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Two Dimensional Hydrodynamics Model with Evaporation for Coastal Systems

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Abstract

Introduction. The use of two-dimensional (2D) hydrodynamic models is relevant, despite the development of numerical methods of marine hydrodynamics focused on the use of three-dimensional spatial models. This is due to the modelling of hydrodynamic processes in shallow and coastal systems in solving practically important problems of predicting the transport of pollutants in suspended and dissolved forms. Evaporation for the Southern of Russia marine coastal systems (the Azov Sea, the Northern Caspian, etc.), and even more so in the coastal areas of the Red Sea, is a significant factor that affects not only the balance of water masses, but also makes changes in the momentum of the system and the distribution of the velocity vector of the aquatic environment. This effect is significant for coastal currents and shallow-water systems.

Materials and Methods. The traditional method of converting the terms of the Navier-Stokes equations containing differentiation by horizontal spatial variables was used, involving the rearrangement of differentiation operations by horizontal spatial coordinates and integration by vertical coordinate when constructing a spatially two-dimensional model of hydrodynamics of marine coastal systems when integrated by vertical coordinate. This made it possible to avoid the appearance of non-physical sources of energy and momentum in the spatially two-dimensional model, which can be essential in traditional 2D models with significant depth differences characteristic of coastal systems. The implementation of the analogue of the law of conservation of the total mechanical energy of the system for the constructed 2D model is investigated.

Results. Using the correct transformation of the 3D model (integration of the Navier-Stokes equations and continuity along a vertical coordinate, taking into account evaporation from a free surface), spatially two-dimensional models of hydrodynamics are constructed, for which the basic conservation laws, including mass and total mechanical energy of the system, are fulfilled. The implementation of an analogue of the law of conservation of total mechanical energy for various types of boundary conditions, including at the bottom, is investigated. The evaporation from the free surface is correctly accounted for not only in the continuity equation, but also in the equations of motion taking into account wind and waves.

Discussion and Conclusion. 2D model of hydrodynamics has been constructed and studied, taking into account evaporation not only in the mass balance equation (continuity), but also in the Navier-Stokes equations of motion. The proposed model can be used for predictive modelling of hydrophysical processes, including the spread of pollutants in the aquatic environment of coastal systems and shallow reservoirs in relation to marine systems such as the Azov Sea, the Northern Caspian Sea, coastal areas of the Red Sea, etc. Spatially two-dimensional models of marine hydrodynamics, without replacing three-dimensional models, can serve as a model basis for operational forecasting of situations in coastal systems and shallow-water objects using computing systems with relatively low performance and a moderate amount of RAM (5–10 Tflops, 2–4 TB, respectively).

Keywords: Coastal Systems, Evaporation, 2D Hydrodynamics Models, Mass Conservation Law, Mechanical Energy Conservation Law

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Научная статья

Двумерная гидродинамическая модель прибрежных систем, учитывающая испарение

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Аннотация

Введение. Несмотря на развитие численных методов морской гидродинамики, ориентированных на использование пространственно-трехмерных моделей, применение двумерных гидродинамических моделей по-прежнему остается актуальным. Прежде всего это касается моделирования гидродинамических процессов в мелководных и прибрежных системах при решении практически важных задач прогнозирования переноса загрязняющих веществ во взвешенной и растворенной формах. Испарение для морских прибрежных систем, располагающихся на Юге России (Азовское море, Северный Каспий и др.), а тем более в прибрежных районах Красного моря, является существенным фактором, который влияет не только на баланс водных масс, но и вносит изменения в импульс системы и распределение вектора скорости водной среды. Этот эффект заметен для прибрежных течений и мелководных систем.

Материалы и методы. В данной работе при построении пространственно-двумерной (2D) модели гидродинамики морских прибрежных систем при интегрировании по вертикальной координате не применялась традиционная методика преобразования членов уравнений Навье-Стокса, содержащих дифференцирование по горизонтальным пространственным переменным, предполагающая перестановку операций дифференцирования по горизонтальным пространственным координатам и интегрирование по вертикальной координате. Это позволило избежать появления в пространственно-двумерной модели нефизических источников энергии и импульса, которые могут иметь существенное значение в традиционных 2D-моделях при значительных перепадах глубин, характерных для прибрежных систем. Дополнительно в работе исследовано выполнение аналога закона сохранения полной механической энергии системы для построенной 2D-модели.

Результаты исследования. С помощью корректного преобразования 3D-модели (интегрирования уравнений Навье-Стокса и неразрывности по вертикальной координате с учетом испарения со свободной поверхности) построены пространственно-двумерные модели гидродинамики, для которых выполняются основные законы сохранения, в том числе массы и полной механической энергии системы. Исследовано выполнение аналога закона сохранения полной механической энергии для различных типов граничных условий, в том числе на дне. Выполнен корректный учет испарения со свободной поверхности не только в уравнении неразрывности, но и в уравнениях движения с учетом ветра и волн.

Обсуждение и заключение. Построена и исследована двумерная модель гидродинамики, учитывающая испарение не только в уравнении баланса масс (неразрывности), но и в уравнениях движения (Навье-Стокса). Предложенная модель может быть использована для прогнозного моделирования гидрофизических процессов, в том числе распространения загрязняющих веществ в водной среде прибрежных систем и мелководных водоемов применительно к таким морским системам, как Азовское море, Северный Каспий, прибрежные районы Красного моря и др. Пространственно-двумерные модели морской гидродинамики, не заменяя трехмерных моделей, могут служить модельной основой для оперативного прогнозирования ситуаций в прибрежных системах и мелководных объектах с использованием вычислительных систем с относительно невысокой производительностью и умеренным объемом оперативной памяти (5–10 Тфлопс, 2–4 ТБ соответственно).

Ключевые слова: прибрежные морские системы, испарение, 2D-модели гидродинамики, законы сохранения массы и полной механической энергии

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Introduction. The use of 2D hydrodynamic models is in demand despite the development of numerical methods of marine hydrodynamics focused on the use of three-dimensional spatial models, the use of two-dimensional hydrodynamic models remains in demand [1‒4]. First of all, this concerns hydrodynamic processes in shallow and coastal systems when solving practically important tasks of operational forecasting of the spread of pollutants in suspended and dissolved forms, the movement of sediments and sediments. Evaporation for Southern Russia marine coastal systems (the Azov Sea, the Northern Caspian, etc.), and even more so for coastal areas of the Red Sea, is a significant factor that affects not only the balance of water masses, but also makes changes in the momentum of the system and the distribution of the velocity vector of the aquatic environment. This effect is very noticeable for coastal currents and shallow-water systems [5‒8]. The aim of the work is to construct a conservative spatially two-dimensional hydrodynamic model for which the laws of conservation of mass balance and total mechanical energy are fulfilled, taking into account the evaporation of water from the free surface of a water body.

Coastal systems are characterized by high intensity of movement of the aquatic environment, large depth differences, a complex shape of the coastline, and in some cases — the presence of various hydraulic structures. Industrial pollution causes the greatest harm to water resources [9‒10]. As a result of the activities of coastal enterprises and the navy, polychlorinated biphenyls, heavy metals, surfactants, easily oxidized organics, polyaromatic hydrocarbons, etc. enter the water. Waste from the petrochemical and oil refining industries is particularly dangerous. Oil pollution is one of the most harmful and intractable emergencies [11–12].

