On impermeable solid walls, the boundary condition of non-slip is physically fulfilled, whereas on the surface (considered either as a movable free surface or as a rigid cover), the condition of free slip applies (wind and atmospheric influences are neglected in this study) [7–9].

Fluid motion is induced by hydrodynamic pressure gradients, while the velocity field is constantly deformed under the action of nonlinear momentum advection and viscosity. When equation (3) is written in dimensionless form, using the velocity scale U and the length scale L , the ratio of advective and viscous forces can be expressed by one dimensionless parameter — the Reynolds number (Re):

$$Re = \frac{UL}{\nu}. \quad (4)$$

Two features in the Navier-Stokes equations are a significant cause of turbulence. At large values of Re, the flow problem can become hydrodynamically unstable and eventually exhibit chaotic behavior. The action of viscous forces (in combination with the boundary conditions of non-slip) introduces rotation (or vorticity) into the velocity field, even if the initial flow field did not contain rotation. Due to the presence of vorticity, the chaotic flow field will always contain vortices or “vortices” [10].

Because of the important role of vorticity, turbulence is, in fact, a three-dimensional phenomenon. From a physical point of view, the growing chaos in turbulent flows is illustrated by the fact that vortices are unstable and tend to break up into smaller vortices. This basically means that turbulent kinetic energy is transferred towards smaller scales until the smallest vortices reach length scales at which their energy is converted into heat under the action of viscosity (the so-called Kolmogorov scales). This ongoing release of turbulent kinetic energy is often referred to as a “3D energy cascade” [11]. The energy flow towards smaller length scales often leaves its mark in the energy density spectrum of turbulent motion. As for the three-dimensional energy cascade, the spectrum of spatial energy density $E \text{ (m}^3/\text{s}^2)$ at small scales of isotropic turbulence (inertial range) should have the following form:

$$E(k) \approx \tau^{2/3} k^{-5/3}, \quad (5)$$

where $\tau \text{ (m}^2/\text{s}^3)$ is the rate of energy dissipation per unit mass; and $k \text{ (m}^{-1})$ is the “wave number” associated with a certain scale of turbulence length. Vorticity plays a vital role in the mechanism of the energy cascade. This can be seen by taking the curvature of equation (3), which gives the vorticity equation:

$$\frac{D\omega_i}{Dt} \equiv \frac{\partial \omega_i}{\partial t} + u_j \frac{\partial \omega_i}{\partial x_j} = \omega_j \frac{\partial u_i}{\partial x_j} + \nu \frac{\partial^2 \omega_i}{\partial x_j^2}, \quad (6)$$

where $\omega \text{ (s}^{-1})$ — is the vorticity vector. This equation shows that the material derivative of the vorticity of a moving fluid particle is determined by the right side of the equation. The first term describes the interaction between the vorticity field and the velocity deformation field; the second one describes the diffusion of vorticity through molecular viscosity [12–14].

The first term is responsible for the vortex stretching and can be rewritten as:

$$\omega_j \frac{\partial u_i}{\partial x_j} = \frac{1}{2} \omega_j \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = \frac{1}{2} \omega_j s_{ij}. \quad (7)$$

If the velocity field is stretched in the direction of the vector of local vorticity (i. e., normal to the corresponding plane of the vortex, (Fig. 2 a)), the local vorticity in this direction will increase; the kinetic energy of rotation will be transmitted to higher frequencies and, consequently, to smaller scales, both in space and in time. This vortex stretching mechanism is responsible for the spectral energy flow in the 3D energy cascade.

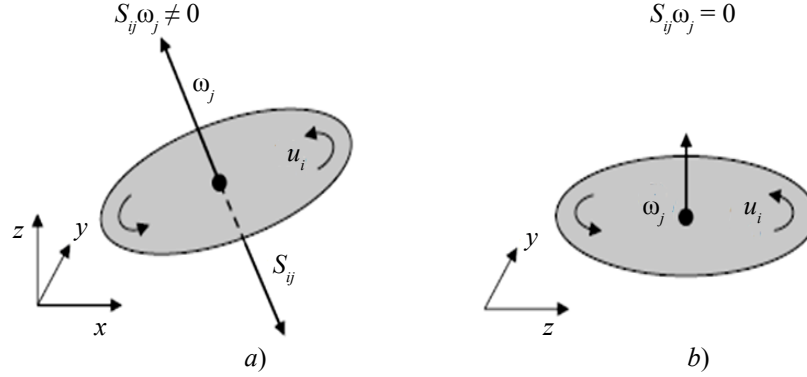


Fig. 2. 2D and 3D vortex dynamics: a — in three-dimensional space, vortices can be stretched in the direction of the local vorticity vector perpendicular to the plane of the vortex; b — a vortex in a two-dimensional plane cannot be stretched in the direction perpendicular to this plane

