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Original Theoretical Research

Sufficient Conditions for the Convergence of Solutions of the Linearized Problem to the Solution of the Original Nonlinear Problem of Multifractional Sediment Transport in Shallow Water

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Abstract
Introduction. This paper examines a two-dimensional spatial model of multifractional sediment transport, specifically focusing on shallow water zones. This process can be described using an initial-boundary value problem for a parabolic equation with nonlinear coefficients. The study employs a temporal grid linearization method with a step size τ, where nonlinear coefficients are calculated with a "lag" at the previous time layer. Previously, the well-posedness conditions for the linearized sediment transport problem were established, and a conservative and stable finite-difference scheme was developed and analyzed, with numerical implementations for both model and real-world problems (the Sea of Azov, the Taganrog Bay, and the Tsimlyansk Reservoir). However, the convergence of solutions of the linearized problem to the solution of the original nonlinear initial-boundary value problem for multifractional sediment transport had not yet been explored. The research results presented in this paper fill this gap. Earlier, the author, together with A.I. Sukhinov, conducted similar studies in the case where sediment fraction composition was not considered. These studies formed the basis for obtaining new results.
Materials and Methods. The derivation of inequalities guaranteeing the convergence of the solutions of a sequence of linearized problems to the solution of the original nonlinear problem is carried out using the method of mathematical induction, with the application of differential equation theory.
Results. The conditions for the convergence of solutions of the linearized multifractional sediment transport problem to the solution of the nonlinear problem in the Banach space L_1 norm at a rate $O(\tau)$ of are determined.
Discussion and Conclusion. The obtained research results can be used for forecasting nonlinear hydrophysical processes, improving their accuracy and reliability due to the new functional capabilities that account for physically significant factors.

Keywords: two-dimensional spatial sediment transport model, multifractional sediment composition, shallow water zone, nonlinear problem, linearized problem

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Оригинальное теоретическое исследование

Достаточные условия сходимости решений линеаризованной задачи к решению исходной нелинейной задачи транспорта многофракционных наносов в зоне мелководья

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Аннотация

Введение. Рассматривается пространственно-двумерная модель транспорта наносов многофракционного состава, ориентированная на зоны мелководья. Для описания этого процесса может быть использована начально-краевая задача для параболического уравнения с нелинейными коэффициентами. Ее исследование проводится с помощью линеаризации на временной сетке с шагом τ, при которой нелинейные коэффициенты рассчитываются «с запаздыванием» на предыдущем временном слое. Для линеаризованной задачи транспорта многофракционных наносов ранее были определены условия корректности, построена и исследована консервативная устойчивая разностная схема, численно реализованная для модельных и реальных задач (Азовское море и Таганрогский залив, Цимлянское водохранилище). Однако вопросы сходимости решений линеаризованной задачи к решению исходной нелинейной начально-краевой задачи транспорта многофракционных наносов пока оставались не рассмотренными. Результаты исследований, представленные в данной работе, восполняют этот пробел. Ранее автором совместно с А.И. Сухиновым удалось провести аналогичные исследования для случая, когда фракционный состав наносов не учитывается. Эти исследования легли в основу для получения нового результата.

Материалы и методы. Получение неравенств, гарантирующих сходимость решений цепочки линеаризованных задач к решению исходных нелинейных задач, проводится методом математической индукции с привлечением теории дифференциальных уравнений.

Результаты исследования. Определены условия сходимости решений линеаризованной задачи транспорта наносов многофракционного состава к решению нелинейной задачи в норме банахового пространства L_1 со скоростью $O(\tau)$.

Обсуждение и заключение. Полученные результаты исследования могут быть использованы при прогнозировании нелинейных гидрофизических процессов, повышения их точности и надежности в силу наличия новых функциональных возможностей учета физически важных факторов.

Ключевые слова: пространственно-двумерная модель транспорта наносов, многофракционный состав наносов, зона мелководья, нелинейная задача, линеаризованная задача

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Introduction. One of the important and complex problems in studying sediment transport in shallow water zones is accounting for its fractional composition. The fractional composition of sediments varies significantly depending on the slope and morphological structure of the seabed, depth, flow velocity, bed surface roughness, and other factors. Differences in fractional composition determine the nature of sediment movement and sedimentation processes. Considering particle size in mathematical modeling of sediment transport enables more accurate and reliable predictions of seabed morphodynamics.

This study examines a nonlinear 2D model of sediment transport that takes into account its fractional composition [1–4]. To analyze this model, a linearization is performed on a time grid with step size , where nonlinear coefficients are computed with a "lag" from the previous time step. Using the method of mathematical induction and differential equation theory, sufficient conditions for the convergence of solutions of the linearized problem to the solution of the original nonlinear initial-boundary value problem are determined. It is worth noting that previous studies have ensured the well-posedness of this problem.