Evaporation is an important process in most oil spills. Light oil changes very dramatically from liquid to viscous. In conditions when the boundary layer of air is stationary (there is no wind) or has low turbulence, the air directly above the water is quickly saturated and evaporation slows down [13]. When the wind speed increases, the evaporation rate increases significantly and is a non-linearly dependent function of wave height. In this paper, a relatively simple evaporation model is used, which allows us to take these effects into account.

Another feature of the obtained spatially two-dimensional models of hydrodynamics is the consideration of the fact that the operations of differentiation by spatial variables in horizontal directions are not, as shown by A.I. Sukhinov, commutative with respect to the operation of integration along a vertical spatial coordinate. In the case of coastal systems, where there is a significant difference in depth, an arbitrary change in the order of these operations, performed for the "convenience and simplicity" of obtaining spatially two-dimensional equations of motion of the aquatic environment, can lead to the appearance of fictitious, physically unreasonable sources of momentum in the Navier-Stokes equations. The method of constructing two-dimensional equations of motion proposed by the authors makes it possible to exclude this negative effect.

Materials and Methods. To simulate the hydrodynamic process with evaporation in an open water area, the equations of conservation of mass, momentum and energy describing the transfer of both liquid and gas phases are used. A rectangular Cartesian coordinate system is introduced. The axis Oz is directed opposite to the direction g from some point on the undisturbed surface of the liquid, the axis Ox is to the east, the axis Oy is to the north. Since the contribution of the centrifugal force is ≈ 0.2 % of the contribution of the gravitational force of attraction to the Earth, the angle ϑ between the vector of the angular velocity of rotation of the Earth and the vertical Oz can be considered complementary to π/2 the latitude of the place.

Results. Let's perform the integration of the 3D continuity equation in the derivation of the 2D model of hydrodynamics

$$u'_x + v'_y + w'_z = 0$$

and the 3D Navier-Stokes equations

$$u'_t + \left(u^2\right)'_x + \left(uv\right)'_y + \left(uw\right)'_z = -\rho^{-1} p'_x - \varphi'_x + \eta\rho^{-1}\left(u''_{xx} + u''_{yy} + u''_{zz}\right) + 2\Omega\left(v\sin\vartheta - w\cos\vartheta\right),$$

$$v'_t + \left(uv\right)'_x + \left(v^2\right)'_y + \left(vw\right)'_z = -\rho^{-1} p'_y - \varphi'_y + \eta\rho^{-1}\left(v''_{xx} + v''_{yy} + v''_{zz}\right) - 2\Omega\, u\sin\vartheta,$$

$$w'_t + \left(uw\right)'_x + \left(vw\right)'_y + \left(w^2\right)'_z = -\rho^{-1} p'_z - \varphi'_z + \eta\rho^{-1}\left(w''_{xx} + w''_{yy} + w''_{zz}\right) + 2\Omega\, u\cos\vartheta$$

for viscous (in linear approximation) incompressible (density) liquid rotating at an angular velocity

$$\mathbf{\Omega} = \Omega\left(\mathbf{j}\cos\vartheta + \mathbf{k}\sin\vartheta\right),$$

where **i, j, k** are the unit orts; $u = u\ (x, y, z, t)$, $v = v\ (x, y, z, t)$, $w = w\ (x, y, z, t)$ are the components of the liquid velocity vector at point (x, y, z) at time t; p is the total hydrostatic pressure; φ is the gravitational potential; η is the first viscosity coefficient in a homogeneous gravity field $\nabla\varphi = -\mathbf{g} = -g\mathbf{k} = \text{const}$; $p_a = p_a\ (x, y, t)$; is the atmospheric pressure,

$p = p_a + + \rho g(\xi - z)$, $\nabla p = g\left(\zeta'_x \mathbf{i} + \zeta'_y \mathbf{j} - \mathbf{k}\right), -h \le z \le \xi$, where $\xi = \xi\ (x, y, z)$ is the elevation of the level of the free surface of the liquid with respect to the undisturbed state;
$h = h\ (x, y, z)$ is the height of the liquid column under the undisturbed surface.

Substituting the expressions for the gravitational potential and pressure into the 3D Navier-Stokes equations, we obtain:

$$u'_x + v'_y + w'_z = 0,$$

$$u'_t + \left(u^2\right)'_x + \left(uv\right)'_y + \left(uw\right)'_z = -g\zeta'_x - \rho^{-1}\left(p_a\right)'_x + \eta\rho^{-1}\left(u''_{xx} + u''_{yy} + u''_{zz}\right) + 2\Omega\left(v\sin\vartheta - w\cos\vartheta\right),$$

$$v'_t + \left(uv\right)'_x + \left(v^2\right)'_y + \left(vw\right)'_z = -g\zeta'_y - \rho^{-1}\left(p_a\right)'_y + \eta\rho^{-1}\left(v''_{xx} + v''_{yy} + v''_{zz}\right) - 2\Omega u\sin\vartheta,$$

$$w'_t + \left(uw\right)'_x + \left(vw\right)'_y + \left(w^2\right)'_z = \eta\rho^{-1}\left(w''_{xx} + w''_{yy} + w''_{zz}\right) + 2\Omega u\cos\vartheta.$$

We integrate the obtained equations along the vertical coordinate z from $-h$ to ξ taking into account the relations for differentiable functions $f = f\,(x, y, z, t)$, $\xi = \xi\ (x, y, t)$, $h = h\ (x, y, t)$:

$$\int_{-h}^{\zeta} f'_t dz = \left(\int_{-h}^{\zeta} f dz\right)'_t - f_s\zeta'_t + f_b\left(-h'_t\right),$$

$$\int_{-h}^{\zeta} f'_x dz = \left(\int_{-h}^{\zeta} f dz\right)'_x - f_s\zeta'_x + f_b\left(-h'_x\right),$$

$$\int_{-h}^{\zeta} f'_y dz = \left(\int_{-h}^{\zeta} f dz\right)'_y - f_s\zeta'_y + f_b\left(-h'_y\right),$$

$$\int_{-h}^{\zeta} f'_z dz = f_s - f_b,$$

where $f_s = f\,(x, y, \xi, t)$, $f_b = f\,(x, y, -h, t)$, are the values of the function f on the surface and bottom, respectively.

We obtain the following equations:

$$\left(U'_x - u_s\zeta'_x - u_b h'_x\right) + \left(V'_x - v_s\zeta'_y - v_b h'_y\right) + \left(w_s - w_b\right) = 0,$$

$$\left(U'_t - u_s\zeta'_t - u_b h'_t\right) + \left(\left(\int_{-h}^{\zeta} u^2 dz\right)'_x - u_s^2\zeta'_x - u_b^2 h'_x\right) + \left(\left(\int_{-h}^{\zeta} uv dz\right)'_y - u_s v_s\zeta'_y - u_b v_b h'_y\right) + \left(u_s w_s - u_b w_b\right) =$$

$$= -gH\zeta'_x - \frac{H}{\rho}\left(p_a\right)'_x + \frac{\eta}{\rho}\left(\left(\left(\int_{-h}^{\zeta} u'_x dz\right)'_x - \left(u'_x\right)_s\zeta'_x - \left(u'_x\right)_b h'_x\right) + \left(\left(\int_{-h}^{\zeta} u'_y dz\right)'_y - \left(u'_y\right)_s\zeta'_y - \left(u'_y\right)_b h'_y\right) + \right.$$

