MATHEMATICAL MODELING

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ

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Simulation of Vehicular Traffic using Macro- and Microscopic Models

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Abstract

To effectively regulate traffic on highways and networks of modern megacities, it is necessary to introduce Intelligent Transport Systems, which include many innovative solutions, in particular, mathematical models for describing the dynamics of traffic flows.

The article is devoted to a brief description of the current state in this area in its development — from the simplest macroscopic and microscopic models that have become classic to modern developments.

Special attention is paid to the original multilane models developed by the authors of the article within both approaches. The macroscopic model is based on the quasigasdynamic approach, while the microscopic one uses the ideology of cellular automata and constitutes a generalization of the Nagel-Schreckenberg model for the multilane case.

The difference in the representation method and the mathematical apparatus for the mac-roscopic and microscopic description of traffic flows is briefly described, followed by the review of the main models at different stages of their development, presented by foreign and Russian authors.

Special attention is paid to the three-phase theory of Boris Kerner and models built in the framework of this theory.

Examples of modern software for traffic modeling are given.

The original quasigasdynamic model of traffic flows, which uses the continuum approximation and is constructed by analogy with the well-known model of gas dynamics, is briefly described. Due to the introduction of the lateral speed, the model is generalized to the multilane case.

An original microscopic model based on the cellular automata theory and representing a generalization of Nagel-Schreckenberg model for the multilane case is described. The model has been further developed by taking into account various driving strategies and behavioral aspects.

The article presents a brief overview of the state of the art in the field of mathematical modeling of traffic flows, as well as original macroscopic and microscopic models developed by the authors for the case of multilane traffic.

Keywords: mathematical modeling, traffic flows, microscopic and macroscopic models, cellular automata, multilane traffic.

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Обзорная статья

Моделирование движения автомобильного транспорта с использованием макро- и микроскопических моделей

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Аннотация

Для эффективного регулирования дорожного движения на магистралях и сетях современных мегаполисов необходимо внедрение Интеллектуальных транспортных систем, включающих в себя множество инновационных решений, в частности, математические модели описания динамики транспортных потоков.

Статья кратко описывает современное состояние транспортных систем и их развитие: от простейших макроскопических и микроскопических моделей, ставших классическими, до современных разработок.

Особое внимание уделяется разработанным авторами статьи оригинальным многополосным моделям в рамках обоих подходов. Макроскопическая модель основана на квазигазодинамическом подходе, а микроскопическая использует идеологию клеточных автоматов и является обобщением модели Нагеля-Шрекенберга на многополосный случай.

Кратко описывается различие в способе представления и математическом аппарате для макроскопического и микроскопического описания транспортных потоков. Дальше следует обзор основных моделей на разных этапах их развития, принадлежащих зарубежным и российским авторам.

Рассматривается трехфазная теория Бориса Кернера и модели, построенные в рамках этой теории.

Приводятся примеры современного программного обеспечения для транспортного моделирования.

Кратко описывается оригинальная квазигазодинамическая модель транспортных потоков, использующая приближение сплошной среды и построенная по аналогии с известной моделью газовой динамики. Благодаря введению скорости перестроения модель обобщена на многополосный случай.

Описывается оригинальная микроскопическая модель, основанная на теории клеточных автоматов, которая является обобщением модели Нагеля-Шрекенберга на многополосный случай. Модель получила дальнейшее развитие путем учета различных водительских стратегий и поведенческих аспектов.

В статье представлен краткий обзор состояния в области математического моделирования транспортных потоков, а также представлены оригинальные макроскопическая и микроскопическая модели, разработанные авторами для случая многополосного движения.

Ключевые слова: математическое моделирование, транспортные потоки, микроскопические и макроскопические модели, клеточные автоматы, многополосное движение.