The existence and uniqueness of solutions to the linearized multifractional sediment transport problem were studied in [3]. The work [4] demonstrated the continuous dependence of the solutions of the linearized multifractional sediment transport problem on the input data. Research on the well-posedness of the linearized sediment transport problem, which does not account for heterogeneous fractional composition, is presented in [5–8].

Materials and Methods. The sediment transport equation, including R fractions, is written as:

$$\left(1-\tilde{\varepsilon}\right)\frac{\partial H}{\partial t}+\sum_{r=1}^{R}div\left(V_r k_r \vec{\tau}_b\right)=\sum_{r=1}^{R}div\left(V_r k_r \frac{\tau_{bc,r}}{\sin\varphi_0}gradH\right)-\sum_{r=1}^{R}\frac{w+w_{g,r}}{\rho_r}c_r,\ r=\overline{1,R}, \quad (1)$$

where $H = H(x,y,t)$ is the water body depth; $\tilde{\varepsilon}$ is the porosity of bottom sediments averaged over all components; t is the time variable, $t\in[0,T]$; V_r volume fraction of the r-th component; $\vec{\tau}_b$ is the vector of tangential shear stress at the bottom of the water body; $\tau_{bc,r}$ is the critical value of tangential stress for the r-th sediment component, $\tau_{bc,r} = a_r\sin\varphi_0$ where a_r is the coefficient for the r-th sediment component, φ_0 is the angle of natural slope of the ground in the water body; w is the vertical component of the velocity vector $\vec{U}$ of the water medium; $w_{g,r}$ is the hydraulic size or settling velocity of the r-th component; ρ_r is the density of the r-th bottom material component; c_r is the concentration of the r-th suspended fraction; k_r is the coefficient determined by the relation:

$$k_r=k_r\left(H,x,y,t\right)=\frac{A\tilde{\omega}d_r}{\left(\left(\rho_r-\rho_0\right)gd_r\right)^{\beta}}\left|\vec{\tau}_b-\frac{\tau_{bc,r}}{\sin\varphi_0}gradH\right|^{\beta-1},$$

($\tilde{\omega}$ is the average wave frequency; d_r is the characteristic size of the r-th component; g is the gravitational acceleration; ρ_0 is the density of the water medium; A and β are the dimensionless constants),

$$k_r\geq k_{0,r}=const>0,\ \forall\left(x,y\right)\in\overline{G},\ r=\overline{1,R},\ 0<t\leq T.$$

We assume that $Ц_T = G\times(0,T)$ is the domain where equation (1) is defined. Let the sediment transport process take place in the domain with the boundary Γ, which represents a piecewise-smooth curve.

The boundary of this cylinder consists of the lateral surface $\Gamma\times[0,T]$ and two bases — $\overline{G}\times\{0\}$ and $\overline{G}\times\{T\}$.

Equation (1) is considered in the domain $G\left(x,y\right)=\left\{0<x<L_x,0<y<L_y\right\}$ (Fig. 1).

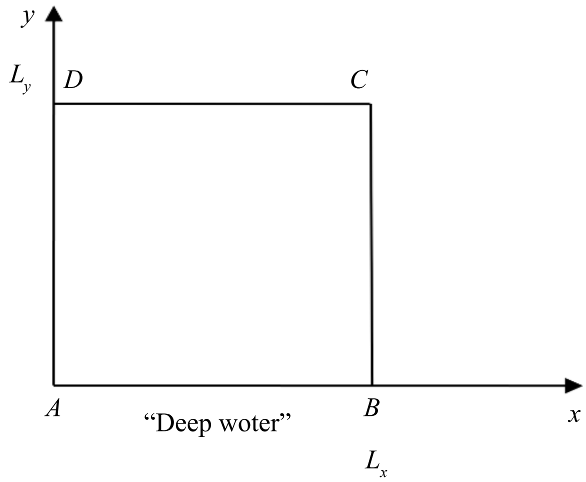

Fig. 1. Computational domain

We supplement equation (1) with initial and boundary conditions:

$$H\left(x,y,0\right)=H_0\left(x,y\right),\ H_0\left(x,y\right)\in C^2\left(G\right)\cap C\left(\overline{G}\right), \quad (2)$$

$$AD:\ H\left(0,y,t\right)=H_1\left(y,t\right), \quad (3)$$

$$BC:\ H\left(L_x,y,t\right)=H_2\left(y,t\right), \quad (4)$$

$$AB:\ H\left(x,0,t\right)=H_3\left(x\right), \quad (5)$$

$$CD:\ H\left(x,L'_y,t\right)=H_4\left(x,t\right)\geq c_0\equiv const>0,L'_y<L_y. \quad (6)$$