$$\left. + \left(\left(u'_z\right)_s - \left(u'_z\right)_b\right)\right) + 2\Omega\left(V\sin\vartheta - W\cos\vartheta\right), \qquad (1)$$

$$\left(V'_t - v_s\zeta'_t - v_b h'_t\right) + \left(\left(\int_{-h}^{\zeta} uv dz\right)'_x - u_s v_s\zeta'_x - u_b v_b h'_x\right) + \left(\left(\int_{-h}^{\zeta} v^2 dz\right)'_y - v_s^2\zeta'_y - v_b^2 h'_y\right) + \left(v_s w_s - v_b w_b\right) =$$

$$= -gH\zeta'_y - \frac{H}{\rho}\left(p_a\right)'_y + \frac{\eta}{\rho}\left(\left(\left(\int_{-h}^{\zeta} v'_x dz\right)'_x - \left(v'_x\right)_s\zeta'_x - \left(v'_x\right)_b h'_x\right) + \left(\left(\int_{-h}^{\zeta} v'_y dz\right)'_y - \left(v'_y\right)_s\zeta'_y - \left(v'_y\right)_b h'_y\right) + \right.$$

$$\left. + \left(\left(v'_z\right)_s - \left(v'_z\right)_b\right)\right) - 2\Omega U\sin\vartheta, \qquad (2)$$

$$\left(W'_t - w_s\zeta'_t - w_b h'_t\right) + \left(\left(\int_{-h}^{\zeta} uw dz\right)'_x - u_s w_s\zeta'_x - u_b w_b h'_x\right) + \left(\left(\int_{-h}^{\zeta} vw dz\right)'_y - v_s w_s\zeta'_y - v_b w_b h'_y\right) +$$

$$+\left(w_s^2-w_b^2\right)=\frac{\eta}{\rho}\left(\left(\left(\int_{-h}^{\zeta} w'_x dz\right)'_x-\left(w'_x\right)_s\zeta'_x-\left(w'_x\right)_b h'_x\right)+\left(\left(\int_{-h}^{\zeta} w'_y dz\right)'_y-\left(w'_y\right)_s\zeta'_y-\left(w'_y\right)_b h'_y\right)+\right.$$

$$\left.+\left(\left(w'_z\right)_s-\left(w'_z\right)_b\right)\right)+2\Omega U\cos\vartheta, \qquad (3)$$

where e $U=\int_{-h}^{\zeta} udz,\ V=\int_{-h}^{\zeta} vdz,\ W=\int_{-h}^{\zeta} wdz$; is the full depth.

By rearranging the terms, we get:

$$U'_x+V'_y+\left(-u_s\zeta'_x-v_s\zeta'_y+w_s\right)-\left(u_bh'_x+v_bh'_y+w_b\right)=0,$$

$$U'_t+\left(\int_{-h}^{\zeta} u^2dz\right)'_x+\left(\int_{-h}^{\zeta} uvdz\right)'_y-u_s\left(\zeta'_t+u_s\zeta'_x+v_s\zeta'_y-w_s\right)-u_b\left(h'_t+u_bh'_x+v_bh'_y+w_b\right)=$$

$$=-gH\zeta'_x+\frac{\eta}{\rho}\left(\left(U'_x-u_s\zeta'_x-u_bh'_x\right)'_x+\left(U'_y-u_s\zeta'_y-u_bh'_y\right)'_y\right)+\left(F_s\right)_x+\left(F_b\right)_x+2\Omega\left(V\sin\vartheta-W\cos\vartheta\right),$$

$$V'_t+\left(\int_{-h}^{\zeta} uvdz\right)'_x+\left(\int_{-h}^{\zeta} v^2dz\right)'_y-v_s\left(\zeta'_t+u_s\zeta'_x+v_s\zeta'_y-w_s\right)-v_b\left(h'_t+u_bh'_x+v_bh'_y+w_b\right)=$$

$$W'_t+\left(\int_{-h}^{\zeta} uwdz\right)'_x+\left(\int_{-h}^{\zeta} vwdz\right)'_y-w_s\left(\zeta'_t+u_s\zeta'_x+v_s\zeta'_y-w_s\right)-w_b\left(h'_t+u_bh'_x+v_bh'_y+w_b\right)=$$

$$=\frac{\eta}{\rho}\left(\left(W'_x-w_s\zeta'_x-w_bh'_x\right)'_x+\left(W'_y-w_s\zeta'_y-w_bh'_y\right)'_y\right)+\left(F_s\right)_z+\left(F_b\right)_z+2\Omega U\cos\vartheta,$$

where the boundary viscous stresses on the surface of the liquid are attributed to the friction force of the wind on the surface

$$\mathbf{F}_s=\left(F_s\right)_x\mathbf{i}+\left(F_s\right)_y\mathbf{j}+\left(F_s\right)_z\mathbf{k}=\left(-H\rho^{-1}\left(p_a\right)'_x+\eta\rho^{-1}\left(-\left(u'_x\right)_s\zeta'_x-\left(u'_y\right)_s\zeta'_y+\left(u'_z\right)_s\right)\right)\mathbf{i}+$$

$$+\left(-H\rho^{-1}\left(p_a\right)'_y+\eta\rho^{-1}\left(-\left(v'_x\right)_s\zeta'_x-\left(v'_y\right)_s\zeta'_y+\left(v'_z\right)_s\right)\right)\mathbf{j}+\left(\eta\rho^{-1}\left(-\left(w'_x\right)_s\zeta'_x-\left(w'_y\right)_s\zeta'_y+\left(w'_z\right)_s\right)\right)\mathbf{k},$$

and viscous stresses at the bottom are attributed to the friction force on the bottom

$$\mathbf{F}_b=\left(F_b\right)_x\mathbf{i}+\left(F_b\right)_y\mathbf{j}+\left(F_b\right)_z\mathbf{k}=\eta\rho^{-1}\left(\left(-\left(u'_x\right)_bh'_x-\left(u'_y\right)_bh'_y-\left(u'_z\right)_b\right)\mathbf{i}+\right.$$

$$\left.+\left(-\left(v'_x\right)_s\zeta'_x-\left(v'_y\right)_s\zeta'_y+\left(v'_z\right)_s\right)\mathbf{j}+\left(-\left(w'_x\right)_s\zeta'_x-\left(w'_y\right)_s\zeta'_y+\left(w'_z\right)_s\right)\mathbf{k}\right).$$

Taking into account the kinematic conditions on the surface and bottom

$$-u_s\zeta'_x-v_s\zeta'_y+w_s=\zeta'_t+\omega\rho^{-1},\ u_bh'_x+v_bh'_y+w_b=-h'_t,$$

where $\omega\rho^{-1}$ is the layer of liquid evaporating per unit of time, we get

$$H'_t+U'_x+V'_y+\frac{\omega}{\rho}=0.$$

The following empirical equation was used to determine the evaporation rate from a unit area:

$$\omega\left(\frac{g}{h}\right)=e\left(P_{us}-P_{set}\right),$$

where P_{us} is the vapor pressure of saturated air, mbar; P_{set} is the partial pressure of water vapor at a given temperature and humidity, mbar; e is the empirical coefficient, g/m^2/h/ mbar, which depends on the intensity of spray formation in the pool.

Consider the two-dimensional problem of determining the evaporation rate from the surface of water when air moves at a constant speed at wind speed V, air humidity f, air temperature T_a, water temperature T_w. The evaporation rate from the surface of the pool W is determined in g/sec/m^2 (Fig. 1).

