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Original article

Application of a Modification of the Grid-Characteristic Method using Overset Grids for Explicit Interface Description to Modelling the Relief of the Ocean Shelf

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Abstract

Introduction. The problem of modelling the propagation of elastic waves is of great practical importance when conducting seismic exploration. Based on it, a model of the environment under study is being built. At the same time, the quality of the constructed model is determined by the accuracy of solving the modelling problem, which ensures constantly increasing requirements for modelling accuracy. For accurate modelling, it is important to correctly describe and take into account the boundaries of the media. At the same time, the quality of the constructed model is determined by the accuracy of solving the modelling problem, which ensures constantly increasing requirements for modelling accuracy.

Materials and Methods. We have studied a modification of the grid-characteristic method on rectangular grids using overset grids to describe the interface of media of complex shape. This approach has previously been used to describe the earth's surface when conducting simulations on land. This paper describes its application in modelling the relief of the ocean shelf.

Results. The use of the overset grid reduces the modelling error, the number of parasitic waves and artifacts and makes it possible to get a more visual picture.

Discussion and Conclusions. Overset grids can be used to describe the interface of media in modelling seismic exploration of the ocean shelf. Their use makes it possible to increase the accuracy of modelling and reduce the number of artifacts compared to using only one grid.

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Keywords: grid-characteristic method, overset grid, chimera grid, shelf seismic exploration

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Научная статья

Применение модификации сеточно-характеристического метода с использованием наложенных сеток для явного выделения границы раздела сред при моделировании рельефа океанического шельфа

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Аннотация

Введение. Задача моделирования распространения упругих волн имеет большое практическое значение при проведении сейсморазведки, поскольку на ее основе выполняется построение модели исследуемой среды. При этом качество построенной модели определяется точностью решения задачи моделирования, что обеспечивает постоянно возрастающие требования к точности моделирования. Для точного моделирования важно корректно описывать и учитывать границы раздела сред. При этом важным фактором остается ресурсоемкость используемого метода моделирования, поскольку использование менее ресурсоемких методов позволяет выполнить больше итераций расчета для инверсии или использовать сетки с меньшим шагом для повышения точности.

Материалы и методы. В данной работе рассматривается модификация сеточно-характеристического метода на прямоугольных сетках, использующая наложенные сетки для описания границы раздела сред сложной формы. Данный подход ранее использовался для описания поверхности земли при проведении моделирования на суше. В данной работе описывается его применение при моделировании рельефа океанического шельфа.

Результаты исследования. Использование наложенной сетки позволяет уменьшить погрешность моделирования, количество паразитных волн и артефактов и получить более наглядную картину.

Обсуждение и заключения. Наложенные сетки могут быть применены для описания границы раздела сред при моделировании сейсморазведки океанического шельфа. Их использование позволяет повысить точность моделирования и снизить количество артефактов по сравнению с использованием только одной сетки.

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Ключевые слова: сеточно-характеристический метод, метод наложенных сеток, метод сеток-химер, шельфовая сейсморазведка

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Introduction. The elastic and acoustic wavespropagation in the medium is studied by many scientific and engineering disciplines. Among the most important practical tasks considered by these disciplines are seismic stability analysis, non-destructive defect detection and seismic exploration. Numerical modeling is widely used in solving all these problems. Two main types of modeling tasks can be distinguished — in fact, the task of moderating the propagation of wave disturbances in a medium with known properties (a direct task) and the task of constructing a model of the medium based on known characteristics of the source signal and receiver readings (an inverse problem). These tasks are not independent, the solution of the inverse problem usually relies on several iterations of solving the direct problem and making refinements to the model.

It is necessary to take into account its features and limitations, as well as the amount of computing resources it requires, when choosing a numerical method used for modeling. The finite element method, the finite difference method and the grid-characteristic method are widely used to simulate the propagation of mechanical waves.

The finite element method [1] uses non-structural meshes, most often tetrahedral, which makes it possible to describe with good accuracy the boundaries of the modeling area, as well as the inhomogeneities and joints between layers lying inside it. This method is based on the approximation of the desired function inside each of the cells using a given basis. It also makes it possible to describe absorbing boundary conditions well using the PML method [2]. Its main disadvantage is its high resource intensity.

The main idea of the finite difference method is to replace differentiation operations in the simulated equation with non-differential expressions determined by the difference scheme used. Finite-difference schemes usually use structural rectangular grids. Their advantage is low resource consumption with good accuracy [3].