Additionally, we assume:

$$AB:\left|\vec{\tau}_b\right|=0, \quad (7)$$

$$grad_{(x,y)}H\in C\left(\overline{Ц_T}\right)\cap C^1\left(Ц_T\right),\ grad_{(x,y)}H_0\in C\left(\overline{G}\right), \quad (8)$$

$$\left.\frac{\partial H}{\partial x}\right|_{x=0}=\left.\frac{\partial H}{\partial x}\right|_{x=L_x}=0, \quad (9)$$

$$k_r \ge k_{0,r} = const > 0,\ \forall(x,y)\in\bar{G}, \tag{10}$$

$$\vec{\tau}_b = \tau_{bx}\vec{i} + \tau_{by}\vec{j}, \left|\tau_{bx}\right| \le c_1,\ \left|\tau_{by}\right| \le c_2,\ c_1 = const,\ c_2 = const. \tag{11}$$

The condition

$$H(x,y,t) \ge c_0 \equiv const > 0,\ 0 \le x \le L_x,\ 0 \le y \le L'_y,\ 0 \le t \le T. \tag{12}$$

ensures that no drying occurs in the considered domain.

Linearization of problem (1)–(6) is performed on a time grid:

$$\omega_\tau = \left\{t_n = n\tau,\ n = 0,1,...,N_t,\ N_t\tau = T\right\},$$

using the methods presented in [5–10].

After linearization, equation (1) and the initial conditions (2) are written as:

$$(1-\tilde{\varepsilon})\frac{\partial H^{(n)}}{\partial t} = \sum_{r=1}^{R} div\left(V_r k_r^{(n-1)}\frac{\tau_{bc,r}}{\sin\varphi_0} gradH^{(n)}\right) - \sum_{r=1}^{R} div\left(V_r k_r^{(n-1)}\vec{\tau}_b\right) - \sum_{r=1}^{R}\frac{w+w_{g,r}}{\rho_r}c_r,$$
$$r = \overline{1,R},\ t_{n-1} < t \le t_n,\ n = 1,2,...,N_t, \tag{13}$$

$$H^{(1)}(x,y,t_0) = H_0(x,y),\ H^{(n)}(x,y,t_{n-1}) = H^{(n-1)}(x,y,t_{n-1}),\ (x,y)\in\bar{G},\ n = 2,...,N. \tag{14}$$

The coefficient $k_r^{(n-1)}$ in equation (14) is determined by the equation:

$$k_r^{(n-1)} = k_r^{(n-1)}\left(H^{(n-1)},x,y,t\right) = \frac{A\tilde{\omega}d_r}{\left((\rho_r-\rho_0)gd_r\right)^{\beta}}\left|\vec{\tau}_b - \frac{\tau_{bc,r}}{\sin\varphi_0}gradH^{(n-1)}(x,y,t_{n-1})\right|^{\beta-1}.$$

Results. We will show that the solution of problem (13), (14), (3)–(6) converges to the solution of the nonlinear problem (1)–(6) in the norm of the space $L_1(G\times[0,T])$ as $\tau \to 0$, $N\tau = T$.

Let us denote the solution of the nonlinear problem as $H_{np}(x,y,t), (x,y)\in\bar{G},$ and the solution of the linearized problem as $H_{lp}(x,y,t), (x,y)\in\bar{G}.$ Note that for each time layer, its own solution function $H_{lp} = (x,y,t)$ is defined, and in general, the linearized problem constructs a family of solutions $\left\{H_{lp}^{(n)}(x,y,t)\right\},\ n = 1,2,...,N,$ that depends on the parameter τ.

We assume that:

1. The function $H_{np} = (x,y,t)$ is bounded on the interval $0 < t \le T$;
2. The derivatives exist and are bounded:

$$\frac{\partial}{\partial x}\left(grad\left(\frac{\partial H_{np}}{\partial t}\right)\right),\ \frac{\partial}{\partial y}\left(grad\left(\frac{\partial H_{np}}{\partial t}\right)\right),\ \frac{\partial}{\partial x}\left(grad\left(\frac{\partial H_{lp}}{\partial t}\right)\right),\ \frac{\partial}{\partial y}\left(grad\left(\frac{\partial H_{lp}}{\partial t}\right)\right);$$

