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Surface Heating Calculation of Composite Material Under the Influence of an Electron Beam on the Surface

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Abstract

Introduction. Modern plasma magnetic confinement systems use tungsten as a material in contact with plasma. Under the influence of high-density plasma irradiation, tungsten undergoes cracking, intense erosion, and macro-particle emission. High-temperature ceramics are considered a promising material for protective coating of plasma components, as they are resistant to thermal loads. One possible solution could be a boron carbide coating, which has a high melting temperature.

Materials and Methods. The impact of an electron beam on samples of rolled tungsten and boron carbide and tungsten composite was studied in experiments on the BETA setup. The heat from the beam propagates into the samples, with the maximum temperature reached at the center and decreasing towards the edges. The modeling area represents a cross-section of the samples, optimal for a task with a cylindrical coordinate system. The numerical implementation is based on the correction scheme and the marching method.

Results. A new model of heating the boron carbide and tungsten composite sample under the influence of surface heating by an electron beam is presented. The model is based on solving the heat conduction equation in an axially symmetric setup with constant values of specific heat capacity, density, and thermal conductivity of metals.

Discussion and Conclusions. An analysis of the model of heating the composite material under the influence of surface heating by an electron beam at constant values of density, thermal conductivity, and specific heat capacity has been conducted. The modeling results are in demand for analyzing experimental results and planning experiments at the Beam of Electrons for Materials Test Applications (BETA) facility, created at the Budker Institute of Nuclear Physics SB RAS.

Keywords: Mathematical modelling, heat equation, boron carbide, tungsten, pulse heating, BETA facility

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Научная статья

Расчет нагрева композитного материала при воздействии на поверхность электронным пучком

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Аннотация

Введение. Современные системы магнитного удержания плазмы используют вольфрам как контактирующий с плазмой материал. Под действием плазменного облучения высокой плотности мощности вольфрам растрескивается, происходит его интенсивная эрозия и эмиссия макрочастиц. Высокотемпературная керамика считается

перспективным материалом для защитного покрытия плазменных компонентов, так как устойчива к термическим нагрузкам. Одним из возможных решений может быть покрытие из карбида бора, который имеет высокую температуру плавления.
Материалы и методы. В экспериментах на установке BETA исследовалось воздействие электронного пучка на образцы прокатанного вольфрама и композита карбида бора и вольфрама. Тепло от пучка распространяется вглубь образцов, максимальная температура достигается в центре и снижается к краям. Область моделирования представляет собой поперечное сечение образцов, оптимальное для задачи с цилиндрической системой координат. Численная реализация основана на схеме стабилизирующей поправки и метода прогонки.
Результаты исследования. Представлена новая модель прогрева образца композита карбида бора и вольфрама при нагреве поверхности электронным пучком. Модель основана на решении уравнения теплопроводности в аксиально-симметричной постановке при постоянных значениях удельной теплоемкости, плотности, теплопроводности металлов.
Обсуждение и заключения. Проведен анализ модели нагрева композитного материала при нагреве поверхности электронным пучком при постоянных значениях плотности, теплопроводности и теплоемкости. Результаты моделирования востребованы для анализа результатов и при планировании экспериментов на стенде Beam of Electrons for materials Test Applications (BETA), созданного в ИЯФ СО РАН.

Ключевые слова: Математическое моделирование, уравнение теплопроводности, карбид бора, вольфрам, импульсный нагрев, стенд BETA

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Introduction. Modern plasma magnetic confinement systems involve the contact of peripheral plasma with the wall. Currently, tungsten is assumed to be the material in contact with the plasma for the ITER (International Thermonuclear Experimental Reactor) divertor and some tokamak plasma components. Studies in recent years have shown that under the influence of high-density plasma irradiation, tungsten experiences cracking, intense erosion, macroscopic particle emission, melting at relatively low temperatures, and other processes that can ultimately lead to accelerated deterioration of tungsten ITER divertor tiles [1–3]. High-temperature ceramics are a promising alternative material for protective coating of plasma components. One of the main advantages of ceramics over other materials in this case is its high resistance to intense thermal loads [4]. To protect ITER tungsten plates from plasma exposure and thus prevent the development of the listed processes, a renewable boron carbide (B4C) coating can be used. It has a high melting temperature (~3000 K), is not susceptible to chemical vapor deposition, and its physical vapor deposition is significantly lower than that of graphite and only slightly increases at temperatures of 1700 K [5]. Additionally, compared to graphite, boron carbide poorly retains hydrogen [6].