2. Theoretical study of the quasi-2D turbulence structures dynamics. Despite the inherent three-dimensionality of turbulence, many areas of turbulent flow in nature are limited to the vertical direction. Such a flow is called shallow; its large-scale turbulence is often regarded as quasi-two-dimensional. Although 2D turbulence is *contradictio in terminis*, this classification still makes sense, since in practice the dynamics of quasi-2D turbulence structures can differ significantly from “normal” 3D turbulence. This can be shown by referring to the vorticity equation (6). In two dimensions, the velocity vector consists of only one component and is perpendicular to the two-dimensional velocity field everywhere (Fig. 2 b). However, stretching of the two-dimensional velocity field is impossible in this perpendicular direction. Consequently, the vortex stretching term vanishes, which leads to:

$$\frac{D\omega_i}{Dt} \equiv v \frac{\partial^2 \omega_i}{\partial x_j^2}. \quad (8)$$

The remaining terms show that, theoretically, vorticity is a conserved quantity in 2D. Moreover, it follows from (8) that the total entropy (a measure of the amount of kinetic energy of rotation) is also approximately conserved, with the exception of a small quadratic dissipation term due to viscosity:

$$\frac{D\Omega}{Dt} = \frac{\partial \Omega}{\partial t} + u_j \frac{\partial \Omega}{\partial x_j} = v \frac{\partial^2 \Omega}{\partial x_j^2} - v \left(\frac{\partial \omega_i}{\partial x_j} \right)^2, \quad (9)$$

where $\Omega = \frac{1}{2} \omega_i^2 (s^{-2})$ is the general enstrophy. The principle of enstrophy conservation is similar to the principle of energy conservation: kinetic energy is also conserved, except for the quadratic dissipation term. This result follows from multiplying equation (3) by u_i :

$$\frac{DE_k}{Dt} + \frac{\partial u_i \rho}{\partial x_i} = \frac{\partial}{\partial x_j} v u_i \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{v}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2, \quad (10)$$

where $E_k = \frac{1}{2} u_i^2 (m^2/s^2)$ is the total kinetic energy. Consequently, in the viscosity limit, the energy balance of a two-dimensional flow in time and space is limited by two conservation laws (9) and (10) (instead of only (10) for the 3D case). The stored values E_k and Ω can be written in spectral form:

$$E_k = \frac{1}{2} u_i^2 = \int_0^\infty E_k(k) dk, \quad (11)$$

$$\Omega = \frac{1}{2} \omega_i^2 = \int_0^\infty k^2 E_k(k) dk. \quad (12)$$

Consequently, Ω can be expressed as the second moment of the spectral distribution of kinetic energy. This means that in a two-dimensional flow, not only the total amount of kinetic energy is preserved, but also its dispersion over all length scales. This fact implies two simultaneous energy cascades. Suppose that the situation with a two-dimensional flow is violated by some forced mechanism with a characteristic length scale, then it is possible to determine the wave number of the impact $k_i \approx L^{-1}$, at which kinetic energy is added to the system. The combination of conservation laws (9) and (10) will cause a redistribution of energy: if kinetic energy is transferred from the k_i scale to higher wave numbers (small scales) $k > k_i$ there must be a compensating energy flow in the direction of lower wavenumbers (large scales) $k < k_i$ in order to maintain dispersion. This energy transfer towards a longer length and time scales is often called backscattering or reverse energy cascade. Consider spectral forms to account for these two simultaneous processes:

$$E(k) = \begin{cases} \tau^{2/3} k^{-5/3}, & \text{if } k < k_i, \\ \eta^{2/3} k^{-3}, & \text{if } k > k_i, \end{cases} \quad (13)$$

where τ (m^2/s^3) is the rate of energy dissipation; and η (s^{-3}) is the rate of energy dissipation per unit mass. The existence of an inverse energy cascade in the region implies that after some initial stage, kinetic energy tends to concentrate in large-scale vortices that are stable and do not decay. This principle is often called “self-organization”.

As a result of experiments and simulations, specific types of vortices are identified, monopolar, dipole and tripolar vortex formations can be distinguished. These configurations are characterized by the fact that neighboring two-dimensional vortices are able to coexist when they have opposite signs of vorticity. On the other hand, two monopolar two-dimensional vortices with the same sign of vorticity are able to combine and form a new, larger vortex. This phenomenon is known as vortex fusion, which is very noticeable and impressive due to the existence of a reverse energy cascade and a natural analogue of the vortex stretching mechanism, which is responsible for their destruction (Fig. 3).

Research results

Numerical simulation of the quasi-2D turbulence structures dynamics. Shallow turbulent flows exhibit many two-dimensional characteristics, which is called quasi-two-dimensional flow behavior. Although the vortex stretching mechanism is not completely excluded in a shallow flow, it is at least very difficult in vertical measurement. If large-scale quasi-two-dimensional coherent structures are present in a shallow flow, it is often observed that they are quite stable and only weakly dissipative.