The grid-characteristic method [4; 5] is in many ways similar to the finite difference method. Instead of directly replacing differentiation with a difference expression, he uses variable substitution, which allows him to move from the original equation to the transfer equation. This transfer equation is solved by searching for the characteristics along which the values are transferred. In this paper, a modification of the grid-characteristic method is used, in which, instead of using characteristics, the transfer equation is solved using difference schemes.

In cases when methods based on structural grids are used for modeling, and in the field of modeling there are boundaries of complex shape, there is a need to somehow adapt the method to describe them. In this paper, overset curved grids are used to describe curved boundaries. The choice of this method is due to its good accuracy and low resource consumption. Earlier in [6] it was shown how this method can be used to describe a free surface of a complex shape, and in this paper it is used to describe the interface of media.

Materials and methods

1. Physical model. The processes of wave propagation in elastic and acoustic media are studied. Let's first consider the elastic medium model. At a point with a radius vector $\vec{x}$ we denote the displacement vector at time t as $u(\vec{x},t)$. Newton's second law on the i axis will have the form [7]:

$$\rho\frac{\partial^2 u_i}{\partial t^2}-\sum_j \frac{\partial \sigma_{ij}}{\partial x_i}-f_i=0.$$

We assume that all offsets are small. Then we can write down Hooke's law:

$$\sigma_{ij}=\sum_{k=1,l=1}^{3,3} C_{ijkl}\in_{kl},$$

where C_{ijkl} is called the stiffness tensor, and ε_{ij} is the Cauchy–Green strain tensor:

$$\in_{i,j}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}+\sum\frac{\partial u_i}{\partial x_l}\frac{\partial u_j}{\partial x_l}\right)\approx\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right).$$

The tensors σ and ε are symmetric and, therefore, contain no more than 6 independent components each. The C_{ijkl} tensor is also symmetric, so the number of its independent components does not exceed 21, and it can be written in Voigt notation [8] as a 6×6 symmetric matrix. In this paper, only isotropic media are considered. In such environments, this matrix is further simplified and has the form [9]:

$$C_{\alpha\beta}=\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0\\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0\\ 0 & 0 & 0 & \mu & 0 & 0\\ 0 & 0 & 0 & 0 & \mu & 0\\ 0 & 0 & 0 & 0 & 0 & \mu\end{bmatrix}.$$

You can enter the displacement velocity vector at point v as a derivative of the displacement vector in time. The system of equations will take the form:

$$\rho\frac{\partial\vec{v}}{\partial t}=(\nabla\cdot\sigma)^T+\vec{f},$$

$$\frac{\partial\sigma}{\partial t}=\lambda(\nabla\cdot\vec{v})\mathbf{I}+\mu\left(\nabla\otimes\vec{v}+\nabla\otimes\vec{v}\right)^T\Big).$$

The parameters λ and μ are called Lame parameters.Together with the density, they set the properties of the medium at the point. The velocities of longitudinal and transverse waves can be expressed through them as follows:

$$c_p=\sqrt{\frac{\lambda+2\mu}{\rho}},$$

$$c_s=\sqrt{\frac{\mu}{\rho}}.$$

The continuity equation and the Euler equation are used to describe acoustic media. Denote the pressure, density and velocity at a point with a radius vector $\vec{x}$ в at time t as p_A (x,t), ρ_A (x,t) and vA (x,t) respectively. The equations have the following form [10]:

$$\frac{\partial \rho_A}{\partial t} + \nabla \cdot \left(\rho_A \vec{v}_A\right) = 0,$$

$$\frac{\partial \vec{v}_A}{\partial t} + \left(\vec{v}_A \nabla\right)\vec{v}_A = -\frac{1}{\rho_A}\nabla p.$$

In this paper, we consider problems in which the media are at rest before the propagation of wave disturbances. In addition, changes in pressure and density caused by wave propagation are assumed to be small compared to their values at rest. Denote the pressure and density at rest at the selected point as p_0 (x,t) and ρ_0 (x,t), and their changes caused by propagating waves, as $p(x,t)$ and $\rho(x,t)$. In the assumptions above, the equations of the acoustic medium can be reduced to this form:

$$\frac{\partial v}{\partial t} + \frac{1}{\rho_0}\nabla p = 0,$$

$$\frac{\partial p}{\partial t} + \rho_0 c^2 \nabla \cdot \vec{v} = 0.$$

In the approximation used, the propagation of acoustic waves can be considered as a special case of the propagation of elastic waves in a medium where only *P*-waves propagate. The parameters of such an elastic medium will be set by the following relations:

$$c_p = c\sqrt{\frac{\lambda}{\rho}},$$

$$c_s = 0,$$

$$\mu = 0.$$

2. Grid-characteristic method. Consider the application of the grid-characteristic method for solving equations describing an elastic medium. Let's collect all the unknowns in these equations into one vector [11]:

$$q = [v_1, v_2, v_3, \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}].$$