Research conducted at the Budker Institute of Nuclear Physics of the Siberian Branch of the Russian Academy of Sciences (BINP SB RAS) focuses on studying surface erosion of tungsten samples under the influence of a laser pulse or an electron beam. Experiments are conducted at the Beam of Electrons for Materials Test Applications (BETA) facility, established at the BINP SB RAS [1]. Modeling of tungsten heating and melting has been performed based on solving a two-phase Stefan problem in the sample domain [2]. The novelty and complexity of solving this problem lie in the necessity to formulate nonlinear boundary conditions describing heating and material evaporation on its surface [7]. Temperature calculations are used to determine the current in the tungsten sample and the evaporated substance [8]. Thermal currents are determined from electrodynamics equations considering electrical resistance and thermoelectromotive force calculated through an energy integral over electrons. The current is considered a potential source of material rotation observed in experiments.

A new research direction at the BINP SB RAS involves studying the heating process of boron carbide and tungsten composite samples [4]. Experimental studies are accompanied by computational experiments. The practical focus of the work requires the model problem formulation to closely match the experimental conditions. The results of calculations from the final model will be used to analyze data obtained at the BETA experimental facility at the BINP SB RAS.

Materials and Methods. In experiments conducted at the BETA facility, samples of rolled tungsten [8] and boron carbide-tungsten composite [4] were subjected to the influence of an axisymmetric electron beam. Electrons with energies of 80–90 keV heat the material in a layer that is thin compared to the characteristic heating depth of the material. The heat absorbed by the surface propagates into the material. The sample dimensions are 25 mm × 25 mm with a typical thickness of 4 mm. Since the sample heats to a depth of several hundred microns in such a short time, the modelling domain represented a cross-section of the sample: an area of 12 mm × 2 mm (Fig. 1).

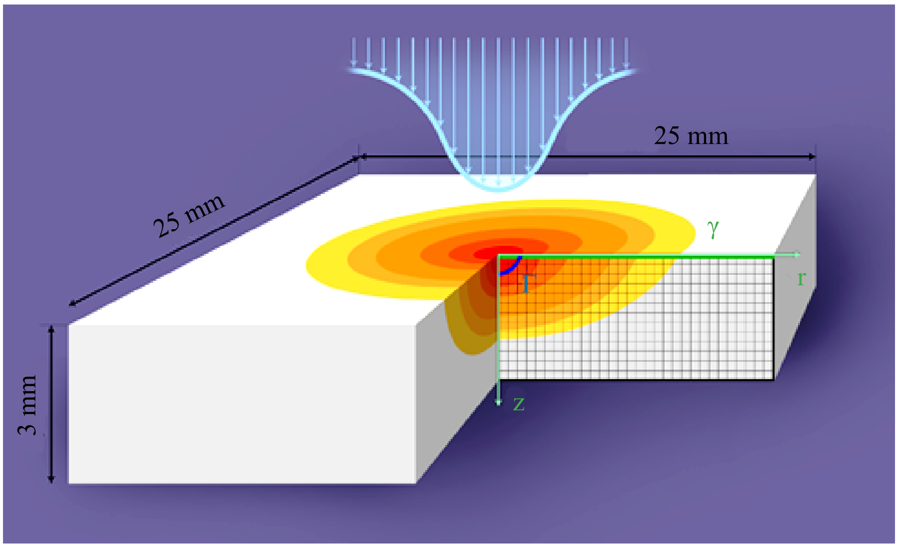

Fig. 1. Experimental setup diagram

The process of heat propagation on the surface and into the depth of the sample is described by the heat conduction equation in an axially symmetric configuration:

$$\begin{cases} c(T)\rho(T)\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\frac{r\lambda(T)\partial T}{\partial r} + \frac{\partial T}{\partial z}\frac{\lambda(T)\partial T}{\partial z}, \\ (n,\nabla T)|_{\gamma} = \frac{W(t,r)}{\lambda(T)}, (n,\nabla T)|_{\Omega-\gamma} = 0, T|_{t=0} = T_0, \end{cases} \quad (1)$$

where Ω represents all the boundaries of the sample; γ denotes the heated surface; $T(t, r, z)$ represents the temperature; $c(T)$ is the specific heat capacity; $\rho(T)$ stands for density; $\lambda(T)$ represents thermal conductivity; $W(t, r)$ is the power density on the surface γ; n denotes the normal to the surface; T_0 is the initial temperature.