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Introduction. World experience shows that in large cities it is necessary to introduce an Intelligent Transport System (ITS) for the effective construction of new transport networks with a complex multi-level structure, as well as for the operational regulation of traffic on them. ITS is a set of systems based on information, communication and management technologies embedded in vehicles and road infrastructure. It combines many innovative solutions: from mathematical models and methods of traffic description to decision support systems for traffic management, not to mention technical and engineering aspects.

The proposed article is devoted to a brief description of classical and modern trends in the field of mathematical modeling of motor traffic flows. Two main directions in this field are considered: macroscopic and microscopic models.

An overview of ready-made software tools for modeling the flows of road transport is also provided.

Special attention is paid to the original multiband models developed by the authors of the article within the framework of both approaches. The macroscopic model considers the transport flow as the movement of a weakly compressible gas and uses the ideology of kinetically consistent difference schemes and a quasi-gas dynamic (QGD) system of equations [1]. Recently, modern ultra-high-performance computing technology has appeared and the popularity of microscopic models has increased significantly. However, due to their cost-effectiveness, macroscopic models do not lose relevance in determining the main characteristics of road traffic necessary for transport planning.

The original microscopic model is based on the theory of cellular Automata (CA), adapted to modeling traffic flows on multi-lane highways and the main elements of the road network (RN) [2]. This approach allows you to take into account many technical parameters of cars and features of driver behavior. Such models can include a detailed description of the movement of cars at intersections and in places of narrowing of roads, overtaking and rebuilding, providing a high degree of compliance with the model of the real situation.

Both proposed approaches have internal parallelism and are suitable for fast and efficient calculations on supercomputers, even for modeling large-scale road networks with several million vehicles.

1. State of the research area. Currently, the theory of traffic flows is an independent scientific direction, which is based on the so-called physics of traffic flows — mathematical and simulation modeling. Mathematical traffic models are used both in research and in practice to justify planning and management decision-making in the transport industry.

Modeling of motor traffic flows began to develop in the USA since the 30s of the 20th century. But due to the increasing volume of transportation everywhere, as well as increasingly accessible computerization, in the 1990s this area began to attract more and more attention. Two main directions of this development have emerged: macroscopic modeling and microscopic modeling, which differ in the way they represent real reality, and in their mathematical description.

In the first case, traffic uses the approximation of a continuous medium and considers the flow of cars similarly to the flow of a weakly compressible gas. The main studied values are the density field (the number of cars per unit length of the road and per lane) and the average speed field, as well as the flow (the number of cars that have passed a given point on the road per unit time). The model consists of a system of partial differential equations and is solved by well-known finite difference methods.

In the case of microscopic modeling, the subject of the study is the movement of one individual car and its interaction with other participants in the movement, the reaction to the environment and its possible changes in this situation.

Such models are described, as a rule, by ordinary differential equations, for the solution of which there are also known numerical methods, for example, the Runge-Kutta method of the second or fourth orders.

Using macroscopic models, it is convenient to describe a fairly dense flow of vehicles when all drivers are forced to adhere to the same strategies and drive at approximately the same speed. With the help of such models, general patterns of traffic are usually investigated. Microscopic models allow us to consider in more detail the movement of the transport unit “driver-car”. This takes into account not only the characteristics of the car itself, but also the behavioral characteristics of the driver, and perhaps even his psychological type. With the help of such models, it is possible to describe not only a sparse flow, but also a dense flow of vehicles thanks to today’s computing capabilities.

One of the first simplest macroscopic models is the Lighthill-Witham-Richards (LWR) model [3]. It is characterized by a single dynamic equation, which is a consequence of the law of conservation of the number of cars:

$$\frac{\partial \rho}{\partial t} + \frac{dQ_e(\rho)}{d\rho}\frac{\partial \rho}{\partial x} = 0,$$

where ρ is the automobile flow density; Q_e is the equilibrium flow.