3. The expression $\frac{w+w_{g,r}}{\rho_r}c_r$ is bounded.

Substituting the function $H_{np} = (x,y,t)$ into equation (1) and the function $H_{lp} = (x,y,t)$ into equation (12), we obtain:

$$(1-\tilde{\varepsilon})\frac{\partial H_{np}}{\partial t} = \sum_{r=1}^{R} div\left(V_r k_{np,r}\frac{\tau_{bc,r}}{\sin\varphi_0} gradH_{np}\right) - \sum_{r=1}^{R} div\left(V_r k_r\vec{\tau}_b\right) - \sum_{r=1}^{R}\frac{w+w_{g,r}}{\rho_r}c_r,\ r = \overline{1,R}. \tag{15}$$

$$(1-\tilde{\varepsilon})\frac{\partial H_{lp}^{(n)}}{\partial t} = \sum_{r=1}^{R} div\left(V_r k_{lp,r}^{(n-1)}\frac{\tau_{bc,r}}{\sin\varphi_0} gradH_{lp}^{(n)}\right) - \sum_{r=1}^{R} div\left(V_r k_{lp,r}^{(n-1)}\vec{\tau}_b\right) - \sum_{r=1}^{R}\frac{w+w_{g,r}}{\rho_r}c_r,$$
$$r = \overline{1,R},\ t_{n-1} < t \le t_n,\ n = 1,2,...,N, \tag{16}$$

where

$$k_{np,r} = k_{np,r}\left(H_{np},x,y,t\right) = \frac{A\tilde{\omega}d_r}{\left((\rho_r-\rho_0)gd_r\right)^{\beta}}\left|\vec{\tau}_b - \frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{np}(x,y,t)\right|^{\beta-1},$$

$$k_{lp,r}^{(n-1)} = k_{lp,r}^{(n-1)}\left(H_{lp}^{(n-1)},x,y,t\right) = \frac{A\tilde{\omega}d_r}{\left((\rho_r-\rho_0)gd_r\right)^{\beta}}\left|\vec{\tau}_b - \frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{lp}^{(n-1)}(x,y,t)\right|^{\beta-1}.$$

Multiplying both sides of equations (15) and (16) by the functions $H_{np}(x,y,t)$, $H_{lp}^{(n)} = H_{lp}^{(n)}(x,y,t),\ (x,y)\in\bar{G},$ respectively, and integrating the resulting expressions over the variables t, $0 < t \le T$ и (x,y) and (x,y) in the domain G followed by some straightforward transformations, we can write:

$$\frac{1}{2}(1-\tilde{\varepsilon})\iint_G\left(H_{np}^2(x,y,T)-H_{np}^2(x,y,0)\right)dxdy=\sum_{r=1}^{R}\int_0^T\left(\iint_G H_{np}\,div\left(V_r k_{np,r}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{np}\right)dxdy\right)dt$$
$$-\sum_{r=1}^{R}\int_0^T\left(\iint_G H_{np}\,div\left(V_r k_{np,r}\vec{\tau}_b\right)dxdy\right)dt-\sum_{r=1}^{R}\int_0^T\left(\iint_G H_{np}\frac{w+w_{g,r}}{\rho_r}c_r\right)dt. \quad (17)$$

$$(1-\tilde{\varepsilon})\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}\frac{\partial H_{lp}^{(n)}}{\partial t}dxdy\right)dt=\sum_{r=1}^{R}\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r k_{lp,r}^{(n-1)}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{lp}^{(n)}\right)dxdy\right)dt-$$
$$-\sum_{r=1}^{R}\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r k_{lp,r}^{(n-1)}\vec{\tau}_b\right)dxdy\right)dt-\sum_{r=1}^{R}\int_0^T\left(\iint_G H_{lp}^{(n)}\frac{w+w_{g,r}}{\rho_r}c_r\right)dt,\ r=\overline{1,R}. \quad (18)$$

Summing both sides of relation (18) over n, $n = 1,...,N$, we obtain:

$$(1-\tilde{\varepsilon})\iint_G\left(\sum_{n=1}^{N}\int_{t_{n-1}}^{t_n}H_{lp}^{(n)}\frac{\partial H_{lp}}{\partial t}dt\right)dxdy=\sum_{r=1}^{R}\sum_{n=1}^{N}\left[\int_{t_{n-1}}\left(\iint_G H_{lp}^{(n)}div\left(V_r k_{lp,r}^{(n-1)}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{lp}^{(n)}\right)dxdy\right)dt\right.$$
$$\left.-\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r k_{lp,r}^{(n-1)}\vec{\tau}_b\right)dxdy\right)dt-\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}\frac{w+w_{g,r}}{\rho_r}c_r dxdy\right)dt\right]. \quad (19)$$