The power distribution over the surface of the heat flow is determined as $W(t,r) = W_{max}(t)\exp(-Ar^2)$. Here $A = \frac{1}{a^2} = 0.03088523\,\text{mm}^{-2}$ is a constant characterizing the radius of the beam a. The axially symmetric configuration of the heat flow determines the optimal formulation of the problem in a cylindrical coordinate system, as the power density distribution over the surface has maximum values at the center of the sample and decreases proportionally to the radius. The heating of the sample occurs at the center of the plate and does not reach its edges. The sample is heated to a depth of no more than 1 mm. Solving the problem in a transverse section is determined by the importance of the material heating process into the depth of the sample. The boundary condition on the heated surface and material parameters contribute significantly to the solution.

Let's introduce a uniform rectangular grid with nodes (i, k): $i = 1\ldots N_r$, $k = 1\ldots N_z$ in the two-dimensional domain (r, z): $r \in [r_0, r_{max}]$, $z \in [z_0, z_{max}]$, $r_0 = z_0 = 0$, $r_{max} = 12$ мм, $z_{max} = 2$ mm, where, for $0 \le z \le 0.1$ mm, there is a layer of boron carbide, and for $0.1 < z \le 2$ mm, there is tungsten. We define the grid nodes using mesh functions:

$$t^n = n\tau,\ n = 1,\ldots,T,$$
$$r_i = ih,\ i = 1,\ldots,N_r,$$
$$z_k = kh,\ k = 1,\ldots,N_z,$$
$$T_{ik}^n = T(t^n, r_i, z_k),$$
$$W_i^n = W_{max}(t^n)\exp(-Ar_i^2).$$

Let's introduce difference operators

$$(\Lambda_{rr})_{ik}^n = \frac{1}{r_i h^2 c\rho}\left[r_{i+\frac{1}{2}}\left(T_{i+1k}^n - T_{ik}^n\right) - r_{i-\frac{1}{2}}\left(T_{ik}^n - T_{i-1k}^n\right)\right], \quad (2)$$

$$(\Lambda_{zz})_{ik}^{n} = \frac{\lambda}{c\rho h^2}\left(T_{ik}^{n} - T_{i-1k}^{n}\right),$$

$$f_{i+\frac{1}{2}j} = \frac{f_{ij} + f_{i+1j}}{2}, \quad f_{ij+\frac{1}{2}} = \frac{f_{ij} + f_{ij+1}}{2}.$$

The numerical implementation of equation (1) is based on the stabilizing correction scheme [9] and the tridiagonal matrix algorithm:

$$\begin{cases} \dfrac{T_{ik}^{n+\frac{1}{2}} - T_{ik}^{n}}{\tau} = (\Lambda_{rr})_{ik}^{n+\frac{1}{2}} + (\Lambda_{zz})_{ik}^{n}, \\ \dfrac{T_{ik}^{n+1} - T_{ik}^{n+\frac{1}{2}}}{\tau} = (\Lambda_{zz})_{ik}^{n+1} - (\Lambda_{zz})_{ik}^{n}. \end{cases} \quad (3)$$

Boundary conditions:

$$\frac{T_{i,3}^{n+1} - 4T_{i,2}^{n+1} + 3T_{i,1}^{n+1}}{2h} = \frac{W_i^{n+1}}{\lambda},$$

$$\frac{T_{N_r-2,k}^{n+\frac{1}{2}} - 4T_{N_r,k}^{n+\frac{1}{2}} + 3T_{N_r-1,k}^{n+\frac{1}{2}}}{2h} = 0,$$

$$\frac{T_{i,N_z-2}^{n+1} - 4T_{i,N_z}^{n+1} + 3T_{i,N_z-1}^{n+1}}{2h} = 0.$$

To verify the correctness of the algorithm, a test problem is solved [10]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial z}\left(k_0 u^{\sigma} \dfrac{\partial u}{\partial z}\right), \ z > 0, \ t > 0, \\ u(z,0) = 0, \ u(0,t) = u_0 t^{\frac{1}{\sigma}}, \end{cases} \quad (4)$$

where $u_0 = \left(\frac{\sigma c^2}{k_0}\right)^{\frac{1}{\sigma}}$, $k_0 = 0{,}5$, $\sigma = 2$, $c = 1$ and the exact solution:

$$u_{\text{exact}}(z,t) = \begin{cases} \left(\dfrac{\sigma c}{k_0}(ct - z)\right)^{\frac{1}{\sigma}}, & \text{at} \quad z \le ct, \\ 0, & \text{at} \quad z > ct. \end{cases}$$

We solve the problem for $x \in [0,1]$ and for the time interval $t \in [0, 0.5]$. Figure 2 presents the solution of problem (4) at different time instants, Fig. 3 shows the graph of the relative error ε as a function of the grid step h.

