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Existence and Uniqueness of the Initial-Boundary Value Problem Solution of Multicomponent Sediments Transport in Coastal Marine Systems

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Abstract

Introduction. This work is devoted to the study of a non-stationary two-dimensional model of sediment transport in coastal marine systems. The model takes into account the complex multi-fractional composition of sediments, the gravity effect and tangential stress caused by the impact of waves, turbulent exchange, dynamically changing bottom topography, and other factors. The aim of the work was to carry out an analytical study of the conditions for the initial-boundary value problem existence and uniqueness corresponding to the specified model.

Materials and Methods. Linearization of the initial-boundary value problem is performed on a temporary uniform grid. The nonlinear coefficients of a quasilinear parabolic equation are taken with a "delay" by one grid step. Thus, a chain of correlated by initial conditions is the final solutions of problems is built. The study of the existence and uniqueness of the problems included in this chain, and therefore the original problem as a whole, is carried out involving the methods of mathematical and functional analysis, as well as methods for solving differential equations.

Results. Earlier, the authors investigated the existence and uniqueness of the initial-boundary value problem of the transport of sediments of a single-component composition. In the present work, the result obtained is extended to the case of multi-fractional sediments.

Discussion and Conclusions. Based on the analysis of the existing results of mathematical modeling of hydrodynamic processes, a non-linear spatial two-dimensional model of sediment transport was previously investigated by the team of authors in the case of bottom sediments consisting of particles having the same characteristic dimensions and density (single-component composition). In this paper, the previous results of the study are extended to the case of sediments of a multicomponent composition, namely, the conditions for the existence and uniqueness of the solution of the initial-boundary value problem corresponding to the considered model are determined.

Keywords: multicomponent sediments' transport, coastal marine system, initial-boundary value problem, solution existence, solution uniqueness.

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Научная статья

Существование и единственность решения начально-краевой задачи транспорта многокомпонентных наносов прибрежных морских систем

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Аннотация

Введение. Настоящая работа посвящена исследованию нестационарной двумерной модели транспорта наносов в прибрежных морских системах. Модель учитывает сложный многокомпонентный состав наносов; действие силы тяжести и тангенциального напряжения, вызванного воздействием волн; турбулентный обмен; динамически изменяемый рельеф дна и другие факторы. Целью работы являлось проведение аналитического исследования условий существования и единственности начально-краевой задачи, соответствующей указанной модели.

Материалы и методы. В работе на временной равномерной сетке выполнена линеаризация начально-краевой задачи, при которой нелинейные коэффициенты квазилинейного параболического уравнения берутся с «запаздыванием» на один шаг сетки. Тем самым строится цепочка задач, связанных по начальным условиям и финальным решениям. Привлекая методы математического и функционального анализа, а также методы решения дифференциальных уравнений, проводится исследование существования и единственности задач, входящих в данную цепочку, а потому и в целом исходной задачи.

Результаты исследования. На основе анализа существующих результатов математического моделирования гидродинамических процессов ранее была исследована нелинейная пространственно-двумерная модель транспорта наносов в случае донных отложений, состоящих из частиц, имеющих одинаковые характерные размеры и плотность (однокомпонентный состав). В настоящей работе предыдущие результаты исследования распространены на случай наносов многокомпонентного состава, а именно определены условия существования и единственности решения начально-краевой задачи, соответствующей рассматриваемой модели.

Обсуждение и заключения. Модель транспорта многокомпонентных наносов может быть полезна для прогноза распространения загрязняющих веществ, а также при исследовании динамики изменения рельефа дна как при антропогенном воздействии, так и в силу естественно протекающих природных процессов в морских системах.

Ключевые слова: транспорт многокомпонентных наносов, прибрежная морская система, начально-краевая задача, существование решения, единственность решения.

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Introduction. It is necessary to use a set of models of different spatial and temporal scales in solving practical tasks related to the environmental assessment of the water body's state [1–6]. The research of mathematical hydrophysical models, which are characterized by a variety of parameters, has been actively developed in recent decades [7–14]. The paper considers 2D mathematical model for calculating the transport of multicomponent sediments in relation to coastal marine systems. The set of convection-diffusion equations for each sediment component (or fraction) forms this mathematical model taking into account turbulent exchange, gravity, tangential stress, dynamically changing bottom relief and other factors [15–17].

The article presents the results of theoretical study of the initial boundary value problem existence and uniqueness based on the constructed model. In accordance with this goal, the initial boundary value problem for quasi-linear equation of parabolic type is considered, for which sufficient conditions for the existence and uniqueness of the solution are determined by methods of mathematical and functional analysis, as well as by methods of solving differential equations.