By transforming the left-hand side of equality (19), we write:

$$\frac{1}{2}(1-\tilde{\varepsilon})\iint_G\left(H_{lp}^2(x,y,T)-H_{lp}^2(x,y,0)\right)dxdy=\sum_{r=1}^{R}\sum_{n=1}^{N}\left[\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r k_{lp,r}^{(n-1)}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{lp}^{(n)}\right)dxdy\right)dt-\right.$$
$$\left.-\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r k_{lp,r}^{(n-1)}\vec{\tau}_b\right)dxdy\right)dt-\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}\frac{w+w_{g,r}}{\rho_r}c_r dxdy\right)dt\right]. \quad (20)$$

Subtracting expression (20) from equality (17) and considering that $H_{np}= (x,y,t)= H_{lp}= (x,y,t)$, $H_{lp}= (x,y,t) = H_{lp}^{(0)}$, we obtain:

$$2(1-\tilde{\varepsilon})\iint_G\left(H_{np}^2(x,y,T)-H_{lp}^2(x,y,T)\right)dxdy=$$
$$=\sum_{r=1}^{R}\int_0^T\left(\iint_G H_{np}div\left(V_r k_{np,r}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{np}\right)dxdy\right)dt-\sum_{r=1}^{R}\int_0^T\left(\iint_G H_{np}div\left(V_r k_{np,r}\vec{\tau}_b\right)dxdy\right)dt-$$
$$-\sum_{r=1}^{R}\int_0^T\left(\iint_G H_{np}\frac{w+w_{g,r}}{\rho_r}c_r\right)dt-\sum_{r=1}^{R}\sum_{n=1}^{N}\left[\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r k_{lp,r}^{(n-1)}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{lp}^{(n)}\right)dxdy\right)dt+\right.$$
$$\left.+\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r k_{lp,r}^{(n-1)}\vec{\tau}_b\right)dxdy\right)dt+\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}\frac{w+w_{g,r}}{\rho_r}c_r dxdy\right)dt\right]. \quad (21)$$

We perform transformations on the right-hand side of equality (21). To do this, we add and then subtract the expressions $H_{lp}^{(n)}div\left(V_r k_{np,r}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{np}\right)$, $H_{lp}^{(n)}div\left(V_r k_{np,r}\vec{\tau}_b\right)$ under the integral sign, respectively, in the fourth and fifth terms of the right-hand side of equality (21).

Combining the terms, we obtain:

$$\frac{1}{2}(1-\tilde{\varepsilon})\iint_G\left(H_{np}^2(x,y,T)-H_{lp}^2(x,y,T)\right)dxdy=$$
$$=\sum_{r=1}^{R}\sum_{n=1}^{N}\int_{t_{n-1}}^{t_n}\left[\left(\iint_G\left(H_{np}-H_{lp}^{(n)}\right)div\left(V_r k_{np,r}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{np}\right)dxdy\right)+\iint_G H_{lp}^{(n)}div\left(V_r\frac{\tau_{bc,r}}{\sin\varphi_0}\cdot\right.\right.$$
$$\left.\cdot\left(k_{np,r}gradH_{np}-k_{lp,r}^{(n-1)}gradH_{lp}^{(n)}\right)\right)dxdy+\iint_G\left(H_{lp}^{(n)}-H_{np}\right)div\left(V_r k_{np,r}\vec{\tau}_b\right)dxdy+$$
$$\left.+\iint_G H_{lp}^{(n)}div\left(V_r\left(k_{lp,r}^{(n-1)}-k_{r,np}\right)\vec{\tau}_b\right)dxdy+\iint_G\left(H_{lp}^{(n)}-H_{np}\right)\frac{w+w_{g,r}}{\rho_r}c_r dxdy\right]dt. \quad (22)$$

We estimate each of the integrals on the right-hand side of equality (22) under the summation sign. For this purpose, we use the reasoning detailed in [9].

Let us introduce the following notation:

$$I_{1,r}^{n}=\int_{t_{n-1}}^{t_n}\left(\iint_G\left(H_{np}-H_{lp}^{(n)}\right)div\left(V_r k_{np,r}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{np}\right)dxdy\right)dt, \tag{23}$$

$$I_{2,r}^{n}=\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r\frac{\tau_{bc,r}}{\sin\varphi_0}\left(k_{np,r}grad\ H_{np}-k_{lp,r}^{(n-1)}gradH_{lp}^{(n)}\right)\right)dxdy\right)dt, \tag{24}$$

$$I_{3,r}^{n}=\int_{t_{n-1}}^{t_n}\left(\iint_G\left(H_{lp}^{(n)}-H_{np}\right)div\left(V_r k_{np,r}\vec{\tau}_b\right)dxdy\right)dt, \tag{25}$$

$$I_{4,r}^{n}=\int_{t_{n-1}}^{t_n}\left(\iint_G H_{lp}^{(n)}div\left(V_r\left(k_{lp,r}^{(n-1)}-k_{np,r}\right)\vec{\tau}_b\right)dxdy\right)dt, \tag{26}$$

$$I_{5,r}^{n}=\int_{t_{n-1}}^{t_n}\left(\iint_G\left(H_{lp}^{(n)}-H_{np}\right)\frac{w+w_{g,r}}{\rho_r}c_r dxdy\right)dt,\ n=1,...,N. \tag{27}$$