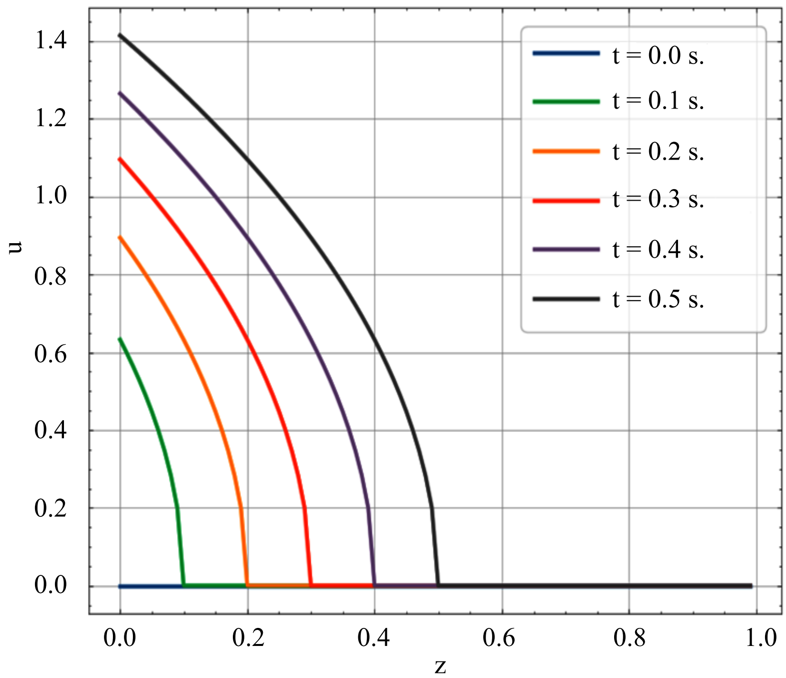

Fig. 2. Solving the problem (4) at different points in time

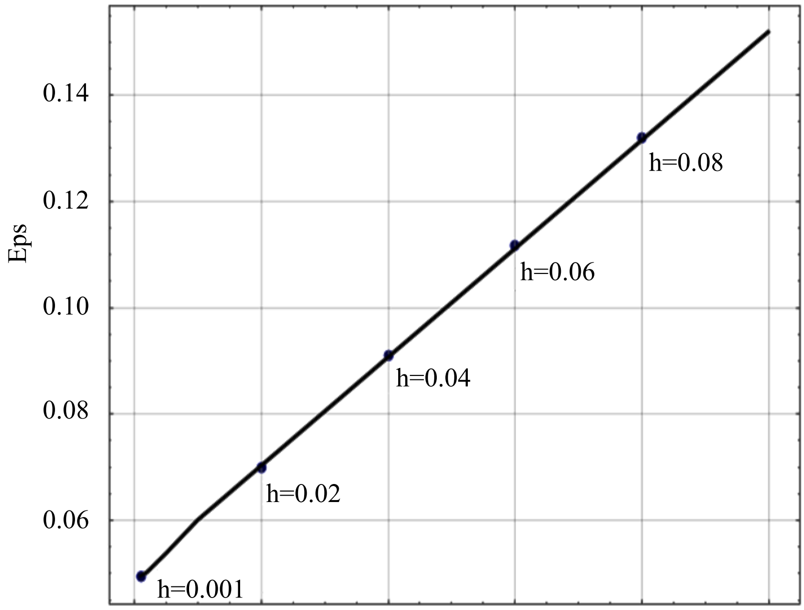

Fig. 3. Graph of the relative error dependency on the grid step

Results. Let's consider the numerical solution of problem (1) for the case of constant coefficients (Table 1).

Table 1

Characteristics of boron carbide and tungsten used in calculations

Units of measurement	Tungsten	Boron Carbide
$c, \frac{J}{kg \cdot K}$	148.34	2153.66
$\rho, \frac{kg}{m^3}$	19051.24	2509.06
$\lambda, \frac{W}{m \cdot K}$	119.55	20.79

The calculation was performed with $\tau = 0.01$, $h = 0.01$. The calculation time was determined by heating the plate surface to 2000 K. Numerical experiments show that boron carbide, having a higher specific heat capacity compared to tungsten, heats up more strongly, but due to its low thermal conductivity, it slows down the propagation of heat inward.