In this model, it is assumed that the flow or average velocity is always in local equilibrium relative to the actual density and instantly changes with it, that is, unreasonably high accelerations occur: $V = V_e(\rho)$, $Q = Q_e(\rho)$. Models of this type, due to the lack of finite acceleration, cannot describe the growth of traffic waves and the instability of the traffic flow.

At the next stage, models appeared that include, in addition to the continuity equation, a second dynamic equation — the acceleration equation, which describes local acceleration as a function of density, velocity, their gradients and other possible external factors. Such a class of models is known as a class of *second-order models*, in contrast to the LWR models, which are called *first-order models*.

The book [4] presents the Payne model [5], for which the acceleration equation has the form:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{V_e(\rho) - V}{\tau} + \frac{V_e'(\rho)}{2\rho\tau}\frac{\partial \rho}{\partial x}$$

with constant relaxation time and the Kerner-Conheuser model [6]:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{V_e(\rho) - V}{\tau} - \frac{c_0^2}{\rho}\frac{\partial \rho}{\partial x} + \frac{\eta}{\rho}\frac{\partial^2 V}{\partial x^2}.$$

Here an analogue of sound velocity $\pm c_0$ and dynamic viscosity η are introduced. This model is purely phenomenological.

The Payne model and many subsequently proposed second-order models, including those with diffusion corrections, have some disadvantages. In particular, with strong spatial inhomogeneities of the initial conditions, negative values of speeds and densities exceeding the maximum permissible may occur, and also, according to these models, cars behind have a noticeable effect on the movement of the car, which is unrealistic in the case of one lane. In the future, a lot of effort was spent on making macro models anisotropic, that is, according to these models, cars should respond only to the situation in front of them. The most well-known models that solve these problems are the Aba-Raskla [7] and Zang [8] models.

Within the framework of the microscopic approach, the simplest model was the model of following the leader [9], which could reproduce only the basic details and features of traffic flows. The simplest representative of this class of models is a continuous-time model of optimal speed:

$$\dot{v} = \frac{v_{\text{opt}}(s) - v}{\tau},$$

which describes the adaptation of the actual speed of the car v to the optimal speed $v_{\text{opt}}(s)$ for the time scale set by the *adaptation time* τ. Its analogue is the discrete-time Newell model [10]:

$$v_\alpha(t + \Delta t) = v_{\text{opt}}(s(t)) = \min\left(v_0, \frac{s}{\Delta t}\right),$$

$$x_\alpha(t + \Delta t) = x_\alpha(t) + \frac{v_\alpha(t) + v_\alpha(t + \Delta t)}{2}\Delta t.$$

Another interesting example of the simplest model of following the leader is the Pipes model [11], based on the safe driving rule developed in California: "the rule for following the vehicle in front at a safe distance is to keep the distance between your car and the car in front of you no less than the shortest length of the car on the every ten miles an hour of the speed at which you are traveling". Translated into mathematical language, this model can be formulated as follows:

$$s_i(t)_{\min} = \frac{l_i}{0.44}\dot{x}_i(t) + l_{i-1},$$

where $s_i(t)$ is the gap between the current and the cars in front, l_i is the length of the i-th car.

The Intelligent Driver Model (IDM) was a further development of the models of following the leader. Continuous-time IDM is the simplest complete and trouble-free model, giving realistic acceleration profiles and plausible behavior in all single-lane traffic situations. The most well-known model of this class is the Driver model [12], which demonstrates realistic behavior during acceleration and braking.

