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Grid-characteristic Method using Superimposed Grids in the Problem of Seismic Exploration of Fractured Geological Media

Ivan A Mitkovets, Nikolay I Khokhlov

Moscow Institute of Physics and Technology (National Research University), 1A, build 1, Kerchenskaya St., Moscow, Russian Federation

k_h@inbox.ru

Abstract

Introduction. Seismic exploration in conditions of heterogeneity of the environment is an urgent topic for the oil and gas industry. Consequently, the development of numerical methods for solving the direct problem of seismic exploration remains relevant as a necessary link in the development and improvement of methods for solving the inverse problem. The Schonberg thin crack model has performed well in the numerical solution of problems requiring explicit consideration of geological inhomogeneities.

Materials and Methods. In this paper, we consider a modification of the grid-characteristic method using superimposed grids. The presented approach makes it possible to conduct computational experiments, explicitly taking into account fractured inhomogeneities with arbitrary spatial orientation. For this, in addition to the basic regular computational grid, there is the concept of superimposed grids. Inhomogeneities, such as cracks, are described within the framework of the superimposed grid and, in turn, have no restrictions associated with the main grid. Thus, by performing an interpolation operation between the superimposed main grids, we can bypass the requirement of alignment of cracks and edges of the main grid.

Results. The proposed approach made it possible to study the dependence of the anisotropy of the seismic response of a fractured cluster on the dispersion of the angles of inclination of the cracks.

Discussion and Conclusions. A modification of the grid-characteristic method using superimposed grids is proposed to explicitly account for fractured inhomogeneities in a heterogeneous geological environment.

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Keywords: grid-characteristic method, superimposed grids, chimeric grids, seismics, seismic exploration, heterogeneous geological environment

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Научная статья

Сеточно-характеристический метод с использованием наложенных сеток в задаче сейсморазведки трещиноватых геологических сред

И.А. Митьковец, Н.И. Хохлов

Московский физико-технический институт (национальный исследовательский университет), Российская Федерация, г. Москва, ул. Керченская, 1А, корп. 1

k_h@inbox.ru

Аннотация

Введение. Сейсморазведка в условия гетерогенности среды является актуальной темой для нефтегазовой промышленности. Следовательно, остается актуальным развитие численных методов решения прямой задачи сейсморазведки как необходимого звена при разработке и усовершенствовании методов решения обратной

задачи. Модель тонкой трещины Шонберга хорошо себя показала при численном решении задач, требующих явного учета геологических неоднородностей.
Материалы и методы. В данной работе авторы рассматривают модификацию сеточно-характеристического метода применением наложенных сеток. Представленный подход позволяет проводить вычислительные эксперименты, явно учитывая трещиноватые неоднородности с произвольной пространственной ориентацией. Для этого помимо основной регулярной вычислительной сетки водится понятие наложенных сеток. Неоднородности, такие как трещины, описываются в рамках наложенной сетки и, в свою очередь не имеют ограничений, связанных с основной сеткой. Таким образом, производя операцию интерполирования между наложенными основными сетками, мы можем обойти требование соосности трещин и ребер основной сетки.
Результаты исследования. Предлагаемый подход позволил произвести исследование зависимости анизотропии сейсмического отклика трещиноватого кластера от дисперсии углов наклона трещин.
Обсуждение и заключения. Предложена модификация сеточно-характеристического метода с применением наложенных сеток для явного учета трещиноватых неоднородностей в гетерогенной геологической среде.

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Ключевые слова: сеточно-характеристический метод, наложенные сетки, химерные сетки, сейсмика, сейсморазведка, гетерогенная геологическая среда

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Introduction. Methods of search and exploration of oil and gas fields include an effective solution to the inverse problem of seismic exploration in a heterogeneous geological environment. This becomes inherently important, given that the known oil and gas deposits are gradually being exhausted, and in order to maintain the production level, it is necessary to search for new deposits or extract minerals from already developed deposits using modern methods. Often, potential sites are located in regions rich in fractured heterogeneities. Additionally, modern technologies for increasing production at the fields provide for the use of such a tool as hydraulic fracturing. Modern approaches to hydraulic fracturing based on multi-stage procedures open up opportunities for the resumption of production even in those fields that have been recognized as exhausted for many years. The geophysical information collected as a result of seismic exploration makes it possible to simulate the process of hydraulic fracturing, which is a critical element for adapting the technology to a specific field. Taking into account the location of existing cracks, as well as taking into account the fracture parameters (which can be determined using seismic exploration), it is possible to control the shape of the resulting rupture. Such control over the shape of the created crack is relevant because of the risk of traffic jams in the formed channels, which can lead to their blocking and, as a result, to a decrease in the efficiency of field operation. For the success of this kind of manipulation, the presence of an accurate picture of the structure of cracks and faults hidden under the earth›s surface is of paramount importance.