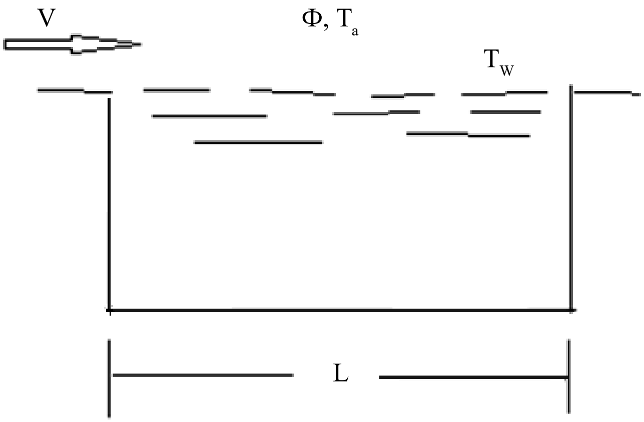

Fig. 1. The boundary between water and air

Let us determine the empirical dependencies for calculating the evaporation rate according to the formula based on the unit area based on experimental data:

$$\omega = \frac{\left(A + B \cdot V\right)\left(P_w - \varphi \cdot P_a\right)}{r_w},$$

where P_w is the saturated steam pressure at water temperature; P_a is the saturated steam pressure at air temperature; r_w is the heat of vaporization (r_w = 2.2582 J/kg at normal atmospheric pressure); A and B are empirical constants. The spread of evaporation rates across different sources is +100 % –80 %.

There are a number of standards that give similar results in the middle of this range: WMO (1966) USSR, Sartori (1989), McMillan (1971), etc. According to the WMO standard (1966) of the USSR, the coefficients A = 0.0369, B = 0.0266. It should be noted that the evaporation rate calculated according to the specified standard for V = 0 m/s is consistent with the evaporation rate determined according to the VDI 2089 standard for a fixed (undisturbed) surface, with an accuracy of 10–15 %.

Calculations can be performed in both laminar and turbulent formulations with calibration of the Schmidt number S_c and the turbulent Schmidt number Sc_t. This number is calibrated depending on the difference in water and wind speeds in the area of the interface between the media. Based on the available tools of the STAR-CCM hydrodynamics package, the velocity in the interface area can be determined as

$$V_b = \nabla V \cdot G^{(1/3)},$$

where ∇V is the velocity gradient determined from the current velocity field; $G^{1/3}$ is the characteristic cell size calculated from its volume. The dependence of the turbulent Schmidt number on the velocity in the interface region for predicting the evaporation rate on waves. For example, with a wave height of 1.5 m, a length of 10 m, and a speed of 3 m/s, we get:

$$Sc_t(V_B) = (-0.333\, V_B^2 + 6.667\, V_B + 3) \cdot 3.5 .$$

The above formula is used to predict the evaporation rate in the presence of waves. We do not consider further refinement of the evaporation process and will continue to obtain a 2D model:

$$U'_t + \left(\int_{-h}^{\zeta} u^2 dz\right)'_x + \left(\int_{-h}^{\zeta} uvdz\right)'_y + \frac{\omega}{\rho} u_s = -gH\zeta'_x + \frac{\eta}{\rho}\Big(\left(U''_{xx} - u_s\zeta''_{xx} - u_b h''_{xx}\right) + \left(U''_{yy} - u_s\zeta''_{yy} - u_b h''_{yy}\right)\Big) +$$
$$+\left(\left(F_s\right)_x - \frac{\eta}{\rho}\left(\left(u_s\right)'_x\zeta'_x + \left(u_s\right)'_y\zeta'_y\right)\right) + \left(\left(F_b\right)_x - \frac{\eta}{\rho}\left(\left(u_b\right)'_x h'_x + \left(u_b\right)'_y h'_y\right)\right) + 2\Omega\left(V\sin\vartheta - W\cos\vartheta\right),$$

$$V'_t + \left(\int_{-h}^{\zeta} uvdz\right)'_x + \left(\int_{-h}^{\zeta} v^2 dz\right)'_y + \frac{\omega}{\rho} v_s = -gH\zeta'_y + \frac{\eta}{\rho}\Big(\left(V''_{xx} - v_s\zeta''_{xx} - v_b h''_{xx}\right) + \left(V''_{yy} - v_s\zeta''_{yy} - v_b h''_{yy}\right)\Big) +$$
$$+\left(\left(F_s\right)_y - \frac{\eta}{\rho}\left(\left(v_s\right)'_x\zeta'_x + \left(v_s\right)'_y\zeta'_y\right)\right) + \left(\left(F_b\right)_y - \frac{\eta}{\rho}\left(\left(v_b\right)'_x h'_x + \left(v_b\right)'_y h'_y\right)\right) - 2\Omega U\sin\vartheta,$$

$$W'_t + \left(\int_{-h}^{\zeta} uwdz\right)'_x + \left(\int_{-h}^{\zeta} vwdz\right)'_y + \frac{\omega}{\rho} w_s = \frac{\eta}{\rho}\Big(\left(W''_{xx} - w_s\zeta''_{xx} - w_b h''_{xx}\right) + \left(W''_{yy} - w_s\zeta''_{yy} - w_b h''_{yy}\right)\Big) +$$
$$+\left(\left(F_s\right)_z - \frac{\eta}{\rho}\left(\left(w_s\right)'_x\zeta'_x + \left(w_s\right)'_y\zeta'_y\right)\right) + \left(\left(F_b\right)_z - \frac{\eta}{\rho}\left(\left(w_b\right)'_x h'_x + \left(w_b\right)'_y h'_y\right)\right) + 2\Omega U\cos\zeta .$$

Isolating in derivatives

$$\left(f_s\right)'_x = \left(f'_x\right)_s + \left(f'_z\right)_s \zeta'_x, \quad \left(f_b\right)'_x = \left(f'_x\right)_b - \left(f'_z\right)_b h'_x,$$
$$\left(f_s\right)'_y = \left(f'_y\right)_s + \left(f'_z\right)_s \zeta'_y, \quad \left(f_b\right)'_y = \left(f'_y\right)_b - \left(f'_y\right)_b h'_y,$$

of complex functions $f_s = f(x, y, \xi, t), t)$, $f_b = f(x, y, -h(x, y, t)\, t)$ terms having the form and dimension of viscous stresses, therefore, changing continuously when crossing the boundaries of the "atmosphere — liquid" and "liquid — bottom" interface, and attributing them to the generalized forces of friction of wind on the surface $\mathbf{F}_s^*$ and liquid on the bottom $\mathbf{F}_b^*$, we obtain:

$$H'_t + U'_x + V'_y + \frac{\omega}{\rho} = 0,$$

$$U'_t + \left(\int_{-h}^{\zeta} u^2 dz\right)'_x + \left(\int_{-h}^{\zeta} uvdz\right)'_y + \frac{\omega}{\rho} u_s = -gH\zeta'_x + \frac{\eta}{\rho}\left(\left(U''_{xx} + U''_{yy}\right) - u_s\left(\zeta''_{xx} + \zeta''_{yy}\right) - u_b\left(h''_{xx} + h''_{yy}\right)\right) +$$
$$+ \left(F_s^*\right)_x + \left(F_b^*\right)_x + 2\Omega\left(V \sin\vartheta - W \cos\vartheta\right), \tag{4}$$