For $n = 1$ for integrals (23)–(27), we obtain:

$$I_{1,r}^{1}\le\frac{1}{2}\tau^2L_xL_yM_{1,r}^{1},\quad I_{2,r}^{1}\le\frac{1}{2}\tau^2M_{2,r}^{1}L_xL_y,\quad I_{3,r}^{1}\le\frac{1}{2}\tau^2L_xL_yM_{3,r}^{1},$$
$$I_{4,r}^{1}\le\frac{1}{2}\tau^2M_{4,r}^{1}L_xL_y,\quad I_{5,r}^{1}\le\frac{1}{2}\tau^2L_xL_yM_{5,r}^{1}, \tag{28}$$

where

$$M_{1,r}^{1}\equiv\max_{t_0\le t\le t_1}\left\{\max_{(x,y)\in\bar{G}}\left|\left(\frac{\partial H_{np}\left(x,y,\xi_1\right)}{\partial t}-\frac{\partial H_{lp}^{(1)}\left(x,y,\xi_2\right)}{\partial t}\right)div\left(V_r k_{np,r}\frac{\tau_{bc,r}}{\sin\varphi_0}gradH_{np}\right)\right|\right\},$$

$$M_{2,r}^{1}\equiv\max_{t_0\le t\le t_1}\left\{\max_{(x,y)\in\bar{G}}\left\{\left|H_{lp}^{(1)}div\left(V_r\frac{\tau_{bc,r}}{\sin\varphi_0}\left(\frac{\partial k_{np,r}\left(x,y,\xi_3\right)}{\partial t}gradH_{np}+k_{lp,r}^{(0)}grad\frac{\partial H_{np}\left(x,y,\xi_4\right)}{\partial t}\right)\right)dxdy\right|\right\}\right\},$$

$$M_{3,r}^{1}\equiv\max_{t_0\le t\le t_1}\left\{\max_{(x,y)\in\bar{G}}\left|\left(\frac{\partial H_{lp}^{(1)}\left(x,y,\xi_5\right)}{\partial t}-\frac{\partial H_{np}\left(x,y,\xi_6\right)}{\partial t}\right)div\left(V_r k_{np,r}\vec{\tau}_b\right)\right|\right\},$$

$$M_{4,r}^{1}\equiv\max_{t_0\le t\le t_1}\left\{\max_{(x,y)\in\bar{G}}\left\{\left|H_{lp}^{(1)}div\left(V_r\left(-\frac{\partial k_{np,r}\left(x,y,\xi_7\right)}{\partial t}\right)\vec{\tau}_b\right)\right|\right\}\right\}.$$

$$M_{5,r}^{1}\equiv\max_{t_0\le t\le t_1}\left\{\max_{(x,y)\in\bar{G}}\left|\left(\frac{\partial H_{lp}^{(1)}\left(x,y,\xi_5\right)}{\partial t}-\frac{\partial H_{np}\left(x,y,\xi_6\right)}{\partial t}\right)\frac{w+w_{g,r}}{\rho_r}c_r\right|\right\},$$

$$t_0<\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \xi_5,\ \xi_6,\ \xi_7\le t_1,\ \ t_0<t\le t_1.$$

Considering the obtained estimates (28) and using the triangle inequality for the magnitudes of the quantities, we obtain an inequality of the form:

$$\iint_G\left(H_{np}^2\left(x,y,t_1\right)-H_{lp}^{(1)2}\left(x,y,t_1\right)\right)dxdy\le\tau^2L_xL_y\sum_{r=1}^{R}M_{1,r}^{*}, \tag{29}$$

where

$$M_{1,r}^{*}=\frac{1}{1-\tilde{\varepsilon}}\left(M_{1,r}^{1}+M_{2,r}^{1}+M_{3,r}^{1}+M_{4,r}^{1}+M_{5,r}^{1}\right).$$

By swapping the functions $H_{np}^{\ 2}(x,y,t_1)$ и $H_{lp}^{(1)2}(x,y,t_1)$ and following reasoning analogous to the one given above, we can obtain an estimate:

$$\iint_G\left(H_{lp}^{(1)2}\left(x,y,t_1\right)-H_{np}^2\left(x,y,t_1\right)\right)dxdy\le\tau^2L_xL_y\sum_{r=1}^{R}M_{1,r}^{*}. \tag{30}$$