Let's put the derivatives for each of the coordinates together. We obtain a matrix equation of the following form:

$$\frac{\partial}{\partial t}\mathbf{q} - \mathbf{A_1}\frac{\partial}{\partial x_1}\mathbf{q} - \mathbf{A_2}\frac{\partial}{\partial x_2}\mathbf{q} - \mathbf{A_3}\frac{\partial}{\partial x_3}\mathbf{q} = 0.$$

We will perform splitting by spatial coordinates [12], that is, we will divide one system into three different ones, one for each axis. Since initially the system of equations is hyperbolic, it is possible to perform diagonalization of matrices A_i:

$$\mathbf{A}_i = \mathbf{\Omega}_i^{-1}\mathbf{\Lambda}_i\mathbf{\Omega}_i,$$

where $\mathbf{\Lambda}_i$ is a diagonal matrix consisting of eigenvalues $\mathbf{A}_i$, and $\mathbf{\Omega}_i$ consists of columns equal to eigenvectors $\mathbf{A}_i$.

Next, you can replace the variable $\omega_i=\Omega_i\, q$, after which the systems along the axes will take the form:

$$\frac{\partial}{\partial t}\omega_i + \mathbf{\Lambda}_i\omega_i = 0.$$

Since the matrices $\mathbf{\Lambda}_i$ are diagonal, we are actually dealing with a set of separate transfer equations for each of the components ω_i. These individual transfer equations can be solved using the method of characteristics [4], but in this paper finite-difference schemes are used to solve them.

Thus, the time step of modeling occurs as follows: first, a transition to new variables and transfer equations is performed, then a time step is performed in these variables, after which a reverse replacement is performed and new values of the original variables in the grid nodes are calculated.

3. The method of overset grids. The method of overset grids, first described in [13], allows us to combine the advantages of using curved grids, which make it possible to describe well the boundaries of complex shapes with low resource consumption of structural grids. When using this method, the main grid is first constructed — a large structural grid covering the entire modeling area. After that, the areas near the borders that need to be described are covered with separate curved grids of smaller size. If the areas covered by overset grids make up an insignificant part of the entire modeling area, then the share of computing resources spent on calculation in overset grids is insignificant, that is, the addition of overset grids in this case does not lead to a noticeable increase in the resource intensity of the calculation.

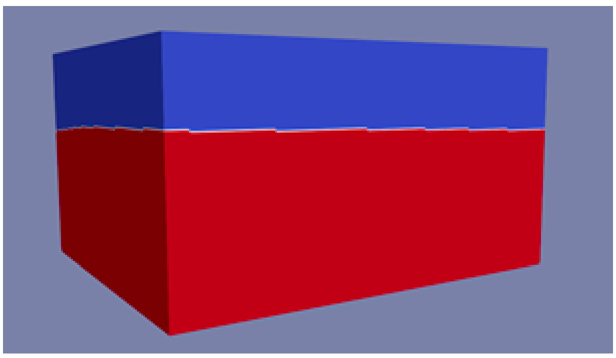
When using overset grids, the time step of modelling is performed in two stages. First, a time step is performed independently on each of the grids. After that, the values are transferred between the grids. In areas that are covered by overset grids, the values in the nodes of the main grid are overwritten by the values from the corresponding overset grids. For each of the overset grids, several nodes closest to the boundary of the layers along all axes are called ghost nodes (phantom nodes). The values in these nodes are not used to calculate new values in the nodes of the main grid, but on the contrary, the values in them are overwritten by the values from the main grid [6]. The number of phantom node layers used is determined based on the difference scheme used, in our case it is equal to two, since a five-point difference scheme is used.