$$V'_t + \left(\int_{-h}^{\zeta} uvdz\right)'_x + \left(\int_{-h}^{\zeta} v^2 dz\right)'_y + \frac{\omega}{\rho} v_s = -gH\zeta'_y + \frac{\eta}{\rho}\left(\left(V''_{xx} + V''_{yy}\right) - v_s\left(\zeta''_{xx} + \zeta''_{yy}\right) - v_b\left(h''_{xx} + h''_{yy}\right)\right) +$$
$$+ \left(F_s^*\right)_y + \left(F_b^*\right)_y - 2\Omega U \sin\vartheta, \tag{5}$$

$$W'_t + \left(\int_{-h}^{\zeta} uwdz\right)'_x + \left(\int_{-h}^{\zeta} vwdz\right)'_y + \frac{\omega}{\rho} w_s = \frac{\eta}{\rho}\left(\left(W''_{xx} + W''_{yy}\right) - w_s\left(\zeta''_{xx} + \zeta''_{yy}\right) - w_b\left(h''_{xx} + h''_{yy}\right)\right) +$$
$$+ \left(F_s^*\right)_z + \left(F_b^*\right)_z + 2\Omega U \cos\vartheta, \tag{6}$$

where

$$\mathbf{F}_s^* = \left(\left(F_s\right)_x - \eta\rho^{-1}\left(\left(u'_x\right)_s \zeta'_x + \left(u'_y\right)_s \zeta'_y + \left(u'_z\right)_s\left(\left(\zeta'_x\right)^2 + \left(\zeta'_y\right)^2\right)\right)\right)\mathbf{i} +$$
$$+ \left(\left(F_s\right)_y - \eta\rho^{-1}\left(\left(v'_x\right)_s \zeta'_x + \left(v'_y\right)_s \zeta'_y + \left(v'_z\right)_s\left(\left(\zeta'_x\right)^2 + \left(\zeta'_y\right)^2\right)\right)\right)\mathbf{j} +$$
$$+ \left(\left(F_s\right)_z - \eta\rho^{-1}\left(\left(w'_x\right)_s \zeta'_x + \left(w'_y\right)_s \zeta'_y + \left(w'_z\right)_s\left(\left(\zeta'_x\right)^2 + \left(\zeta'_y\right)^2\right)\right)\right)\mathbf{k},$$
$$\mathbf{F}_b^* = \left(\left(F_b\right)_x - \eta\rho^{-1}\left(\left(u'_x\right)_b h'_x + \left(u'_y\right)_b h'_y + \left(u'_z\right)_b\left(\left(h'_x\right)^2 + \left(h'_y\right)^2\right)\right)\right)\mathbf{i} +$$
$$+ \left(\left(F_b\right)_y - \eta\rho^{-1}\left(\left(v'_x\right)_b h'_x + \left(v'_y\right)_b h'_y + \left(v'_z\right)_b\left(\left(h'_x\right)^2 + \left(h'_y\right)^2\right)\right)\right)\mathbf{j} +$$
$$+ \left(\left(F_b\right)_z - \eta\rho^{-1}\left(\left(w'_x\right)_b h'_x + \left(w'_y\right)_b h'_y + \left(w'_z\right)_b\left(\left(h'_x\right)^2 + \left(h'_y\right)^2\right)\right)\right)\mathbf{k},$$

they are equal in magnitude and directed opposite to the forces acting from the side of the column of liquid on the column of atmospheric air above it and the section of the bottom below it. The terms that change abruptly when crossing the boundaries of the "atmosphere — liquid" and "liquid — bottom" interface are left to the account of the forces of internal viscous friction.

In case of

$$W \cos\vartheta << V \sin\vartheta$$

the solutions of equations (4) and (5) do not depend on the solution of equation (6), which we exclude

$$H'_t + U'_x + V'_y + \frac{\omega}{\rho} = 0, \tag{7}$$

$$U'_t + \left(\int_{-h}^{\zeta} u^2 dz\right)'_x + \left(\int_{-h}^{\zeta} uvdz\right)'_y + \frac{\omega}{\rho} u_s = -gH\zeta'_x + \frac{\eta}{\rho}\left(\Delta U - u_s \Delta\zeta - u_b \Delta h\right) +$$
$$+ \left(F_s^*\right)_x + \left(F_b^*\right)_x + 2\Omega V \sin\vartheta,$$

$$V'_t + \left(\int_{-h}^{\zeta} uvdz\right)'_x + \left(\int_{-h}^{\zeta} v^2 dz\right)'_y + \frac{\omega}{\rho} v_s = -gH\zeta'_y + \frac{\eta}{\rho}\left(\Delta V - v_s \Delta\zeta - v_b \Delta h\right) +$$
$$+ \left(F_s^*\right)_y + \left(F_b^*\right)_y - 2\Omega U \sin\vartheta, \tag{8}$$

where $\Delta = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is two-dimensional Laplace operator.

Introducing coefficients C_{uu}, C_{uv}, C_{vv}, C_u, C_v:

$$\int_{-h}^{\zeta} u^2 dz = C_{uu} H^{-1} U^2, \quad \int_{-h}^{\zeta} uv dz = C_{uv} H^{-1} UV, \quad \int_{-h}^{\zeta} v^2 dz = C_{vv} H^{-1} V^2, \; u_s = C_u H^{-1} U, \, v_s = C_v H^{-1} V,$$

equations (7)–(9) can be rewritten as

$$H'_t + U'_x + V'_y + (\omega/\rho) = 0,$$

$$U'_t + \left(C_{uu} U^2/H\right)'_x + \left(C_{uv} UV/H\right)'_y + (\omega/\rho) C_u (U/H) = -gH\zeta'_x + (\eta/\rho)\left(\Delta U - C_u (U/H)\Delta\zeta - u_b \Delta h\right) +$$
$$+ \left(F_s^*\right)_x + \left(F_b^*\right)_x + 2\Omega V \sin\vartheta,$$

$$V'_t + \left(C_{uv} UV/H\right)'_x + \left(C_{vv} V^2/H\right)'_y + (\omega/\rho) C_v (V/H) = -gH\zeta'_y + (\eta/\rho)\left(\Delta V - C_v (V/H)\Delta\zeta - v_b \Delta h\right) +$$
$$+ \left(F_s^*\right)_y + \left(F_b^*\right)_y - 2\Omega U \sin\vartheta. \tag{9}$$

Due to the Cauchy-Bunyakovsky inequality, there are the following restrictions for the coefficients C_{uu} and C_{vv}:

$$U^2 = \left(\int_{-h}^{\zeta} u dz\right)^2 \le H \int_{-h}^{\zeta} u^2 dz \Rightarrow C_{uu} \ge 1, \; V^2 = (v)^2 \le H \int_{-h}^{\zeta} v^2 dz \Rightarrow C_{vv} \ge 1,$$

and due to the positive semi-definiteness of the quadratic form

$$H \int_{-h}^{\zeta} (u - v)^2 dz = H\left(\int_{-h}^{\zeta} u^2 dz - 2\int_{-h}^{\zeta} uv dz + \int_{-h}^{\zeta} v^2 dz\right) = C_{uu} U^2 - 2C_{uv} UV + C_{vv} V^2 \ge 0,$$

— restriction for C_{uv}

$$C_{uv}^{\;2} \le C_{uu} C_{vv}.$$

The next stage of the study is to obtain and analyze the balance equation of total mechanical energy with certain simplifications.