Separately, it should be noted the microscopic Prigogine models based on the kinetic Boltzmann theory [13, 14]. The model introduces a distribution function type function in kinetic theory $f(x,u,t)$, which denotes the number of cars located at time t at a point in space between x and $x+dx$ and having a speed between u and $u+du$. The concept of the desired distribution is also introduced, which is an idealization of the goal to which this traffic flow aspires. The real and desired distributions may differ for many different reasons: road conditions, weather conditions, interaction with other cars, etc. By themselves, these reasons may also change over time and, consequently, the real distribution will approach the desired for some relaxation time. Based on these assumptions , an equation of the Boltzmann equation type is written for the real distribution:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = \left(\frac{\partial f}{\partial t}\right)_{rel} + \left(\frac{\partial f}{\partial t}\right)_{int},$$

where $\left(\frac{\partial f}{\partial t}\right)_{rel}$ is the transition of the real distribution to the desired one in the absence of interaction of cars, $\left(\frac{\partial f}{\partial t}\right)_{int}$ is the change in the real distribution arising from interactions between cars.

The members on the right side can be set in various ways, the distribution function can also have a more complex form. Due to this, there are a sufficient number of varieties of this model: for example, in the Paveri-Fontana model [15], in addition to the real one, the “desired” speed of this car is introduced. Prigozhin’s approach was subsequently developed in the works of Helbing et al. [16, 17].

In the future, models of both macro- and microscopic types developed in the direction of taking into account the human factor. There were models with a safe speed of movement [18], non-equilibrium models with realistic acceleration [19], models describing traffic at complex road junctions [20, 21], describing mixed flows consisting of heterogeneous vehicles [22, 23], etc.

Modern studies of the dynamics of traffic flows are mainly on the path of complicating existing models. One can cite, for example, publications [24–26] devoted to macroscopic models of hydrodynamic type.

In the field of microscopic modeling, a separate specific direction has been rapidly developing recently, using the theory of cellular automata. These models can be divided into two groups: deterministic and stochastic. An example of a deterministic model is the 184 Wolfram Rule. This model belongs to the class of elementary cellular automata. This is a group of 256 (2^{2^3}) one-dimensional models with the number of neighbors 3, they can be found in the Wolfram Atlas on the website [27].

One of the first realistic stochastic models of traffic flows is the well—known Nagel-Schreckenberg model [28]. This model requires detailed consideration, since many modern models developed by researchers around the world are based on it.

The route in the Nagel-Schreckenberg model is represented as a one-dimensional lattice, each cell of which can be either empty or contain a particle denoting a vehicle. The particles move from one cell to another (free) in one direction. In the case of single-lane traffic, they cannot overtake each other. The whole system is the space, time, speed are discrete. The speed shows how many cells the car moves in one time step. Acceleration occurs instantly between steps. At each time step, the system status is updated according to certain rules:

1. Acceleration. The speed of i-th car is increased by one if the maximum allowed speed is not reached: $V_i \to \min(V_i+1, V_{\max})$.

2. Deceleration. The speed of the car is reduced by one if there is a threat of collision with the car in front: $V_i \to \min(V_i, D_i - 1)$, where D_i is the distance to the car in front.

3. Random disturbances. If the speed of the car is positive, then it can be reduced by one with some probability: $V_i \to \max(V_i - 1, 0)$ with probability p.

4. Motion. Each car moves forward by the number of cells corresponding to its new speed after completing the previous steps: $X_i \to X_i + V_i$.

To simplify the recording, we assume that speed and distance are measured in cells, and time is dimensionless. For this reason, the values can be added, subtracted and compared with each other.

To date, there are more complex and detailed CA models. The article [29] presents an interesting generalization of the theory of cellular automata for the case of maritime transport in application to maritime transport. In this case, the space discretization rules are supplemented by mapping rules. The authors of the article [30] investigate the capacity of a motorway with two entrances and one intermediate exit between them also using a model of cellular automata. The aim of the research is to maximize the throughput of the system by establishing the optimal flow for two entrances. The paper [31] presents a numerically reliable model of cellular automata aimed at accurately reproducing deceleration and acceleration in accordance with realistic reactions of drivers when considering vehicles with different deceleration capabilities.

It is possible to model large road networks using cellular automata. As an example, we will give a model created by A. P. Buslaev [32] and colleagues at MADI. Their approach is based on ring structures of cellular automata with common cells, for which there is competition. Similar ring structures may have different topologies, the movement along them simulates the movement along the UDS with intersections.