Materials and methods

1. Initial boundary value problem of multicomponent sediment transport. Let's write down the equation of multicomponent sediments transport [16, 17]:

$$(1-\varepsilon_r)\frac{\partial H}{\partial t}+\operatorname{div}\left(V_r k_r \vec{\tau}_b\right)=\operatorname{div}\left(V_r k_r \frac{\tau_{bc,r}}{\sin\varphi_0}\operatorname{grad}H\right)+\frac{w_{g,r}}{\rho_r}c_r,\quad r=\overline{1,R}. \tag{1}$$

Where $H=H(x,y,t)$ is the reservoir depth; ε_r is the porosity r-th component in the sediments composition; V_r is the r-th component's volume fraction; $\vec{\tau}_b$ is the tangential tangential stress vector at the reservoir bottom; $\tau_{bc,r}$ is the critical value of the tangential stress for r-th sediment component, $\tau_{bc,r}=a_r\sin\varphi_0$, a_r is the coefficient for r-th sediment component, φ_0 is the angle of the natural slope of the soil in the reservoir; $w_{g,r}$ is the hydraulic size or deposition rate of r-th component; ρ_r is the density of r-th bottom material component; $k_r=k_r(H,x,y,t)$ is nonlinear coefficient determined by the ratio:

$$k_r=\frac{A\tilde{\omega}d_r}{\left((\rho_r-\rho_0)gd_r\right)^{\beta}}\left|\vec{\tau}_b-\frac{\tau_{bc,r}}{\sin\varphi_0}\operatorname{grad}H\right|^{\beta-1},$$

where $\tilde{\omega}$ is the average wave frequency; d_r is the characteristic size of r-th component; g is the acceleration of gravity; ρ_0 is the aquatic environment density; A and β are the dimensionless constants.

Let the sediment transport process take place in an area D, $D(x,y)=\{0<x<L_x, 0<y<L_y\}$ with boundary S, representing a piecewise smooth line. We assume that a three-dimensional cylinder $Ц_T=D\times(0,T)$ of height T with a base D is the domain of equation (1). The boundary of this cylinder consists of a side surface $S\times[0,T]$ and two bases — $\overline{D}\times\{0\}$ and $\overline{D}\times\{T\}$.

Equation (1) is considered with the initial condition:

$$H(x,y,0)=H_0(x,y), \tag{2.1}$$

$$H_0(x,y)\in C^2(D)\cap C(\overline{D}), \tag{2.2}$$

$$\operatorname{grad}_{(x,y)}H_0\in C(\overline{D}), \tag{2.3}$$

$$(x,y)\in\overline{D} \tag{2.4}$$

and conditions on the region border $\overline{D}$:

$$\left|\vec{\tau}_b\right|\Big|_{y=0}=0, \tag{3}$$

$$H(L_x,y,t)=H_2(y,t),\ 0\le y\le L_y, \tag{4}$$

$$H(0,y,t)=H_1(y,t),\ 0\le y\le L_y, \tag{5}$$

$$H(x,0,t)=H_3(x),\ 0\le x\le L_x. \tag{6}$$

$$H(x,L_y,t)=0,\ 0\le x\le L_x. \tag{7}$$

Let assume:

$$\operatorname{grad}_{(x,y)}H\in C(\overline{Ц}_T)\cap C^1(Ц_T),$$

$$\tau_{bx}=\tau_{bx}(x,y,t),$$

$$k_r\ge k_{0r}=\text{const}>0,\ \forall(x,y)\in\overline{D},\ 0<t\le T,$$

2. The initial boundary value problem linearization of multicomponent sediment transport. Let's build a time grid ω_τ, with step τ: $\omega_\tau=\{t_n=n\tau,\ n=0,1,...,N,\ N\tau=T\}$.

If n=1, then the reservoir depth $H^{(1)}(x,y,t_0)$ is known and is determined from the initial condition, i. e. $H^{(1)}(x,y,t_0)=H_0(x,y)$. If $n=2,...,N$, then the reservoir depth $H^{(n)}(x,y,t_{n-1})$ will also be known, since problem (1)–(7) is solved for the time interval, $t_{n-2}<t\le t_{n-1}$, i. e. $H^{(n)}(x,y,t_{n-1})=H^{(n-1)}(x,y,t_{n-1})$.