Since the nodes of the main and overset grids do not match, and for each grid at each time, the function values are known only in the nodes, interpolation is used to calculate new values during rewriting. Since there are no changes in the geometry of the computational domain in the problems considered in this paper, the grid nodes remain stationary during the calculation. This allows you to transfer part of the calculations required for interpolation to the preprocessing stage to save computing resources during the calculation. During this preprocessing, for all nodes of the main grid, the values in which will be overwritten, there are nodes of the overset grid, the values in which will participate in the calculation of new values in the node of the main grid and the summation weight, and, conversely, for the phantom nodes of the overset grids, the source nodes of the values from the main grid and their weights are searched. Thus, only weighted summation with known coefficients is performed during the calculation.

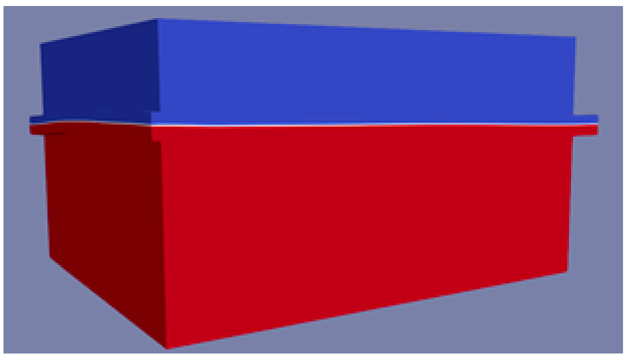
Commonly used methods of multidimensional interpolation include the nearest neighbor method, the inverse distance method [14], the natural neighborhood method (also known as the Sibson method [15]) and local basis decomposition methods. In the calculation in this paper, the method of local decomposition by radial type functions (Radial Basis Function, RBF) was used.

Computational experiment. The propagation of wave disturbances from a point source was simulated. The size of the simulated area was 1080×1320 meters, the upper boundary of the area coincided with the water surface, and the lower one was at a depth of 720 meters. The source of the disturbances was located near the water surface. Receivers were also located near the surface. The depth map was taken from the dataset [16]. The density of the shelf medium was assumed to be equal to 2400 kg /m^3, the velocities of longitudinal and transverse waves in it were 2850 m/s and 1650 m/s, respectively. The speed of waves in the water was assumed to be equal to 1500 m/ s, and the density of water was 1050 kg /m^3. The Riker pulse was used as the source signal. The time step of the simulation was 1 ms.

Two main approaches to modelling this area were considered. In the first approach, only one rectangular grid of 180×220×120 nodes with a step of 6 meters along all axes was used. Depending on whether the nodes are above or below the bottom surface, they were assigned the physical properties of the water or the bottom material. When using this approach, the interface of the media actually had a ladder structure. In the second approach, an additional curved grid of 192×234×11 nodes with a step of about 6 meters was used to describe the interface of the media, the shape of which repeated the interface. Figure 1 shows an illustration of the description of the interface in these ways is shown in Figure 1.



a)

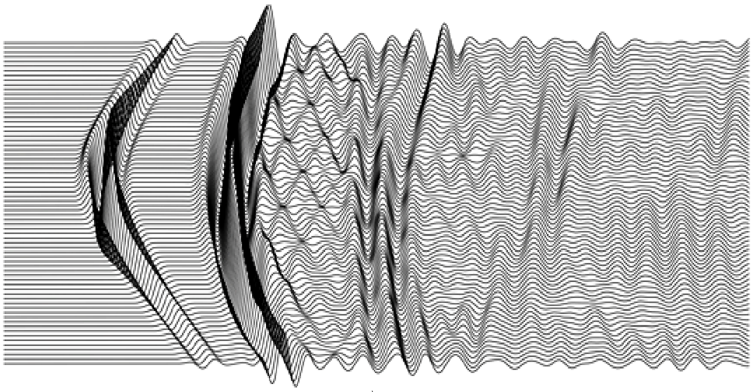


b)

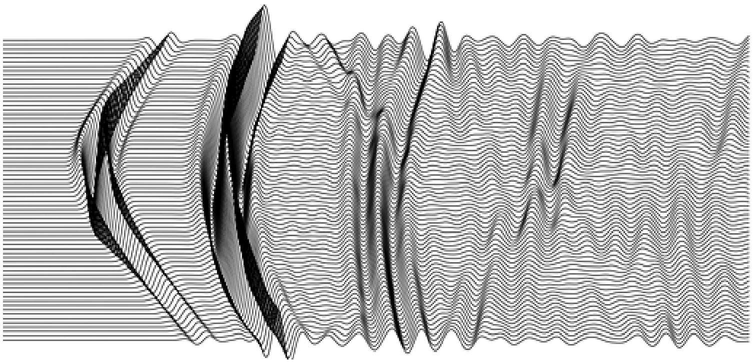
Fig. 1. Modelling of the interface:
a) without using overset grid; b) using overset grid

In this case, there are two ways to use overset grids. You can use two overset grids, each of which will be on one side of the border and set a contact condition between them, or you can limit yourself to one grid in which there will be nodes with different properties. In this experiment, the second approach was used, since nodes with different properties are still present in the main grid, but if the areas on different sides of the border were described by two different main grids, then using the first approach would be preferable.