A typical quasi-2D problem, characterized by both small 3D turbulence and 2DCS. The latter are clearly distinguishable large-scale structures that remain intact for a relatively long time during passage through the flow region.

Figure 3 shows an example of vortex fusion in 2D modeling of large vortices (LES), it demonstrates graphs of vorticity contours at four stages of the fusion process.

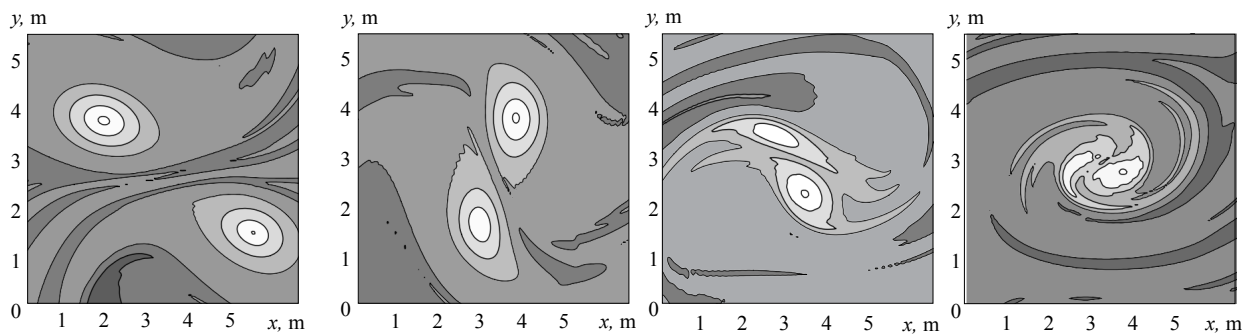


Fig. 3. Results of 2D modeling of the LES vortex fusion due to the existence of an inverse energy cascade: vorticity contours graphs

The term “coherent structures” is used to consider related large-scale turbulent fluid masses that spread uniformly throughout the water depth and contain phase-correlated vorticity, with the exception of a thin bottom boundary layer. In the case of internal instability with transverse shear, separation does not occur: due to the difference in lateral velocities, hydrodynamic instabilities will arise, which will gradually develop into 2DCS. Differences in lateral velocity can be caused, for example, by river mergers or differences in depth and irregularities (complex channels).

Discussion and conclusions. In all cases, the generation and development of 2DCS sequences requires a certain transit time and a certain spatial distance from their source. There are three different regions of 2DCS development, based on the ratio of the distance of the vortex propagation x and the depth of the water H . In the “near field” region ($x/H < 1$) three-dimensional small-scale turbulence prevails, but the average transverse shift present is usually two-dimensional, mainly due to the shape of the geometry. In the “far field” ($x/H > 10$) 2DCS are well developed in the horizontal direction and eventually dissipate due to friction on the bottom. Both medium flow and large-scale turbulence have a pronounced 2D character. The “middle field” ($1 < x/H < 10$) is characterized by the interaction between the growing 2DCS, the average flow and 3D turbulence of the bottom, which leads to the effects of the average secondary flow and 3D effects inside the 2DCS, for example, areas of ascending and descending fluid flows.

The development of two-dimensional turbulence in shallow channels often serves as a good illustration of the processes that control quasi-two-dimensional turbulence, including the merging of individual vortices. 2DCS usually grow when moving in a downstream direction. Eventually the 2DCS will decay in the far field area. The main mechanism controlling this decay is the loss of energy due to friction on the bottom. The larger the size of the vortex relative to the depth, the faster the direct dispersion of its kinetic energy occurs. This fact limits the maximum size of the vortex λ that can be detected in real shallow flows. In cases of very shallow flow, even the formation of 2DCS can already be suppressed by friction against the bottom.

Among the many shallow flow configurations that can contain 2DCS, there are several main types: traces, mesh turbulence, jets and mixing layers. These basic configurations, based on general studies of 3D turbulence, have their analogues in the theory of shallow flow. In three-dimensional cases, the turbulence regime of these types of flows is determined by the ratio between advective and viscous forces, which is expressed by the Reynolds number (Re). In the corresponding quasi-two-dimensional cases, this friction is more important than the molecular viscosity, therefore, the behavior of these shallow flows is determined by the ratio between horizontal transverse shear and bottom friction. These two values, respectively, determine the production and dissipation of kinetic energy of 2DCS.

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SV Protsenko: making calculations, drawing conclusions, text preparation, working with sources, making graphic materials. EA Protsenko: basic concept formulation, research objectives and tasks computational analysis, conclusions correction, text revision.

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