From inequalities (29) and (30), we get the following inequality:

$$\iint_G \left| H_{lp}^{(1)2}(x,y,t_1) - H_{np}^2(x,y,t_1) \right| dxdy \le \tau^2 L_x L_y \sum_{r=1}^{R} M_{1,r}^*. \tag{31}$$

We transform the left-hand side of inequality (31):

$$\iint_G \left| H_{lp}^{(1)2}(x,y,t_1) - H_{np}^2(x,y,t_1) \right| dxdy = \iint_G \left| H_{lp}^{(1)}(x,y,t_1) - H_{np}(x,y,t_1) \right| \cdot \\ \cdot \left| H_{lp}^{(1)}(x,y,t_1) + H_{np}(x,y,t_1) \right| dxdy. \tag{32}$$

Next, we will assume that an inequality of the form (12) holds for the functions $H_{np} = (x,y,t)$ and $H_{lp}^{(1)}$, i. e.

$$H_{np}(x,y,t) \ge c_0 > 0,\ (x,y) \in \bar{G},\quad 0 \le t \le T, \tag{33}$$

$$H_{lp}^{(1)}(x,y,t) \ge c_0 > 0,\ (x,y) \in \bar{G},\quad 0 \le t \le T. \tag{34}$$

Considering expressions (33)–(34), we obtain:

$$\iint_G \left| H_{lp}^{(1)2}(x,y,t_1) - H_{np}^2(x,y,t_1) \right| dxdy \ge 2c_0 \iint_G \left| H_{lp}^{(1)}(x,y,t_1) - H_{np}(x,y,t_1) \right| dxdy. \tag{35}$$

From relations (31), (32), and (35), we get the following estimate:

$$\iint_G \left| H_{lp}^{(1)}(x,y,t_1) - H_{np}(x,y,t_1) \right| dxdy \le \frac{1}{2c_0} \tau^2 L_x L_y \sum_{r=1}^{R} M_{1,r}^*. \tag{36}$$

The required estimate for $n = 1$ is obtained, since inequality (36) is equivalent to a relation of the form:

$$\left\| H_{lp}^{(1)}(x,y,t_1) - H_{np}(x,y,t_1) \right\|_{L_1(G\times[t_0,t_1])} \le \frac{1}{2c_0} \tau^2 L_x L_y \sum_{r=1}^{R} M_{1,r}^*, \tag{37}$$

where

$$\left\| Q(x,y,t) \right\|_{L_1(G\times[t_0,t_1])} \equiv \int_{t_0}^{t_1} \left(\iint_G \left| Q(x,y,t) \right| dxdy \right) dt.$$

For $n = 2$ for integrals (23)–(27), we obtain:

$$I_{1,r}^2 \le \frac{1}{2}\tau^2 L_x L_y \left(\frac{1}{c_0} \tau M_{1,r}^* M_{1,r}^2 + M_{2,r}^2 \right),\quad I_{2,r}^2 \le \frac{1}{2}\tau^2 L_x L_y M_{3,r}^2 M_{4,r}^2,\quad I_{3,r}^2 \le \frac{1}{2}\tau^2 L_x L_y \left(\frac{1}{c_0} \tau M_{1,r}^* M_{5,r}^2 + M_{6,r}^2 \right), \\ I_{4,r}^2 \le \frac{1}{2}\tau^2 L_x L_y M_{3,r}^2 M_{7,r}^2,\quad I_{5,r}^2 \le \frac{1}{2}\tau^2 L_x L_y \left(\frac{1}{c_0} \tau M_{1,r}^* M_{8,r}^2 + M_{9,r}^2 \right), \tag{38}$$

where

$$M_{1,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{(x,y)\in\bar{G}} \left| div\left(V_r k_{np,r} \frac{\tau_{bc,r}}{\sin\varphi_0} gradH_{np} \right) \right| \right\},$$

$$M_{2,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{(x,y)\in\bar{G}} \left| \left(\frac{\partial H_{np}(x,y,\xi_1)}{\partial t} - \frac{\partial H_{lp}^{(2)}(x,y,\xi_2)}{\partial t} \right) div\left(V_r k_{np,r} \frac{\tau_{bc,r}}{\sin\varphi_0} gradH_{np} \right) \right| \right\},$$

$$M_{3,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{(x,y)\in\bar{G}} \left\{ \left| H_{lp}^{(2)} \right| \right\} \right\},$$