Results. Using the receiver readings obtained during the simulation, synthetic seismograms of the vertical component of the displacement velocity were constructed (Fig. 2).



a)



b)

Fig. 2. Synthetic seismograms of the vertical component of the displacement velocity:
a) without using overset grid; b) using overset grid

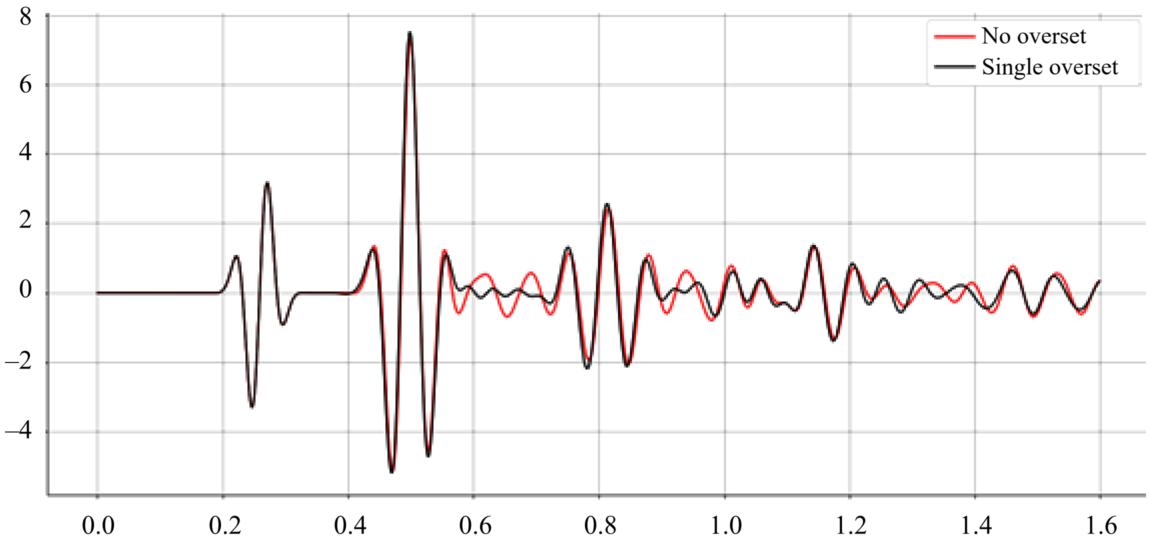


Fig. 3. Comparison of seismogram readings on one receiver

It can be seen that these seismograms are qualitatively the same, but on the seismogram, which was built using a overset grid to describe the interface of the media, the waves are visible better, and the number of artifacts and noise is less. Figure 3 shows a comparison of the readings of one of the receivers.

This comparison also confirms the conclusion made: the use of the overset grid makes it possible to reduce the modelling error, the number of parasitic waves and artifacts and get a more visual picture.

At the same time, the increase in the calculation time when adding the overset grid was insignificant and did not exceed 10 % of the calculation time using a single grid.

Discussion and Conclusions. In this study, it is shown that overset grids, previously used to isolate a free boundary when modelling near the earth's surface on land, can also be used to describe the interface of media when modelling seismic exploration of the ocean shelf. Their use makes it possible to increase the accuracy of modelling and reduce the number of artifacts compared to using only one grid. At the same time, the use of overset grids does not lead to a significant increase in the resource intensity of the computing complex. In addition, the overset grids do not require modification of the main computational grid or significant changes to the modelling process. From this we can conclude that the use of overset grids in such tasks is often justified, since it allows you to increase accuracy with little cost.

The proposed method can also be used to describe the boundaries of geological layers in a solid medium, since there is no fundamental difference between these tasks. However, additional modifications of the method may be required there to take into account cases when several interface boundaries meet at one point. The study of this issue has not yet been conducted, but it can serve as a topic for subsequent works.

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