The data in the process of seismic exploration, obtained on a variety of seismic sensors located at an insignificant depth in the earth›s surface, are interpreted by modern methods of computational mathematics to recreate a model of the geological environment in the studied area. However, it is difficult to verify the results obtained in this way due to the lack of an opportunity to obtain a detailed and qualitative model of the geological environment by alternative methods. Thus, in order to develop the capabilities and accuracy of modern methods for solving the inverse problem of seismic exploration, it is critically important to develop the capabilities and accuracy of methods for solving the direct problem of seismic exploration.

Several techniques are known that take into account the presence of fractured structures when modeling the propagation of elastic perturbation waves in genuine geological formations. One of the most common is a mathematical approach based on the linear sliding model proposed by Schonberg (LSM), which was described in an article published in 1980 [1], and received further experimental confirmation in other sources [2–3]. Nevertheless, when modeling areas with faults, the use of anisotropic models [4] turns out to be most effective at large wavelengths, although it does not take into account most of the characteristics. An alternative method for modeling a zone with faults is the explicit approach [5], which has its advantages. Other techniques were also studied, including the addition of additional nodes, as shown in [6–7], as well as the use of additional computational grids to describe the wave propagation process inside the fault [8].

The authors present a new version of the grid-characteristic method [9], which uses the technique of overset grids. The first concepts of this approach were outlined in the source [10]. One of the initial works devoted to the application of overset (or adaptive) computational grids was the work of authors named Berger and Joseph [11], as well as Steger and Benek [12–13]. The idea of using overset grids has been successfully developed and is currently being used to solve various problems, as shown in studies [14–18]. The innovativeness of the methodology proposed by the authors lies in the use of overset grids in solving the problems of seismic exploration of fractured areas and in organizing these grids around cracks in such a way that the Jacobian of the transformation tends to unity. In this study, the authors focus on 2D geological models.

Materials and Methods

1. The equation of elasticity. One of the key equations in the linear theory of elasticity is considered to be the Hooke equation, which provides a connection between the stress tensor and the strain tensor [19–20]. The equations of conservation of mass and momentum are also used to describe the process of wave propagation in a medium. A more complex representation of elastic media is possible using extended equations that take into account nonlinear and inhomogeneous characteristics of the medium. These equations may contain nonlinear relationships between stress and strain, and a variety of physical processes, such as anisotropy or energy dissipation, can also be taken into account. Within the framework of the study, a model of a linear elastic and isotropic medium was chosen to solve the generalized problem of modelling the propagation of a seismic wave in the ground. This model has been studied in a number of previous papers [21–24].

Newton's second law is fulfilled at each point of a linear elastic medium:

$$\rho \vec{\boldsymbol{v}}_t = (\nabla \cdot \mathbf{T})^{\mathrm{T}}, \tag{1}$$

where **T** is Cauchy stress tensor, ρ is the density of the medium, ***v*** is the velocity of movement of the medium.

Hooke's law in tensor form has the form:

$$\mathbf{T} = \lambda\, tr(\varepsilon)\mathbf{I} + 2\mu\varepsilon, \tag{2}$$

$$\varepsilon = \frac{1}{2}\left(\nabla \otimes \boldsymbol{u} + (\boldsymbol{u} \otimes \nabla)^{\mathrm{T}}\right), \tag{3}$$

where ***u*** is the displacement tensor, **I** is the unit tensor, λ and μ are Lame parameters, the elastic deformation characteristics, ε is the strain tensor, ⊗ is the tensor product operator $(\nabla \otimes \boldsymbol{v})_{i,j} = \nabla_i \boldsymbol{v}_j$.

Taking into account the effect of an external force $\vec{f}$, from equations (1)–(3) it is possible to obtain a system of equations for a linear elastic isotropic medium in the following form:

$$\mathbf{T} = \lambda(\nabla \cdot \vec{\boldsymbol{v}})\mathbf{I} + \mu\left(\nabla \otimes \vec{\boldsymbol{v}} + (\nabla \otimes \vec{\boldsymbol{v}})^{\mathrm{T}}\right), \tag{4}$$

$$\rho \vec{\boldsymbol{v}}_t = (\nabla \cdot \mathbf{T})^{\mathrm{T}} + \vec{f}. \tag{5}$$

The system of equations (4) and (5) can be represented as a system of differential equations, here and further in the work we assume the absence of an external force:

$$\frac{\partial}{\partial t}\mathbf{T}_{ij} = \lambda\left(\sum_k \frac{\partial v_k}{\partial x_k}\right) I_{ij} + \mu\left(\nabla_i v_j + \nabla_j v_i\right), \tag{6}$$

$$\rho \frac{\partial}{\partial t}\boldsymbol{v}_j = \frac{\partial \mathbf{T}_{ji}}{\partial x_i}. \tag{7}$$