In the early 2000s, an alternative theory of traffic flows appeared, namely, Boris Kerner's three-phase theory was proposed. The first works date back to 2002, however, the main provisions of the theory were formulated later in books [33] and [34]. Unlike previous theories, where two main phases of traffic flows (free movement and dense flow) were considered, here the author considers the existence of three phases: free flow, synchronized movement and a wide moving cluster, that is, two phases are distinguished in a dense flow. This makes it possible to predict and explain the empirical properties of the transition from free to dense traffic, as well as the features of the resulting spatial-temporal traffic structures. The author himself calls his theory empirical, qualitative, based on observational data, which allows the creation of various mathematical models within the framework of this theory. The author and other researchers have created models based on cellular automata [35, 36]. In particular, the Kerner-Klenov model [37, 38] introduces the concepts of acceleration and synchronization distance to correspond to the theory of three phases. Due to the mathematical description of stochastic acceleration with delay and the adaptation effect inside the synchronized flow, in the developed model, the transition from free to dense flow is an $F \rightarrow S$ transition (according to Kerner's theory of three phases) in a metastable free flow, which is observed in all empirical data. Kerner and Klenov also proposed a deterministic model [39]. In [40, 41], variants of macroscopic models implementing the three-phase theory are proposed.

In general, models corresponding to Kerner's three-phase theory are characterized by the ability to describe instabilities that inevitably arise in real traffic. Such models demonstrate one of the main theses of the theory of three phases: transitions between phases from free flow to synchronized and from synchronized to wide moving clusters can occur under the influence of random processes and at different values of the flow, and not be tied clearly to its specific value of the flow. Most of the models that exist today do not have this property.

Currently, the theory of three phases is gaining more and more followers, as evidenced by many publications, for example, [42, 43]. The article [44] presents a recently modified KKW (Kerner-Klenov-Wolf) model, which includes various types of vehicles. Variable sensitivity of the driver to speed fluctuations is introduced. Conclusions are drawn about the effect of changes in the speed of one or more vehicles on the overall flow rate at different intensity of the initial flow.

Domestic developments in the field of transport modeling correspond to the main global trends. The work on Buslaev networks carried out at MADI was mentioned above, stochastic models are also studied there, as well as the application of queuing theory to solve transport problems. MIPT, together with foreign colleagues, actively conducts research based on the theory of three Kerner phases, develops models of cellular automata [35, 37–39], hydrodynamic models [45, 46], simulation models, develops numerical methods for finding equilibria in large transport networks [47]. It should be noted the work of a team of authors from Lomonosov Moscow State University [48, 49], dedicated to the organization of traffic and the study of flow instability based on hydrodynamic models. Solving problems of optimal control of traffic flows at the RN [50], including using genetic algorithms, is actively engaged in the FRS "Informatics and Managemen" of the Russian Academy of Sciences. A two-dimensional quasi-gas dynamic model of transport flows and a multi-band CA model developed at the Keldysh IAM of the Russian Academy of Sciences will be presented below.

2. Software for modeling traffic flows. There are a huge number of software solutions for transport modeling. The packages reviewed in the collection [51] continue to evolve. The most well-known among commercial packages are:

– PTV Vision Traffic Suite [52];

– Aimsun (TSS-Transport Simulation Systems) [53].

There is also free and open source software, for example:

– MATSim [54, 55];

– Eclipse SUMO [56, 57].

PTV Vision Traffic Suite includes products:

– PTV Visum (strategic planning, calculation of transport demand, analysis of the transport network of cities, megacities, countries and regions based on macro modeling);

– PTV Vissim (traffic simulation, hypothesis testing on traffic management);

– PTV Viswalk (simulation of pedestrian flows, planning of mass events, development of evacuation plans);

– PTV Vistro (work at the network level, taking into account several types of intersections at once — regulated and unregulated, optimization of regulation modes).