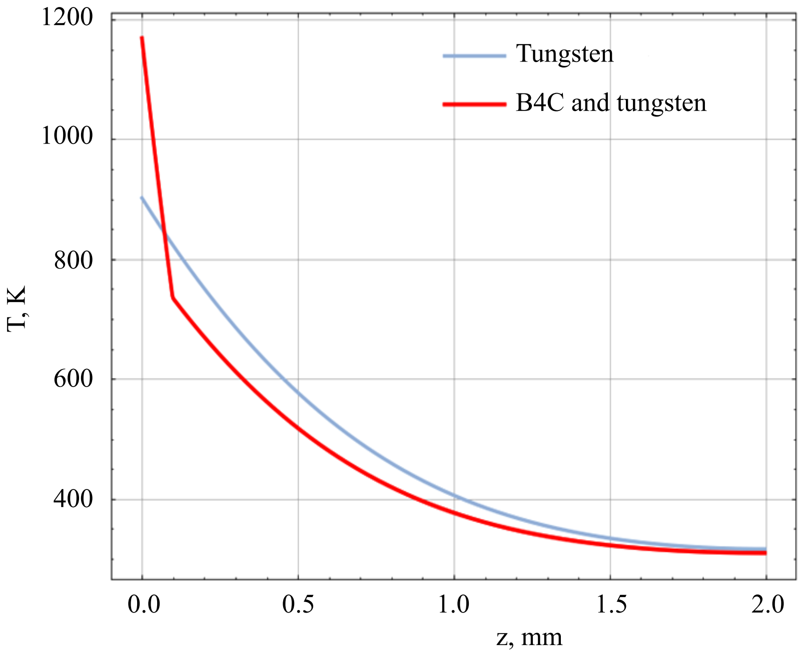

Fig. 4. Temperature distribution within the sample

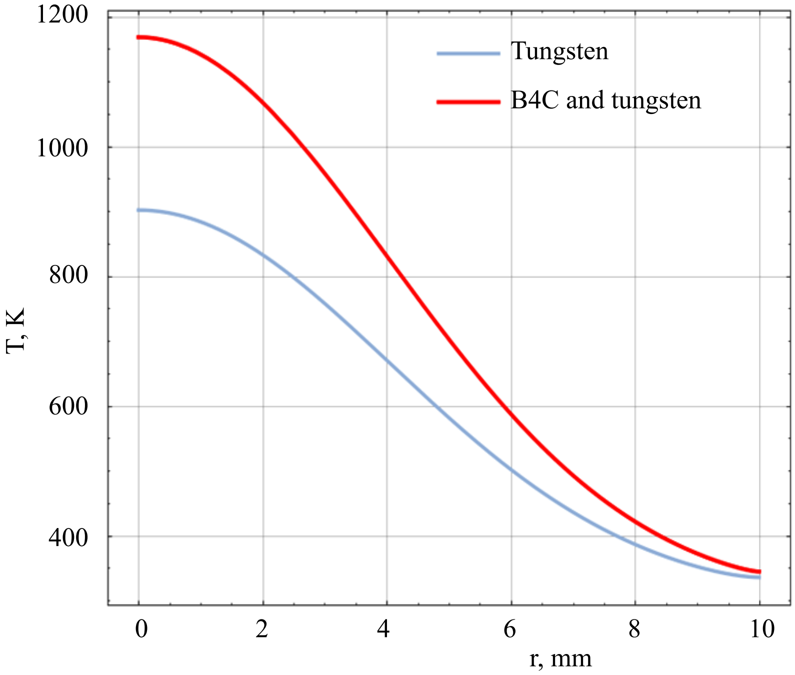

Fig. 5. Temperature distribution on the surface of the sample

Thus, the application of a boron carbide protective coating allows reducing the temperature affecting the tungsten, thereby preventing the material from heating to critical values ($T > 800$ K [11, 12]), which can lead to cracking.

Discussion and Conclusions. An analysis of the heating model of the composite material under electron beam heating at constant values of density, thermal conductivity, and specific heat was conducted. The results of the conducted modelling demonstrate that boron carbide, having a higher specific heat compared to tungsten, heats up more intensely, but due to its low thermal conductivity, it slows down the spread of heat inward.

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G.G. Lazareva — results discussion.

V.A. Popov — problem formulation.

Conflict of interest statement

The authors do not have any conflict of interest.

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