Denote:

$$k_r^{(n-1)}\equiv\frac{A\tilde{\omega}d_r}{\left((\rho_r-\rho_0)gd_r\right)^{\beta}}\left|\vec{\tau}_b-\frac{\tau_{bc,r}}{\sin\varphi_0}\operatorname{grad}H^{(n-1)}(x,y,t_{n-1})\right|^{\beta-1},\quad n=1,2,...,N. \tag{8}$$

After linearization, equation (1) and the initial condition will take the form:

$$(1-\varepsilon_r)\frac{\partial H^{(n)}}{\partial t}=\operatorname{div}\left(V_r k_r^{(n-1)}\frac{\tau_{bc,r}}{\sin\varphi_0}\operatorname{grad}H^{(n)}\right)-\operatorname{div}\left(V_r k_r^{(n-1)}\vec{\tau}_b\right)+\frac{w_{g,r}}{\rho_r}c_r,\quad r=\overline{1,R}, \tag{9}$$

$$t_{n-1} < t \le t_n,\ n = 1, 2, ..., N,$$

$$H^{(1)}(x, y, t_0) = H_0(x, y),\quad H^{(n)}(x, y, t_{n-1}) = H^{(n-1)}(x, y, t_{n-1}),\quad (x, y) \in \overline{D},\quad n = 2, ..., N. \tag{10}$$

Boundary conditions (3)–(7) are assumed to be fulfilled for all time intervals $t_{n-1} < t \le t_n$, $n = 1, 2, ..., N$.

Research results

1. Investigation of the linearized initial boundary value problem of multicomponent sediments transport solution existence. Let 's put $n = i,\ i = 1, 2, ..., N$ in the equation (9).

We have:

$$(1 - \varepsilon_r)\frac{\partial H^{(i)}}{\partial t} = \operatorname{div}\left(V_r k_r^{(i-1)} \frac{\tau_{bc,r}}{\sin\varphi_0} \operatorname{grad} H^{(i)}\right) - \operatorname{div}\left(V_r k_r^{(i-1)} \vec{\tau}_b\right) + \frac{w_{g,r}}{\rho_r} c_r,\quad r = \overline{1, R}. \tag{11}$$

Equation (11) is supplemented by conditions (10) and (3)–(7).

If $i = 1$, then based on the assumptions made earlier, we can write:

$$V_r k_r^{(0)} \frac{\tau_{bc}}{\sin\varphi_0} \in C^1(Ц_\infty),\quad V_r k_r^{(0)} \vec{\tau}_b \in C^1(Ц_\infty). \tag{12}$$

From [18] it can be concluded that if condition (12) is met, the solution of the initial boundary value problem (11), (10), (2) − (7), $t_0 < t \le t_1$, $i = 1$, exists and belongs to the class:

$$H^{(1)}(x, y, t) \in C^2(Ц_{t_1}) \cap C(\overline{Ц}_{t_1}),\ \operatorname{grad}_{(x,y)} H^{(1)} \in C(\overline{Ц}_{t_1}).$$

If $i = 2$, then the initial-boundary value problem will have an initial condition $H^{(2)}(x, y, t_1) \equiv H^{(1)}(x, y, t_1)$. Its smoothness coincides with the smoothness of the initial condition for equation (11) of the number $i = 1$:

$$H^{(2)}(x, y, t) \in C^2(Ц_{t_2}) \cap C(\overline{Ц}_{t_2}),\ \operatorname{grad}_{(x,y)} H^{(2)} \in C(\overline{Ц}_{t_2}).$$

It is obvious that the conditions from [18] and the solution of the problem are again applicable (11), (10), (2) − (7) for the number $i = 2$ exists.

Further, if i, $i = 3, ..., N$, then for each case we will have a mixed problem for a linear equation of parabolic type. The initial and boundary conditions have a smoothness sufficient for the functions existence $H^{(i)}(x, y, t)$, $t_{i-1} < t \le t_i$, $i = 1, 2, ..., N$ класса $C^2(Ц_{t_i}) \cap C(\overline{Ц}_{t_i})$, $\operatorname{grad}_{(x,y)} H^{(i)} \in C(\overline{Ц}_{t_i})$, which are solutions to initial boundary value problems (11), (10), (2)–(7) [19].