The system of differential equations of the theory of linear elasticity (6) and (7) can be represented in matrix form, which is convenient to use when studying the characteristic methods of computational mathematics. We introduce the notation $\vec{u} = (u_1,\ldots u_5)^T = (v_1, v_2, \boldsymbol{T}_{11}, \boldsymbol{T}_{22}, \boldsymbol{T}_{12})^T$, then the system of equations in the Cartesian coordinate system takes the form:

$$\frac{\partial u}{\partial t} + \mathbf{A_1}\frac{\partial \mathbf{u}}{\partial x_1} + \mathbf{A_2}\frac{\partial \mathbf{u}}{\partial x_2} = 0, \tag{8}$$

$$\mathbf{A_1} = -\begin{bmatrix} 0 & 0 & \rho^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho^{-1} \\ \lambda + 2\mu & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{9}$$

$$\mathbf{A}_2 = -\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \rho^{-1} \\ 0 & 0 & 0 & 0 & \rho^{-1} & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & \lambda+2\mu & 0 & 0 & 0 & 0 \\ \mu & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (10)$$

To describe the elastic properties of the medium, it is convenient to use sound wave velocities: longitudinal C_p and transverse C_s. The longitudinal velocity reflects the velocity of the wave in the direction of force, the transverse velocity in the perpendicular direction. They are calculated through the Lame parameters and the density of the medium as follows:

$$C_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad (11)$$

$$C_s = \sqrt{\frac{\mu}{\rho}}. \quad (12)$$

In this paper, the model of a two-shore thin Schonberg crack is used to simulate the propagation of elastic waves in the presence of cracks. In the case when the crack is oriented along the OY axis, the boundary conditions have the form:

$$\boldsymbol{T}_{xx}^0 = \boldsymbol{T}_{xx}^1,$$

$$\boldsymbol{T}_{xy}^0 = \boldsymbol{T}_{xy}^1,$$

$$\frac{\partial \boldsymbol{T}_{xx}^0}{\partial t} = K_T\left(\upsilon_x^1 + \upsilon_x^0\right),$$

$$\frac{\partial \boldsymbol{T}_{xy}^0}{\partial t} = K_N\left(\upsilon_y^1 + \upsilon_y^0\right).$$

where the indices 0 and 1 are used to mark the ratio of the magnitude to the left and right sides of the crack, respectively, K_T and K_N are the crack parameters, which in our case of a thin liquid-filled crack are equal:

$$K_T = \infty,$$

$$K_N = 0.$$

2. Grid-characteristic method. To obtain a numerical solution of a system of equations describing a linear elastic medium, a grid-characteristic method is used. This method was first presented in [25–27]. In the context of this study, which is limited to the numerical solution of a system of hyperbolic equations, it can be argued that for the matrix $\mathbf{A}_j$ there are always N eigenvalues and N linearly independent eigenvectors. This in turn confirms the possibility of the existence of an inverse matrix for $\mathbf{\Omega}_j$. Then, taking into account the splitting by components, equation (8) takes the form:

$$\frac{\partial \boldsymbol{u}}{\partial t} = \mathbf{\Omega}_j^{-1}\mathbf{\Lambda}_j\mathbf{\Omega}_j\frac{\partial \boldsymbol{u}}{\partial x_j}, \; j = 1,2, \quad (13)$$

where $\mathbf{\Omega}_j$ consists of columns that are the eigenvectors of the matrix $\mathbf{A}_j$, and $\mathbf{\Lambda}_j$ is a diagonal matrix consisting of the eigenvalues of the matrix $\mathbf{A}_j$. At the same time, $\mathbf{\Lambda}_j$ for any j has the same form:

$$\Lambda = \text{dia g}\sqrt{(\lambda+2\mu)/\rho}, -\sqrt{(\lambda+2\mu)/\rho}, \sqrt{\mu/\rho}, -\sqrt{\mu/\rho}, -\sqrt{\mu/\rho}, \; 0,0,0. \quad (14)$$

Taking into account expressions (11) and (12), equation (14) can be reduced to the form:

$$\Lambda = \text{diag}\{C_p, -C_p, C_s, -C_s, C_s, -C_s, 0,0,0\}.$$

Let's make a characteristic replacement of the variables $\boldsymbol{v} = \mathbf{\Omega}\boldsymbol{u}$ ($\boldsymbol{u} = \mathbf{\Omega}^{-1}\boldsymbol{v}$) in equation (13), b multiply on the left by the matrix $\mathbf{\Omega}^{-1}$:

$$\frac{\partial \boldsymbol{v}}{\partial t} + \Lambda\frac{\partial \boldsymbol{v}}{\partial x} = 0.$$

Thus, the calculation of the value for each element of the vector u at the subsequent time step, denoted as n+1, is carried out provided that the value of $\boldsymbol{v}^{(n+1)}$ is known: $\boldsymbol{u}^{n+1} = \mathbf{\Omega}^{-1}\boldsymbol{v}^{n+1}$.