Aimsun has now evolved from a myostimulator into a fully integrated traffic simulation application that combines ride demand forecasting, macroscopic functions and a mesoscopic-microscopic hybrid simulator.

PTV and Aimsun products are implemented for the Windows operating system.

MATSim is based on a multi-agent approach for large-scale transport modeling, consists of several modules that can be combined or used separately. Modules can be replaced with custom implementations.

SUMO is an academic development for modeling transport systems involving cars, public transport and pedestrians. The programs are based on a microscopic approach. SUMO includes many auxiliary tools that automate the main tasks and allow you to import a network, calculate a route, visualize, as well as calculate emissions of pollutants and calculate noise. SUMO can be supplemented with customizable models and provide interfaces for remote control of modeling. The distinctive features of SUMO are portability and extensibility. Versions of the package have been developed for a number of popular operating systems, in particular, for Linux.

It should be noted that there are also software packages for implementing the concept of BIM, 3D modeling and creating digital counterparts in the field of integrated design of roads and transport infrastructure, in particular, products of Bentley Systems [58], including OpenRoads and OpenCities Planner.

Thus, the world has already accumulated quite a lot of experience in modeling traffic flows, effective software tools have been developed that become an integral part of both short-term and long-term transport planning, and lay the foundation for intelligent transport systems.

3. Quasi-gas dynamic model of transport flows. As mentioned above, many macroscopic models describe the movement of vehicles by analogy with the gas dynamic flow. Consequently, the basis of the models is a system of equations of gas dynamics. The authors of this article some time ago developed a two-dimensional multiband macroscopic model for describing traffic flows, constructed by analogy with the QGD system of equations [59]. The QGD system was created to describe gas-dynamic flows in a wide range of Mach numbers, including well-proven in modeling substantially subsonic flows. Therefore, it was natural to use it when constructing a model of traffic flows in the approximation of a continuous medium. The equations of the QGD system, unlike traditional gas-dynamic equations, contain additional diffusion terms in the right part. In the case of transport flows, they can be considered as a natural viscosity, which allows smoothing solutions at large gradients and implementing numerical algorithms by end-to-end counting, without distinguishing features.

A distinctive feature of the multiband model is the presence in the system of an equation for the "transverse" component of the velocity, which makes sense of the speed of lane-to-lane rearrangement. Therefore, the model can be used to simulate traffic on the highway, taking into account its real geometry. Multibandness and the change in the number of bands is taken into account by specifying a specific computational domain, and not using sources in the right-hand sides of the equations. A detailed description of the QGD model of transport flows is contained in [1, 60, 61]. The system of equations of the proposed model looks like this:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho U)+\frac{\partial}{\partial y}(\rho V)= =\frac{\partial}{\partial x}\frac{\tau_x}{2}\left(\frac{\partial}{\partial x}\left(\rho U^2+P_x\right)-f_x+\frac{\partial}{\partial y}(\rho UV)\right)+ +\frac{\partial}{\partial y}\frac{\tau_y}{2}\left(\frac{\partial}{\partial y}\left(\rho V^2+P_y\right)-f_y+\frac{\partial}{\partial x}(\rho UV)\right), \tag{1}$$

$$\frac{\partial \rho U}{\partial t}+\frac{\partial}{\partial x}\left(\rho U^2+P_x\right)-f_x+\frac{\partial}{\partial y}(\rho UV)= =\frac{\partial}{\partial x}\frac{\tau_x}{2}\left(\frac{\partial}{\partial x}\left(\rho U^3+3P_xU\right)-3f_xU\right)+\frac{\partial}{\partial y}\frac{\tau_y}{2}\left(\frac{\partial}{\partial y}\left(\rho UV^2+P_yU\right)-f_yU\right)+ +\frac{\partial}{\partial x}\frac{\tau_x}{2}\left(\frac{\partial}{\partial y}\left(\rho U^2V+P_yV\right)-f_yV\right)+\frac{\partial}{\partial y}\frac{\tau_y}{2}\left(\frac{\partial}{\partial x}\left(\rho U^2V+P_xV\right)-f_xV\right), \tag{2}$$