When $C_{uu} \equiv C_{uv} \equiv C_{vv} \equiv 1$ for a simplified model, we get:

$$H'_t + U'_x + V'_y + (\omega/\rho) = 0,$$

$$U'_t + \left(U^2/H\right)'_x + \left(UV/H\right)'_y + (\omega/\rho) C_u (U/H) = -gH\zeta'_x + (\eta/\rho)\left(\Delta U - C_u (U/H)\Delta\zeta - u_b \Delta h\right) +$$
$$+ \left(F_s^*\right)_x + \left(F_b^*\right)_x + 2\Omega V \sin\vartheta, \tag{10}$$

$$V'_t + \left(UV/H\right)'_x + \left(V^2/H\right)'_y + (\omega/\rho) C_v (V/H) = -gH\zeta'_y + (\eta/\rho)\left(\Delta V - C_v (V/H)\Delta\zeta - v_b \Delta h\right) +$$
$$+ \left(F_s^*\right)_y + \left(F_b^*\right)_y - 2\Omega U \sin\vartheta. \tag{11}$$

The law of conservation of total mechanical energy is fulfilled — the sum of the potential energy in the resulting gravity field and the positive definite quadratic form of the integrals U and V, acceptable as an estimate of the kinetic energy of a column of liquid.

Multiplying (10) by U/H:

$$(U/H) U'_t + (U/H)\left(U^2/H\right)'_x + (U/H)\left(UV/H\right)'_y + (\omega/\rho) C_u \left(U^2/H^2\right) + gU\zeta'_x =$$
$$= (\eta/\rho)(U/H)\left(\Delta U - C_u (U/H)\Delta\zeta - u_b \Delta h\right) + (U/H)\left(\left(F_s^*\right)_x + \left(F_b^*\right)_x\right) + 2\Omega \sin\vartheta (UV/H),$$

multiplying (11) by V/H:

$$(V/H) V'_t + (V/H)\left(UV/H\right)'_x + (V/H)\left(V^2/H\right)'_y + (\omega/\rho) C_v \left(V^2/H^2\right) + gV\zeta'_y =$$
$$= (\eta/\rho)(V/H)\left(\Delta V - C_v (V/H)\Delta\zeta - v_b \Delta h\right) + (V/H)\left(\left(F_s^*\right)_y + \left(F_b^*\right)_y\right) - 2\Omega \sin\vartheta (UV/H)$$

and taking into account the ratios

$$(U/H) U'_t = \left(U^2/(2H)\right)'_t + \left(U^2/\left(2H^2\right)\right) H'_t,$$

$$(V/H) V'_t = \left(V^2/(2H)\right)'_t + \left(V^2/\left(2H^2\right)\right) H'_t,$$

$$(U/H)\left(U^2/H\right)'_x = \left(U^2/\left(2H^2\right)\right) U'_x + \left((U/H)\left(U^2/(2H)\right)\right)'_x,$$

$$(U/H)(UV/H)'_y = (U^2/(2H^2))V'_y + ((V/H)(U^2/(2H)))'_y,$$
$$(V/H)(UV/H)'_x = (V^2/(2H^2))U'_x + ((U/H)(V^2/(2H)))'_x,$$
$$(V/H)(V^2/H)'_y = (V^2/(2H^2))V'_y + ((V/H)(V^2/(2H)))'_y,$$

we will get

$$(U^2/(2H))'_t + ((U/H)(U^2/(2H)))'_x + ((V/H)(U^2/(2H)))'_y + (U^2/(2H^2))(H'_t + U'_x + V'_y + (\omega/\rho)) + \tag{12}$$
$$+ (\omega/\rho)(C_u - 1/2)(U/H)^2 + g\left((U\zeta)'_x - \zeta U'_x\right) =$$
$$= (\eta/\rho)(U/H)(\Delta U - C_u(U/H)\Delta\zeta - u_b\Delta h) + (U/H)((F_s^*)_x + (F_b^*)_x) + 2\Omega\sin\vartheta(UV/H),$$

$$(V^2/(2H))'_t + ((U/H)(V^2/(2H)))'_x + ((V/H)(V^2/(2H)))'_y + (V^2/(2H^2))(H'_t + U'_x + V'_y + (\omega/\rho)) + \tag{13}$$
$$+ (\omega/\rho)(C_v - 1/2)(V/H)^2 + g\left((V\zeta)'_y - \zeta V'_y\right) =$$
$$= (\eta/\rho)(V/H)(\Delta V - C_v(V/H)\Delta\zeta - v_b\Delta h) + (V/H)((F_s^*)_y + (F_b^*)_y) - 2\Omega\sin\vartheta(UV/H).$$

Adding (12) and (13), we come to

$$((U^2 + V^2)/(2H))'_t + ((U/H)((U^2 + V^2)/(2H) + gH\zeta))'_x + ((V/H)((U^2 + V^2)/(2H) + gH\zeta))'_y +$$
$$+ (\omega/(\rho H))((2C_u - 1)U^2 + (2C_v - 1)V^2)/(2H) - g\zeta(U'_x + V'_y) =$$
$$= (\eta/\rho)\left((U/H)\Delta U + (V/H)\Delta V - \left(C_u(U/H)^2 + C_v(V/H)^2\right)\Delta\zeta - ((U/H)u_b + (V/H)v_b)\Delta h\right) +$$
$$+ \left(U((F_s^*)_x + (F_b^*)_x) + V((F_s^*)_y + (F_b^*)_y)\right)/H.$$

Above the fixed ($h_0' \equiv 0$) bottom is performed

$$-g\zeta(U'_x + V'_y) = g\zeta(\zeta'_t + (\omega/\rho)) = \left(g(\zeta^2 - h^2)/2\right)'_t + (\omega/(\rho H))gH\zeta =$$
$$= (gH(\zeta - h)/2)'_t + (\omega/(\rho H))(gH(\zeta - h)/2 + gH^2/2).$$

As a result, we come to an equation that is an analogue of the equation of the balance of total mechanical energy in differential form

$$(\mathrm{K} + \Pi)'_t + ((U/H)(\mathrm{K} + \Pi + P))'_x + ((V/H)(\mathrm{K} + \Pi + P))'_y + (\omega/(\rho H))(\Pi + P) + \tag{14}$$
$$+ (\omega/(\rho H))((2C_u - 1)U^2 + (2C_v - 1)V^2)/(2H) =$$
$$= (\eta/\rho)\left((U/H)\Delta U + (V/H)\Delta V - \left(C_u(U/H)^2 + C_v(V/H)^2\right)\Delta\zeta - ((U/H)u_b + (V/H)v_b)\Delta h\right) +$$
$$+ \left(U((F_s^*)_x + (F_b^*)_x) + V((F_s^*)_y + (F_b^*)_y)\right)/H,$$

where $K = (U^2 + V^2)/(2H)$, $\Pi = gH(\xi - h)/2$, $\mathrm{P} = gH^2/2$, $\Pi + P = gH\xi$.