3. Overset grids. Direct interpolation is carried out from the main grid to the external nodes of the overset grid at the start of each stage of the computational process, which is limited to a certain time step, after performing calculations in the nodes of the main computational grid. This action is necessary to take into account the changes in the intensity and vector displacement of the medium caused by the wave propagation process within the main grid. After that, new values are calculated in the nodes of the overset grid. At the end of each time step, reverse interpolation occurs from the nodes of the overset grid to the main one. This process ensures synchronization of changes that took place during this stage of calculations. This approach is due to the need to take into account the influence of various inhomogeneities represented in the overset grids when performing calculations at the next time stage [28].

The authors used the bilinear interpolation function to find the values of the desired functions at a point by the values of the function at four known points:

$$F_{x,y} = b_1 + b_2 x + b_3 y + b_4 xy.$$

Application of overset grids

1. Implementation. A continuous medium can be represented using a single basic rectangular grid when applying a overset grid to determine the position of an inclined crack . At the same time, cracks can be defined using overset grids, which are arranged according to the orientation of each individual crack. The method of calculating a straight crack on a regular rectangular grid and its application within the framework of the grid-characteristic method were described in detail in the scientific work [9]. It is important to note that the application of the overset grid is not mandatory to account for cracks that are coaxial with the edges of the main computational grid.

Figure 1 shows the location of the grids and cracks used, and the nodes of the overset grid involved in the interpolation between the grids are also indicated. In the figure, the borders of the main rectangular regular grid representing the environment are represented in black. The edges of the overset grid are highlighted in blue. Additional «ghost» nodes of the overset grid are marked in green, where interpolation from the main grid is performed. The orange color highlights the part of the nodes of the overset grid from where the interpolation occurs. The red line indicates the location of the crack.

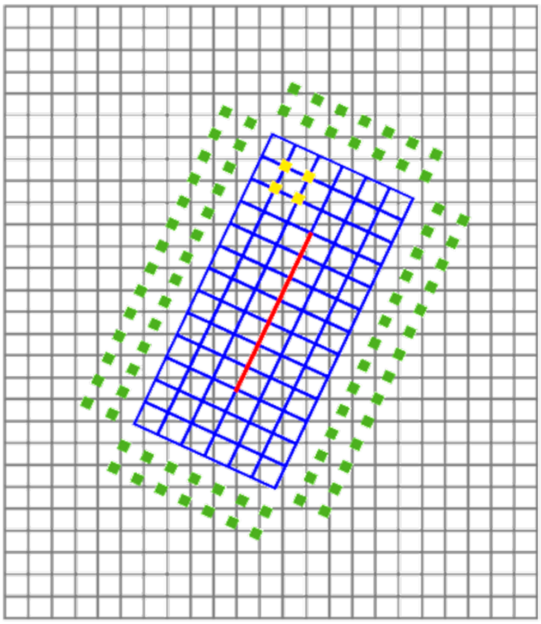


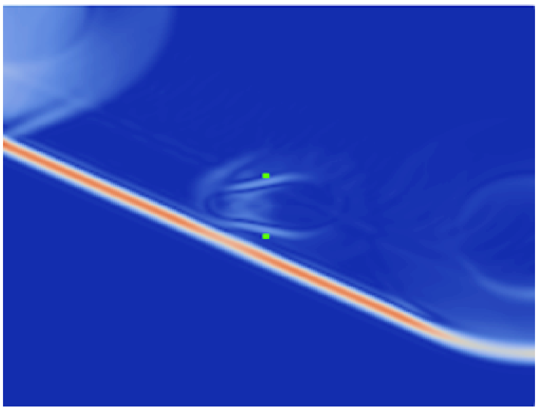
Fig. 1. Using the overset grid to account for the crack

2. Verification. To assess the accuracy of the proposed modification, we will compare the readings obtained on virtual receivers during the simulation of the interaction of an elastic wave with a crack. In one case, the crack coincides with the horizontal axis of the main grid, and in the other, it is rotated relative to this axis using the overset grid. To make such a turn, it is necessary to correctly process the data received from virtual receivers, as well as correctly rotate the crack and the receivers and the elastic wave source.