$$\frac{\partial \rho V}{\partial t}+\frac{\partial}{\partial y}\left(\rho V^2+P_y\right)-f_y+\frac{\partial}{\partial x}(\rho UV)= =\frac{\partial}{\partial x}\frac{\tau_x}{2}\left(\frac{\partial}{\partial x}\left(\rho U^2V+P_xV\right)-f_xV\right)+\frac{\partial}{\partial y}\frac{\tau_y}{2}\left(\frac{\partial}{\partial y}\left(\rho V^3+3P_yV\right)-3f_yV\right)+ +\frac{\partial}{\partial x}\frac{\tau_x}{2}\left(\frac{\partial}{\partial y}\left(\rho V^2U+P_yU\right)-f_yU\right)+\frac{\partial}{\partial y}\frac{\tau_y}{2}\left(\frac{\partial}{\partial x}\left(\rho V^2U+P_xU\right)-f_xU\right). \tag{3}$$

The following designations are used here: ρ is the traffic flow density; U is the longitudinal, along the road, speed component; V is the transverse speed component (speed of rebuilding); $P=\lambda\rho^{\beta}/\beta$ is the pressure analog; $f=a\cdot\rho$ is the force of acceleration or deceleration, where $a=(U_{eq}-U)/T$ is the acceleration.

The equilibrium longitudinal velocity is calculated according to the parabolic fundamental diagram:

$$U_{eq}=U_{free}(1-\rho/\rho_{jam})/T. \tag{4}$$

$T=t_0(1+r\rho/(\rho_{jam}-r\rho))$ can be considered as a relaxation time. The equations are also supplemented by a number of phenomenological constants.

The above system contains an equation for the transverse velocity, similar to the equation of the longitudinal velocity. However, the test calculations have shown that it is more convenient to use an algebraic equation instead of the differential equation (3):

$$V_1=k_u\rho\frac{\partial U}{\partial y}-k_\rho U\frac{\partial \rho}{\partial y}+k_{des}\frac{U^2}{(x_{des}-x)^2}(y_{des}-y),$$

where the first term corresponds to the driver's desire to drive at a higher speed, the second — the desire to drive in a lane with a lower density and the third — to achieve a certain goal. Here k_u, k_ρ, k_{des} are the constants; (x_{des}, y_{des}) are the coordinates of the driver's target. The use of equation (4) simplifies the solution process and increases the stability of the difference scheme.

It should be noted that in some cases of an inhomogeneous, but not very complex route, qualitatively correct results can be obtained using a one-dimensional CGD model [60, 62]:

$$\frac{\partial \rho}{\partial t}+\frac{\partial Q}{\partial x}=\frac{\partial}{\partial x}\frac{\tau}{2}\frac{\partial\left(\frac{Q^2}{\rho}+P\right)}{\partial x}+F_\rho, \tag{5}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left[\frac{Q^2}{\rho}+P\right]=f+\frac{\partial}{\partial x}\frac{\tau}{2}\frac{\partial\left(\frac{Q^3}{\rho^2}+P\frac{Q}{\rho}\right)}{\partial x}+F_U. \tag{6}$$

In these equations, written in conservative form, the traffic flow is: $Q = \rho \cdot U$. The source terms are on the right side F_ρ and F_U are equal to zero on a homogeneous road and are not equal to zero if there are entrances or exits from the main road or there is a change in the number of lanes.

The proposed models are numerically implemented using finite-difference schemes. The system is approximated by explicit second-order difference schemes in space. Note that the structure of an explicit computational algorithm fits well on the architecture of multiprocessor computing systems with distributed memory and, if necessary, a large amount of calculations can be parallelized with sufficiently high efficiency [62, 63].