For a positive function $E = E(x, y, t) > 0$, satisfying the transfer equation

$$E'_t + (U/H)E'_x + (V/H)E'_y = 0,$$

equation (4) is also valid for generalizing estimates of kinetic energy

$$K = E \cdot (U^2 + V^2)/(2H).$$

If we consider the boundary ∂G of the region G to be fixed then

$$\iint_G (\mathrm{K} + \Pi)'_t\, dxdy = \left(\iint_G (\mathrm{K} + \Pi)\, dxdy\right)'_t,$$

using the Green function

$$\iint_G \left(((U/H)(\mathrm{K} + \Pi + P))'_x + ((V/H)(\mathrm{K} + \Pi + P))'_y\right) dxdy = \oint_{\partial G} (\mathrm{K} + \Pi + P)(Udx - Vdy)/H =$$

$$= \oint_{\partial G} (\mathrm{K} + \Pi + P)(U\mathbf{i} + V\mathbf{j}, \mathbf{n}) dl / H,$$

$$\iint_G ((U/H)\Delta U + (V/H)\Delta V) dxdy =$$

$$= \oint_{\partial G} (\nabla \mathrm{K}, \mathbf{n}) dl - \iint_G H\left(|\nabla (U/H)|^2 + |\nabla (V/H)|^2\right) dxdy + \iint_G (\mathrm{K}/H)\Delta H dxdy,$$

where $\mathbf{n}$ is the external normal to the boundary ∂G of the region G and assuming $C_u \equiv C_v \equiv C$, we obtain the balance equation of the analogue of the total mechanical energy of the liquid in integral form:

$$\left(\iint_G (\mathrm{K} + \Pi) dxdy\right)'_t + \oint_{\partial G} (\mathrm{K} + \Pi + P)(U\mathbf{i} + V\mathbf{j}, \mathbf{n}) dl/H + \iint (\omega/(\rho H))((2C-1)\mathrm{K} + \Pi + P) dxdy = \tag{15}$$

$$= (\eta/\rho)\left(\oint_{\partial G} (\nabla \mathrm{K}, \mathbf{n}) dl - \iint_G H\left(|\nabla (U/H)|^2 + |\nabla (V/H)|^2\right) dxdy - (2C-1)\iint_G (\mathrm{K}/H)\Delta\zeta dxdy + \right.$$

$$\left. + \iint_G ((\mathrm{K}/H) - ((U/H)u_b + (V/H)v_b))\Delta h dxdy\right) +$$

$$+ \iint_G \left(U\left(\left(F_s^*\right)_x + \left(F_b^*\right)_x\right) + V\left(\left(F_s^*\right)_y + \left(F_b^*\right)_y\right)\right) dxdy / H.$$

If the conditions of «sticking» are met on the bottom surface

$$u_b \equiv v_b \equiv w_b \equiv 0,$$

then the term

$$\iint_G ((U/H)u_b + (V/H)v_b)\Delta h dxdy = 0$$

there is no balance equation (15), and above the bottom surface, which is a harmonic function

$$\Delta h \equiv 0 \tag{16}$$

there is also no term

$$\iint_G (\mathrm{K}/H)\Delta h dxdy = 0$$

and the model

$$\zeta'_t + U'_x + V'_y + (\omega/\rho) = 0 \ \text{ or } \ H'_t + U'_x + V'_y + (\omega/\rho) = 0, \tag{17}$$

$$U'_t + \left(U^2/H\right)'_x + (UV/H)'_y + (\omega/\rho)C(U/H) = -gH\zeta'_x + (\eta/\rho)(\Delta U - C(U/H)\Delta\zeta) + \tag{18}$$

$$+ \left(F_s^*\right)_x + \left(F_b^*\right)_x + 2\Omega V \sin\vartheta,$$

$$V'_t + (UV/H)'_x + \left(V^2/H\right)'_y + (\omega/\rho)C(V/H) = -gH\zeta'_y + (\eta/\rho)(\Delta V - C(V/H)\Delta\zeta) + \tag{19}$$

$$+ \left(F_s^*\right)_y + \left(F_b^*\right)_y - 2\Omega U \sin\vartheta$$

turns out to be strictly dissipative due to the action of internal viscous friction forces.

The corresponding (17)–(19) system of equations in averaged values of velocities $\bar{u} = U/H$ and $\bar{v} = V/H$ will have the form:

$$\zeta'_t + \left(H\bar{u}\right)'_x + \left(H\bar{v}\right)'_y + (\omega/\rho) = 0 \ \text{ or } \ H'_t + \left(H\bar{u}\right)'_x + \left(H\bar{v}\right)'_y + (\omega/\rho) = 0, \tag{20}$$

$$\left(H\bar{u}\right)'_t + \left(H\bar{u}^2\right)'_x + \left(H\bar{u}\bar{v}\right)'_y + (\omega/\rho)C\bar{u} = -gH\zeta'_x + (\eta/\rho)\left(\Delta\left(H\bar{u}\right) - C\bar{u}\Delta\zeta\right) + \tag{21}$$

$$+ \left(F_s^*\right)_x + \left(F_b^*\right)_x + 2\Omega H\bar{v} \sin\vartheta,$$

$$\left(H\bar{v}\right)'_t + \left(H\bar{u}\bar{v}\right)'_x + \left(H\bar{v}^2\right)'_y + (\omega/\rho)C\bar{v} = -gH\zeta'_y + (\eta/\rho)\left(\Delta\left(H\bar{v}\right) - C\bar{v}\Delta\zeta\right) + \tag{22}$$

$$+\left(F_s^*\right)_y+\left(F_b^*\right)_y-2\Omega H\overline{u}\sin\vartheta$$

or, by virtue of the continuity equation:

$$\zeta_t'+\left(H\overline{u}\right)_x'+\left(H\overline{v}\right)_y'+\left(\omega/\rho\right)=0 \text{ or } H_t'+\left(H\overline{u}\right)_x'+\left(H\overline{v}\right)_y'+\left(\omega/\rho\right)=0, \quad (23)$$

$$\overline{u}_t'+\overline{u}\overline{u}_x'+\overline{v}\overline{u}_y'+\left(\omega/\rho\right)\left(C-1\right)\left(\overline{u}/H\right)=-g\zeta_x'+\left(\eta/\left(\rho H\right)\right)\left(\Delta\left(H\overline{u}\right)-C\overline{u}\Delta\zeta\right)+ \quad (24)$$
$$+\left(\left(F_s^*\right)_x+\left(F_b^*\right)_x\right)/H+2\Omega\overline{v}\sin\vartheta,$$

$$\overline{v}_t'+\overline{u}\overline{v}_x'+\overline{v}\overline{v}_y'+\left(\omega/\rho\right)\left(C-1\right)\left(\overline{v}/H\right)=-g\zeta_y'+\left(\eta/\left(\rho H\right)\right)\left(\Delta\left(H\overline{v}\right)-C\overline{v}\Delta\zeta\right)+ \quad (25)$$
$$+\left(\left(F_s^*\right)_y+\left(F_b^*\right)_y\right)/H-2\Omega\overline{u}\sin\vartheta.$$

Other spatially two-dimensional hydrodynamic models of coastal systems and shallow waters can also be obtained.