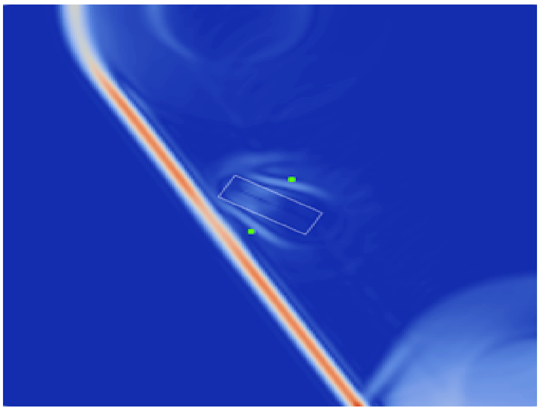
Figure 2 (a, b) shows the wave patterns at one of the time points for the compared productions, as well as the location of the receivers and the overset grid.

The initial conditions assumed the presence of elastic perturbations of the plane front, given by a Gaussian function with a width of 10 meters. The front of the initiated plane wave was rotated relative to the simulated crack by an angle of 30 degrees. The length of the thin crack was 52 meters. In the calculation using the overset grid, the wave-crack-receivers system was rotated by an angle of 30 degrees. The receivers were placed on a line perpendicular to the simulated crack and passing through its center, at a distance of 30 meters from it. The longitudinal velocity of the elastic wave in the medium was $C_p = 3000$ м/с, the transverse velocity was $C_s = 1500$ м/с, the time step was $dt = 0.0002$ seconds, the spatial step of the grids was h= 2.0.

Figures 3 and 4, for receivers above the crack and behind it, respectively, show a comparison of the values of the components of the displacement velocity of the medium obtained as a result of the experiments described above.

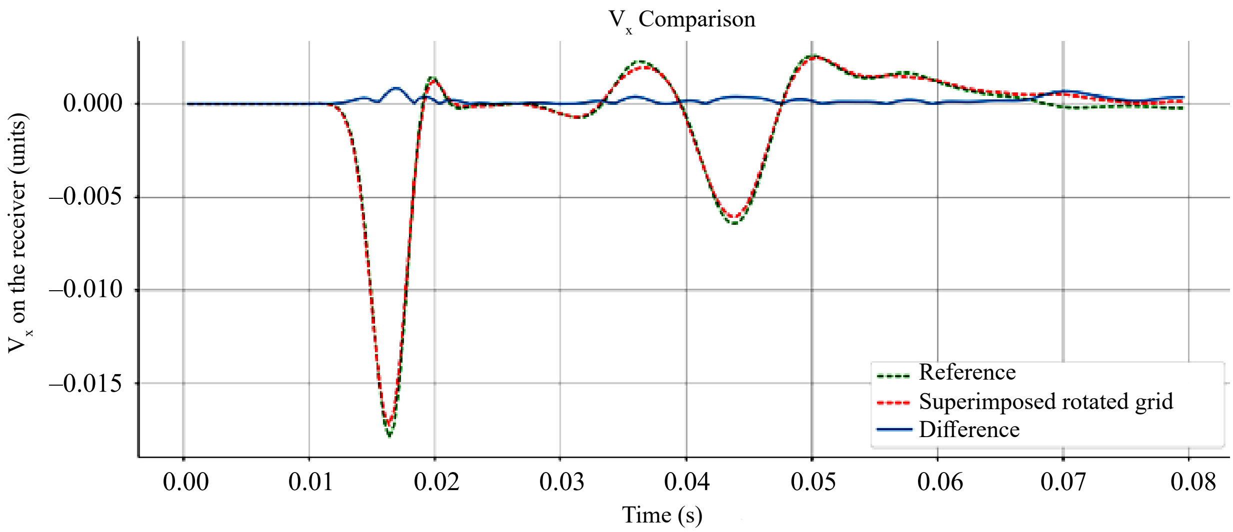


a)



b)

Fig. 2. Wave patterns at one of the time points: a) without using the superimposed grid (the position of the receivers is marked in green); b) using the superimposed grid marked in white, green is receivers