2. The linearized initial boundary value problem solution uniqueness investigation of multicomponent sediments transport. Let's write equation (11) for $n = 1$:

$$(1 - \varepsilon_r)\frac{\partial H^{(1)}}{\partial t} = \operatorname{div}\left(V_r k_r^{(0)} \frac{\tau_{bc,r}}{\sin\varphi_0} \operatorname{grad} H^{(1)}\right) - \operatorname{div}\left(V_r k_r^{(0)} \vec{\tau}_b\right) + \frac{w_{g,r}}{\rho_r} c_r,\quad r = \overline{1, R}. \tag{13}$$

Let us assume the existence of two different solutions to it:

$$H' = H'(x, y, t),\quad H'' = H''(x, y, t),\quad (x, y) \in \overline{D},\quad t_0 < t \le t_1.$$

Denote:

$$w^{(1)}(x, y, t) \equiv H'(x, y, t) - H''(x, y, t),\ t_0 < t \le t_1,\quad w^{(1)}(x, y, t) \not\equiv 0,\quad w^{(1)}(x, y, t_0) \equiv 0.$$

The initial boundary value problem for the function $w(x, y, t) \equiv w^{(1)}(x, y, t)$ will have the form:

$$(1 - \varepsilon_r)\frac{\partial w}{\partial t} = \operatorname{div}\left(V_r k_r^{(0)} \frac{\tau_{bc,r}}{\sin\varphi_0} \operatorname{grad} w\right),\quad r = \overline{1, R}, \tag{14}$$

$$w(x, y, 0) = 0,\quad (x, y) \in \overline{D}, \tag{15}$$

$$\left|\vec{\tau}_b\right| \Big|_{y=0} = 0, \tag{16}$$

$$w(x, L_y, t) = 0,\quad 0 \le x \le L_x, \tag{17}$$

$$w(0, y, t) = 0,\quad 0 \le y \le L_y, \tag{18}$$

$$w(L_x, y, t) = 0,\quad 0 \le y \le L_y, \tag{19}$$

$$w(x, 0, t) = 0,\quad 0 \le x \le L_x. \tag{20}$$

We multiply both parts of equation (14) by the function $w(x,y,t)\not\equiv 0,\ t_0 < t \le t_1,\ (x,y)\in \overline{D},$ and then perform integration over variables $t,\ t_0 < t \le t_1$ and (x,y) in the domain *D*. We will get:

$$\int_{t_0}^{t_1}\left((1-\varepsilon_r)\iint_D w\frac{\partial w}{\partial t}\,dxdy\right)dt = \int_{t_0}^{t_1}\left(\iint_D w\,div\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\,\mathrm{grad}\,w\right)dxdy\right)dt,\ r=\overline{1,R}. \tag{21}$$

After a series of transformations of equality (21), we obtain:

$$\frac{1}{2}(1-\varepsilon_r)\left[\iint_D w^2(x,y,t_1)dxdy - \iint_D w^2(x,y,t_0)dxdy\right] = \int_{t_0}^{t_1}\left(\iint_D w\,div\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\,\mathrm{grad}\,w\right)dxdy\right)dt. \tag{22}$$

Equality (22) under condition (15) is written as:

$$\frac{1}{2}(1-\varepsilon_r)\iint_D w(x,y,t_1)^2 dxdy = \int_{t_0}^{t_1}\left(\iint_D w\,div\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\,\mathrm{grad}\,w\right)dxdy\right)dt. \tag{23}$$

Let:

$$R(w) \equiv \int_{t_0}^{t_1}\left(\iint_D w\,div\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\,\mathrm{grad}\,w\right)dxdy\right)dt. \tag{24}$$

There is equality:

$$\begin{aligned}
&\iint_D\left[w\left(\frac{\partial}{\partial x}\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\right)\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial y}\left(\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\right)\frac{\partial w}{\partial y}\right)\right]dxdy = \\
&= \iint_D\left[\frac{\partial}{\partial x}\left(w\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\right)\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial y}\left(w\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\right)\frac{\partial w}{\partial y}\right)\right]dxdy - \\
&- \iint_D\left[\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\right)\left(\frac{\partial w}{\partial x}\right)^2+\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\right)\left(\frac{\partial w}{\partial y}\right)^2\right]dxdy.
\end{aligned} \tag{25}$$

On the other hand, taking into account the boundary conditions (16)−(20) and the Ostrogradsky-Gauss theorem [19], we have:

$$\iint_D\left[\frac{\partial}{\partial x}\left(w\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\right)\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial y}\left(w\left(V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\right)\frac{\partial w}{\partial y}\right)\right]dxdy = 0. \tag{26}$$

From the equalities (25) and (26) we find:

$$R(w) \equiv -\int_{t_0}^{t_1}\left(\iint_D V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]dxdy\right)dt. \tag{27}$$

Taking into account (27), equality (22) will be written as:

$$\frac{1}{2}(1-\varepsilon_r)\iint_D w^2(x,y,t_1)dxdy = -\int_{t_0}^{t_1}\left(\iint_D V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]dxdy\right)dt. \tag{28}$$