Introducing simplifications

$$\int_{-h}^{\zeta}u_x'dz\to H\left(U/H\right)_x',\quad \int_{-h}^{\zeta}u_y'dz\to H\left(U/H\right)_y',\quad \int_{-h}^{\zeta}v_x'dz\to H\left(V/H\right)_x',\quad \int_{-h}^{\zeta}v_y'dz\to H\left(V/H\right)_y'$$

at stage (1)–(3) and reasoning similarly to the above, we come to the following model

$$\zeta_t'+U_x'+V_y'+\left(\omega/\rho\right)=0, \quad (26)$$

$$U_t'+\left(U^2/H\right)_x'+\left(UV/H\right)_y'+\left(\omega/\rho\right)C\left(U/H\right)=-gH\zeta_x'+\left(\eta/\rho\right)\left(\left(H\left(U/H\right)_x'\right)_x'+\left(H\left(U/H\right)_y'\right)_y'\right)+$$
$$+\left(F_s^*\right)_x+\left(F_b^*\right)_x+2\Omega V\sin\vartheta, \quad (27)$$

$$V_t'+\left(UV/H\right)_x'+\left(V^2/H\right)_y'+\left(\omega/\rho\right)C\left(V/H\right)=-gH\zeta_y'+\left(\eta/\rho\right)\left(\left(H\left(V/H\right)_x'\right)_x'+\left(H\left(V/H\right)_y'\right)_y'\right)+$$
$$+\left(F_s^*\right)_y+\left(F_b^*\right)_y-2\Omega U\sin\vartheta \quad (28)$$

or in averaged values of velocities

$$H_t'+\left(H\overline{u}\right)_x'+\left(H\overline{v}\right)_y'+\left(\omega/\rho\right)=0, \quad (29)$$

$$\overline{u}_t'+\overline{u}\overline{u}_x'+\overline{v}\overline{u}_y'+\left(\omega/\rho\right)\left(C-1\right)\left(\overline{u}/H\right)=-g\zeta_x'+\left(\eta/\left(\rho H\right)\right)\left(\left(H\overline{u}_x'\right)_x'+\left(H\overline{u}_y'\right)_y'\right)+$$
$$+\left(\left(F_s^*\right)_x+\left(F_b^*\right)_x\right)/H+2\Omega\overline{v}\sin\vartheta, \quad (30)$$

$$\overline{v}_t'+\overline{u}\overline{v}_x'+\overline{v}\overline{v}_y'+\left(\omega/\rho\right)\left(C-1\right)\left(\overline{v}/H\right)=-g\zeta_y'+\left(\eta/\left(\rho H\right)\right)\left(\left(H\overline{v}_x'\right)_x'+\left(H\overline{v}_y'\right)_y'\right)+$$
$$+\left(\left(F_s^*\right)_y+\left(F_b^*\right)_y\right)/H-2\Omega\overline{u}\sin\vartheta, \quad (31)$$

taking into account the equalities and assuming that the analogue of the total mechanical energy balance equation is fulfilled

$$\iint_G\left(\left(U/H\right)\left(\left(H\left(U/H\right)_x'\right)_x'+\left(H\left(U/H\right)_y'\right)_y'\right)+\left(V/H\right)\left(\left(H\left(V/H\right)_x'\right)_x'+\left(H\left(V/H\right)_y'\right)_y'\right)\right)dxdy=$$
$$=\oint_{\partial G}H\left(\nabla\left(\mathrm{K}/H\right),\mathbf{n}\right)dl-\iint_G H\left(\left|\nabla\left(U/H\right)\right|^2+\left|\nabla\left(V/H\right)\right|^2\right)dxdy,$$

in the form

$$\left(\iint_G\left(\mathrm{K}+\Pi\right)dxdy\right)_t'+\oint_{\partial G}\left(\mathrm{K}+\Pi+P\right)\left(U\mathbf{i}+V\mathbf{j},\mathbf{n}\right)dl/H+\iint\left(\omega/\left(\rho H\right)\right)\left(\left(2C-1\right)\mathrm{K}+\Pi+P\right)dxdy=$$
$$=\left(\eta/\rho\right)\left(\oint_{\partial G}H\left(\nabla\left(\mathrm{K}/H\right),\mathbf{n}\right)dl-\iint_G H\left(\left|\nabla\left(U/H\right)\right|^2+\left|\nabla\left(V/H\right)\right|^2\right)dxdy-\left(2C-1\right)\iint_G\left(\mathrm{K}/H\right)\Delta\zeta dxdy\right)+$$
$$+\iint_G\left(U\left(\left(F_s^*\right)_x+\left(F_b^*\right)_x\right)+V\left(\left(F_s^*\right)_y+\left(F_b^*\right)_y\right)\right)dxdy/H.$$

Another family of models can be obtained by leaving on account of the forces of internal viscous friction only the terms that do not interfere with obtaining a balance equation with strict dissipation of the analogue of the total mechanical energy of the system due to the action of internal viscous friction forces and transferring the remaining terms to the evaporation intensity, where an excess type term is added (under the surface of the liquid convex upwards) or insufficient (under the surface of the liquid convex downwards) Laplace pressure:

$$U'_t + \left(U^2/H\right)'_x + \left(UV/H\right)'_y + (\omega/\rho)^* C(U/H) = -gH\zeta'_x + (\eta/\rho)\left(\Delta U - (1/2)(U/H)\Delta H\right) + \left(F_s^*\right)_x + \left(F_b^*\right)_x + 2\Omega V \sin\vartheta, \quad (32)$$

$$V'_t + \left(UV/H\right)'_x + \left(V^2/H\right)'_y + (\omega/\rho)^* C(V/H) = -gH\zeta'_y + (\eta/\rho)\left(\Delta V - (1/2)(V/H)\Delta H\right) + \left(F_s^*\right)_y + \left(F_b^*\right)_y - 2\Omega U \sin\vartheta,$$

$$H'_t + U'_x + V'_y + (\omega/\rho)^* - (\eta/\rho)\left(\left(1-(2C)^{-1}\right)\Delta H - \Delta h\right) = 0 \quad (33)$$

or

$$\zeta'_t + U'_x + V'_y + (\omega/\rho)^* - (\eta/\rho)\left(\left(1-(2C)^{-1}\right)\Delta\zeta - (2C)^{-1}\Delta h\right) = 0.$$

The equation of the analogue of the total mechanical energy balance for the model (32)–(34) differs from (15) by replacing (ω/ρ) with

$$(\omega/\rho)^* - (\eta/\rho)\left(\left(1-(2C)^{-1}\right)\Delta H - \Delta h\right) = (\omega/\rho)^* - (\eta/\rho)\left(\left(1-(2C)^{-1}\right)\Delta\zeta - (2C)^{-1}\Delta h\right). \quad (34)$$

In the course of the work, a two-dimensional model of the hydrodynamic process was constructed and studied, taking into account the essential features of coastal systems, based on the balance of mass, energy and momentum. The proposed model can be used for predictive modeling of hydrophysical processes, including the spread of pollutants in the aquatic environment of marine and coastal systems.

Discussion and Conclusion. The peculiarity of the obtained spatially two-dimensional models of hydrodynamics takes into account the fact that the operations of differentiation by spatial variables in horizontal directions are not commutative with respect to the operation of integration along a vertical spatial coordinate. In coastal systems, where there is a significant difference in depth, an arbitrary change in the order of these operations, performed to obtain spatially two-dimensional equations of motion of the aquatic environment, can lead to the appearance of fictitious, physically unreasonable sources of momentum in the Navier-Stokes equations. The method of constructing two-dimensional equations of motion developed by the authors makes it possible to eliminate this negative effect, and maintaining the order of operations ensures that evaporation from a free surface is correctly accounted for not only in the continuity equation, but also in the equations of motion taking into account wind and waves.

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