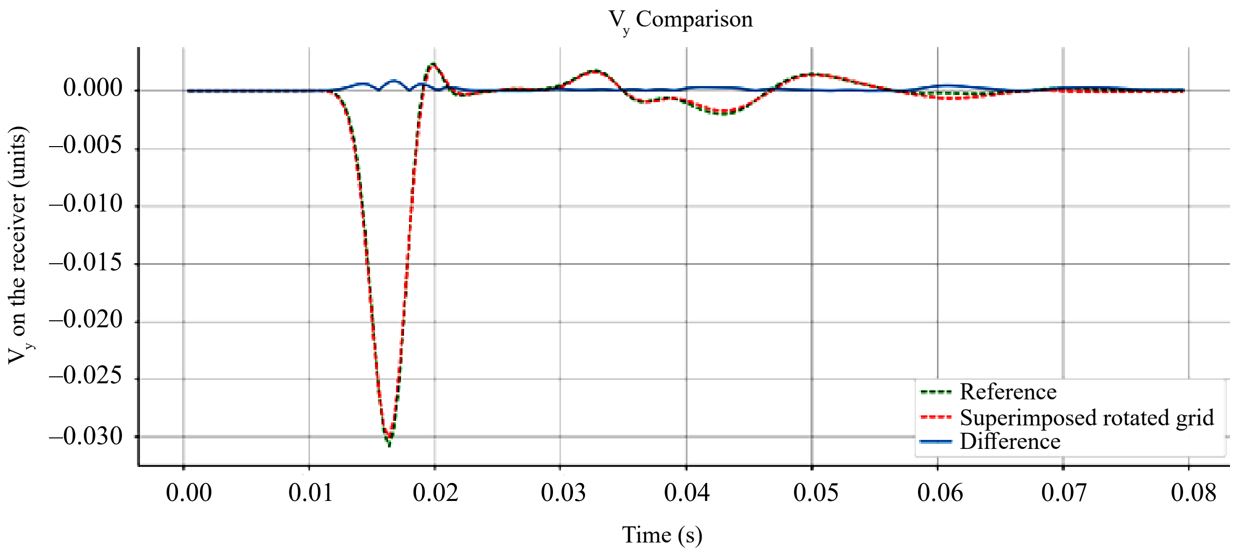


Fig. 3. Comparison of signals on receivers above the crack

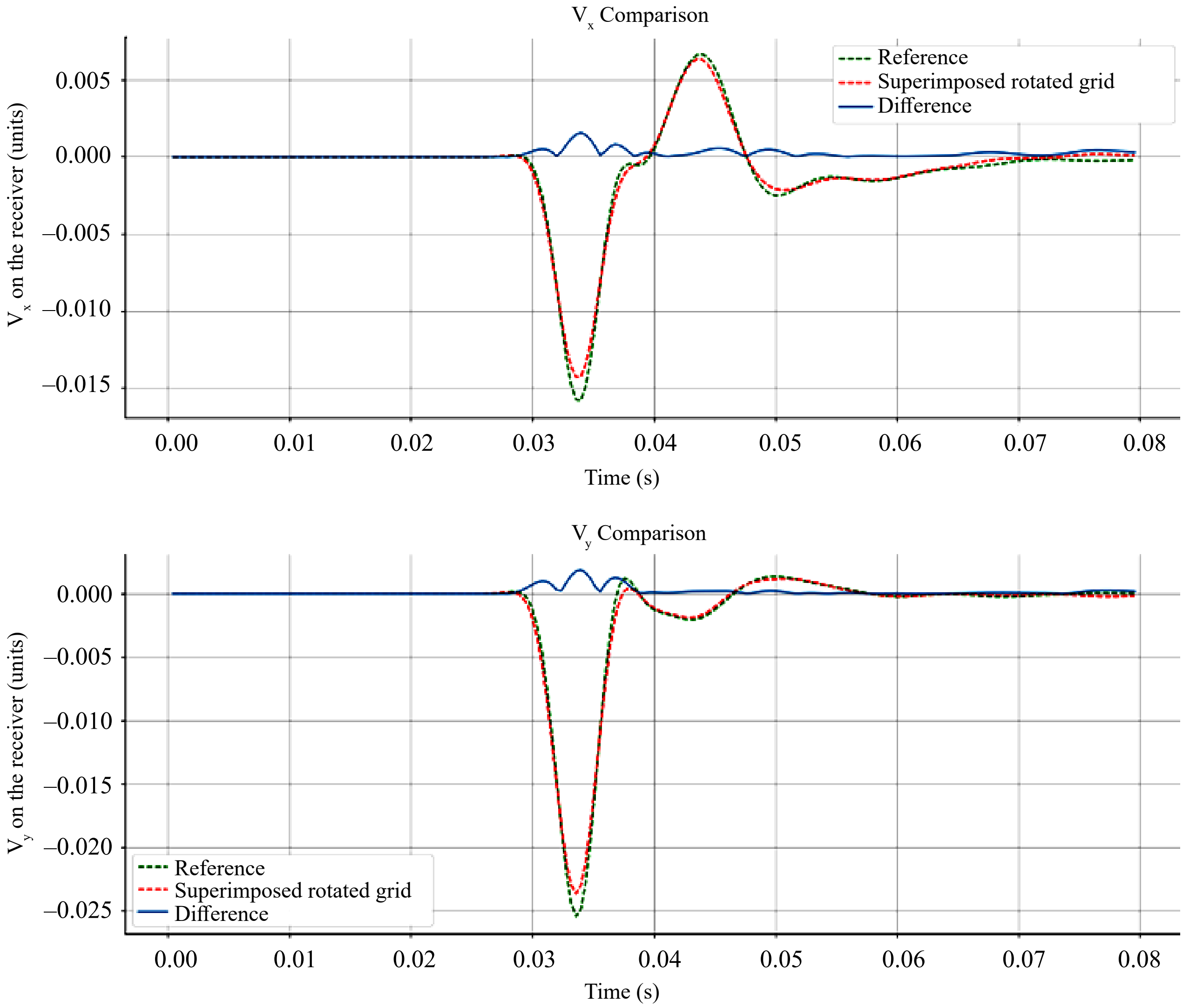


Fig. 4. Comparison of signals on receivers under the crack

Research results. The analysis of the anisotropy of the seismic response of a fractured cluster in an elastic geological medium, interconnected with the dispersion of the slope of cracks, was carried out. A similar analysis related to the anisotropy of the seismic response of a fractured cluster, depending on the variable distance between the cracks and the frequency of the source, was carried out in [29]. The problem of numerical simulation of the seismic response from fractured clusters of subvertical cracks using the grid-characteristic method has been considered in a number of scientific articles [30–33].

Within the framework of this study, a fixed distance of 30 meters was used between vertically and horizontally adjacent cracks. A total of 128 cracks were placed, which were organized into 8 layers and 16 columns. Each crack was inclined at an arbitrary angle relative to the vertical and had a length of 10 meters. In each individual experiment, the angles of rotation of the