Next, we transform the right side of equality (28). By involving the Poincare inequality [20], we obtain:

$$-\int_{t_0}^{t_1}\left[\iint_D V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\left(\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right)dxdy\right]dt \le -V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\int_{t_0}^{t_1}\left[\iint_D\left(\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right)dxdy\right]dt. \tag{29}$$

From the inequality (29) follows the assessment:

$$-\int_{t_0}^{t_1}\left[\iint_D V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\left(\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right)dxdy\right]dt \le -\pi^2 V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}\right)\int_{t_0}^{t_1}\left[\iint_D w^2 dxdy\right]dt. \tag{30}$$

From the equalities (33) and (35) the inequality is obtained:

$$\frac{1}{2}(1-\varepsilon_r)\iint_D w^2(x,y,t_1)dxdy \le -\pi^2 V_r k_r^{(0)}\frac{\tau_{bc,r}}{\sin\varphi_0}\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}\right)\int_{t_0}^{t_1}\left[\iint_D w^2 dxdy\right]dt. \tag{31}$$

Since $w(x,y,t) \not\equiv 0$, then $w^2(x,y,t^*) > 0$ is done. Due to the function continuity $w^2(x,y,t)$ in some neighborhood of the point t^* at $t_0 < t^* \le t_1$, we have $\iint_D w^2(x,y,t)dxdy > 0$, and therefore:

$$-\pi^2 V_r k_r^{(0)} \frac{\tau_{bc,r}}{\sin\varphi_0}\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}\right)\int_{t_0}^{t_1}\left[\iint_D w^2 dxdy\right]dt < 0. \quad (32)$$

From the resulting inequalities (31) and (32), a contradictory inequality will follow:

$$\frac{1}{2}(1-\varepsilon_r)\iint_D w^2(x,y,t_1)dxdy < 0. \quad (33)$$

Therefore, the identity $w(x,y,t_1) \equiv 0$. is valid. Due to the arbitrariness of the time step τ, $\tau > 0$, we have:

$$w(x,y,t) = 0, \quad t_0 < t \le t_1.$$

Obviously, when $w(x,y,t) \equiv 0$ in case of $(x,y) \in \overline{D}$, $t_{n-1} \le t \le t_n$, $n = 2,...,N$.

Thus, the first step of induction at $n = 1$. Similarly , arguments are constructed for $n = s$, $s = 2,..,N$, which leads to equality:

$$w(x,y,t_s) \equiv 0.$$

The result of the reasoning is the following theorem.

Theorem. Let the equations (11) be given:

$$(1-\varepsilon_r)\frac{\partial H^{(n)}}{\partial t} = \operatorname{div}\left(V_r k_r^{(n-1)}\frac{\tau_{bc,r}}{\sin\varphi_0}\operatorname{grad} H^{(n)}\right) - \operatorname{div}\left(V_r k_r^{(n-1)}\vec{\tau}_b\right) + \frac{w_{g,r}}{\rho_r}c_r, \quad r = \overline{1,R},$$

$$t_{n-1} < t \le t_n, \; n = 1,2,...,N,$$

in a rectangular area:

$$D(x,y) = \{0 < x < L_x, 0 < y < L_y\},$$

where $k_r^{(n-1)} \equiv \dfrac{A\tilde{\omega}d_r}{((\rho_r-\rho_0)gd_r)^\beta}\left|\vec{\tau}_b - \dfrac{\tau_{bc,r}}{\sin\varphi_0}\operatorname{grad}H^{(n-1)}(x,y,t_{n-1})\right|^{\beta-1}$ with initial and boundary conditions (11), (3)–(7).

Then, if the conditions are met $k_r^{(n-1)} \ge k_{0r} > 0$, $k_r^{(n-1)} \in C^1(\overline{D})$, then $\forall n$, $n = 1,2,...,N$ the function $H^{(n)}(x,y,t)$, $t_{n-1} < t \le t_n$, $n = 1,2,...,N$ of the class $\operatorname{grad}_{(x,y)} H^{(n)} \in C(\overline{Ц_T})$ will be the solution of the equation of the number n in the cylinder $Ц_T = D \times (0,T)$ and this solution is the only one.

Discussion and conclusions. The novelty of this work is determined by the formulation of a non-stationary spatial-two-dimensional mathematical problem of sediment transport, taking into account their complex multicomponent composition. The linearization of the corresponding initial-boundary value problem is performed on a grid in time and for an arbitrary time step $t_{n-1} < t \le t_n$, $n = 1,2,...,N$, the conditions for the initial-boundary value problem solution existence and uniqueness are obtained.

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