4. Multiband model based on the theory of cellular automata. The second model proposed by the authors earlier and which is promising for implementation in an interactive program is a multiband model using the ideology of cellular automata. A detailed description of this model is given in [2, 60, 64]. Here we will describe it briefly.

The calculated area is a two-dimensional lattice. The number of cells in the transverse direction corresponds to the number of lanes on the considered section of the highway, and the width of the cell is equal to the width of the real road lane (Fig. 1).

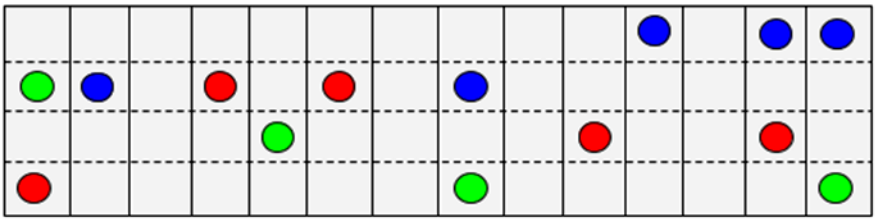

Fig. 1. Calculation area in the CA model

The number of cells along the road depends on the specific task, taking into account that the longitudinal size of the cell is equal to the average length of the car plus the width of the gap between the cars at maximum flow density, that is, in a "traffic jam". In the literature, a length of 7.5 m is given as the standard cell size for passenger cars. The time in such models is discrete, the system is updated at each time step. With standard calculations, this step is equal to 1 s, although in more developed and realistic models this value may vary. At any given time, the grid cells can be in one of two states: the cell is either occupied (which corresponds to the presence of a car in it), or empty. Figure 1 shows the state of the computational domain at some point in time. The different color of the movement elements corresponds to different selected goals. At the next moment in time, the state of the cells is updated in two stages according to certain rules.

At the first stage, each driver checks whether he wants to change lanes and has the opportunity to do so. It is rebuilt if:

– it is necessary to achieve his goal (for example, to drive up to the exit from the road) or it is necessary to go around an obstacle;

– he gets an advantage after rebuilding — goes at a higher speed or with a lower density;

– there is a possibility for rebuilding — rebuilding is allowed and the neighboring cell is empty;

– the security conditions have been met.

After the chosen decision regarding the realignment and the action performed in accordance with it, forward movement takes place along the selected lane according to the Nagel-Schreckenberg single-lane traffic rules [28] given in section 1 of this article.

It should be noted that the initial, simplest version of the rebuilding strategy is described here. In more complex modifications of the model [65], the rules for rebuilding depend on the type of road element (X-shaped intersection, T-shaped intersection, U-turn, narrowing/widening section, etc.), road signs and markings. Various driving strategies and behavioral aspects are also taken into account. The concepts of "aggressive", "cautious", "polite" driver are introduced into the model. The percentages of a particular type of driver may change during the calculation process. A "slow start" algorithm has also been developed.

To implement the model, a CAM-2D software package has been developed [66], which has an integrated web interface and a visualization module in addition to computing modules. The parallel version is designed for calculations of road networks on the CPU of multiprocessor systems using MPI technology [62, 63].

Conclusion. The article presents an overview of works in the field of traffic flow modeling, covering a wide range of approaches — macro- and microscopic models, as well as models of cellular automata. Special attention is paid to the original developments of the authors of the article both in the field of macroscopic and microscopic (namely, cellular automata) modeling. Both developments have their advantages, such as, for example, the ability to simulate the movement of motor transport taking into account the actual geometry of the road, even in the case of macro modeling. The models have been repeatedly tested in calculations and, in addition, allow for effective implementation on supercomputers, since they have internal parallelism. The latter property is a special advantage in the conditions of traffic simulation on transport networks of multimillion megacities.

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