cracks corresponded to a normal distribution with an average value of 45 degrees and a variance varying from 0 to 20 degrees. The scheme of the problem and the location of the crack cluster in the simulated half-space are shown in Figure 5.

In a number of computational experiments, a 50-meter-long plane wave source was used, which fell vertically and was set by the Riker function. The time integration step was $3 \cdot 10^{-4}$ seconds, the total number of steps was 3000. To register the seismic response in the experiments, 300 receivers were used, evenly distributed at a depth of 6 meters from the surface of the simulated area. In order to isolate the seismic response from the wave passing through the receivers, during the processing of the results, the readings of the receivers during the experiment, less than 0.0801 seconds, were ignored. The longitudinal velocity of the elastic wave in the medium was $C_p = 3000$ m/s, the transverse velocity was $C_s = 1500$ m/s, the spatial step of the grids was $h = 2.0$.

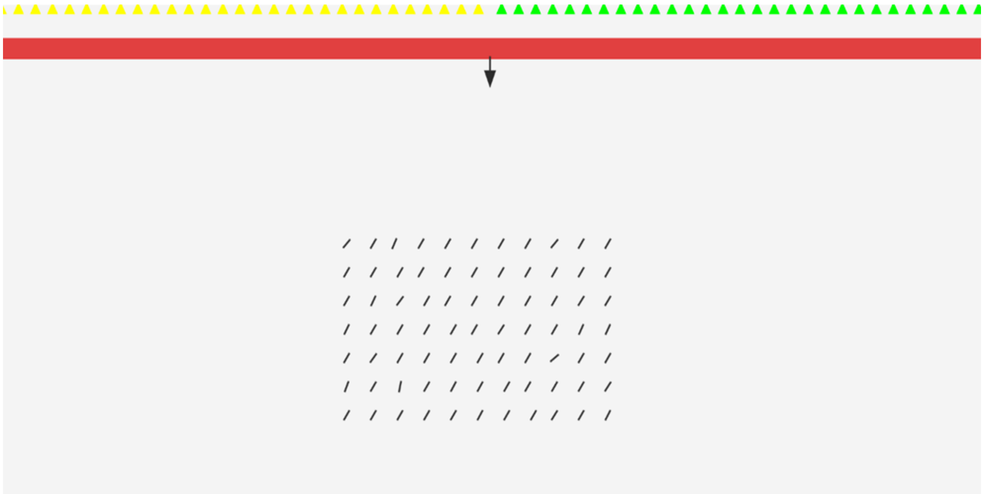


Fig. 5. The scheme of the computational experiment. Yellow and green triangles indicate the “left“ and “right” groups of receivers

To assess the anisotropy of the seismic response, the receivers used were divided into two equal groups: a group of receivers with index L was located to the left of the middle (along the axis O_x) of the main overset grid, and a group of receivers with index R was located to the right. Let the displacement velocity of the medium in the projection on the O_x axis, registered at the end of the i-th computational step, on the j-th virtual receiver in the group L be denoted $V^{i,j}_{x,L}$ (for the projection on the O_y axis respectively $V^{i,j}_{y,L}$). Then the anisotropy of the seismic response A in a given experiment can be calculated using the following formulas:

$$E_R = \sum_{i=1}^{150} \sum_{j=267}^{3000} \left(\left(V^{i,j}_{x,L} \right)^2 + \left(V^{i,j}_{y,L} \right)^2 \right),$$

$$E_L = \sum_{i=1}^{150} \sum_{j=267}^{3000} \left(\left(V^{i,j}_{x,R} \right)^2 + \left(V^{i,j}_{y,R} \right)^2 \right),$$

$$A = \frac{E_L - E_R}{E_L + E_R}.$$

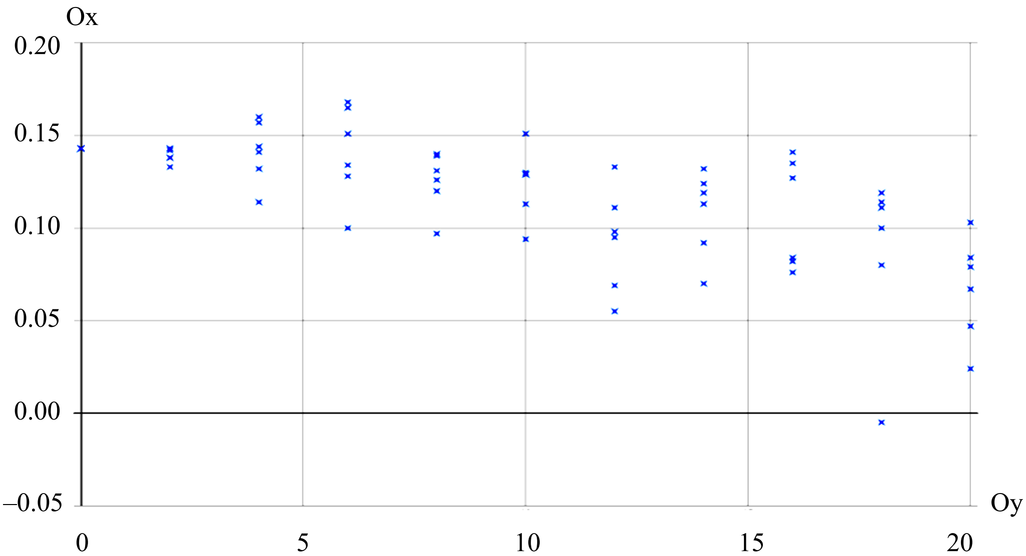


Fig. 6. Dependence of anisotropy on the dispersion of the slope of cracks in the cluster

Discussion and conclusions. A modification of the grid-characteristic method using overset grids is proposed to explicitly account for fractured inhomogeneities in a heterogeneous geological environment.

A verification study was carried out, which showed the high accuracy of the proposed approach and does not introduce a significant error compared to using the classical implementation of the thin Schonberg crack in the grid-characteristic method and at the same time expands a number of applied problems available for numerical modeling by this method.

The dependence of the anisotropy of the seismic response on the fractured cluster depending on the dispersion of the angle of inclination of the cracks in the latter is investigated. The obtained dependence shows a significant spread of results, which demonstrates the complexity of the inverse problem of seismic exploration of heterogeneous geological structures.

In conclusion, it can be concluded that the use of overset grids makes it possible to explicitly take into account geological inhomogeneities, such as cracks, when numerically solving the problem of modeling the propagation of elastic waves in a geological medium. The presented approach to the description of inhomogeneities has a high potential in computational mathematics and arouses interest in its further study.

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About the Authors:

Ivan A Mitkovets, PhD student, Moscow Institute of Physics and Technology (National Research University) (bldg. 1, 1A, Kerchenskaya St., Moscow, 117303, RF), ORCID, mitkovets@phystech.edu

Nikolay I Khokhlov, Associate Professor, Head of the Department of Computer Science and Computational Mathematics, Moscow Institute of Physics and Technology (National Research University) (bldg. 1, 1A, Kerchenskaya St., Moscow, 117303, RF), Candidate of physical and mathematical Sciences, ORCID, Math-Net.Ru, elibrary.ru, ResearcherID, k_h@inbox.ru

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Об авторах:

Иван Анатольевич Митьковец, аспирант, Московский физико-технический институт (национальный исследовательский университет) (РФ, 117303, г. Москва, ул. Керченская, 1А, корп. 1), ORCID, mitkovets@phystech.edu

Николай Игоревич Хохлов, доцент, заведующий кафедрой информатики и вычислительной математики, Московский физико-технический институт (национальный исследовательский университет) (РФ, 117303, г. Москва, ул. Керченская, 1А, корп. 1), кандидат физико-математических наук, ORCID, Math-Net.Ru, elibrary.ru, ResearcherID, k_h@inbox.ru

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