$$M_{4,r}^2 = M_{21,r}^2 + M_{22,r}^2 + M_{23,r}^2 + M_{24,r}^2 + M_{25,r}^2 + M_{26,r}^2,$$

$$M_{5,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{(x,y)\in\bar{G}} \left| div\left(V_r k_{np,r} \vec{\tau}_b \right) \right| \right\},$$

$$M_{6,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{(x,y)\in\bar{G}} \left| \left(\frac{\partial H_{np}(x,y,\xi_1)}{\partial t} - \frac{\partial H_{lp}^{(2)}(x,y,\xi_2)}{\partial t} \right) div\left(V_r k_{np,r} \vec{\tau}_b \right) \right| \right\},$$

$$M_{7,r}^2 = M_{31,r}^2 + M_{32,r}^2 + M_{33,r}^2,$$

$$M_{8,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{(x,y)\in \bar{G}} \left| \frac{w + w_{g,r}}{\rho_r} c_r \right| \right\},$$

$$M_{9,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{(x,y)\in \bar{G}} \left| \left(\frac{\partial H_{np}(x,y,\xi_1)}{\partial t} - \frac{\partial H_{lp}^{(2)}(x,y,\xi_2)}{\partial t} \right) \frac{w + w_{g,r}}{\rho_r} c_r \right| \right\},$$

$$M_{7,r}^2 = M_{31,r}^2 + M_{32,r}^2 + M_{33,r}^2,$$

$$M_{11,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{x\in BC} \left| \tau_{bx} \frac{\partial H_2(y,t)}{\partial x} + \tau_{bx}^{(1)} \frac{\partial H_2(y,t_1)}{\partial x} \right| \right\}, \quad M_{12,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{y\in BC} \left| \tau_{by} \frac{\partial H_2(y,t)}{\partial y} + \tau_{by}^{(1)} \frac{\partial H_2(y,t_1)}{\partial y} \right| \right\},$$

$$M_{13,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{x\in CD} \left| \frac{\partial H_4(x,t)}{\partial x} + \frac{\partial H_4(x,t_1)}{\partial x} \right| \right\}, \quad M_{14,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{x\in CD} \left| \frac{\partial H_4(x,t_1)}{\partial y} + \frac{\partial H_4(x,t_1)}{\partial y} \right| \right\},$$

$$M_{15,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{x\in AD} \left| \frac{\partial H_1(y,t)}{\partial x} + \frac{\partial H_1(y,t_1)}{\partial x} \right| \right\}, \quad M_{16,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{y\in AD} \left| \frac{\partial H_1(y,t_1)}{\partial y} + \frac{\partial H_1(y,t_1)}{\partial y} \right| \right\},$$

$$M_{21,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{y\in BC} \left| V_r \frac{\tau_{bc,r}^2}{\sin^2\varphi_0} \frac{A\tilde{\omega} d_r}{\left((\rho_r - \rho_0) g d_r\right)^3} \left[\frac{\partial}{\partial x}\left(\frac{\partial H_2(y,\xi_4)}{\partial t} \right) \left(\frac{\tau_{bc,r}}{\sin\varphi_0} M_{11,r}^2 - 2\tau_{bx} \right) + \right.\right.\right.$$

$$\left.\left.\left. + \frac{\partial}{\partial y}\left(\frac{\partial H_2(y,\xi_4)}{\partial t} \right) \left(\frac{\tau_{bc,r}}{\sin\varphi_0} M_{12,r}^2 - 2\tau_{by} \right) \right] \frac{\partial H_2(y,t_1)}{\partial x} \right| \right\},$$

$$M_{22,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{y\in BC} \left| k_{np,r} \frac{\partial}{\partial x}\left(\frac{\partial H_2(y,\xi_4)}{\partial t} \right) \right| \right\},$$

$$M_{23,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{x\in CD} \left| V_r \frac{\tau_{bc,r}^2}{\sin^2\varphi_0} \frac{A\tilde{\omega} d_r}{\left((\rho_r - \rho_0) g d_r\right)^3} (t - t_1) \left[\frac{\partial}{\partial x}\left(\frac{\partial H_4(x,\xi_5)}{\partial t} \right) \left(\frac{\tau_{bc,r}}{\sin\varphi_0} M_{13,r}^2 - 2\tau_{bx} \right) + \right.\right.\right.$$

$$\left.\left.\left. + \frac{\partial}{\partial y}\left(\frac{\partial H_4(x,\xi_5)}{\partial t} \right) \left(\frac{\tau_{bc,r}}{\sin\varphi_0} M_{14,r}^2 - 2\tau_{by} \right) \right] \frac{\partial H_4(x,t_1)}{\partial y} \right| \right\},$$

$$M_{24,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{x\in CD} \left| k_{np,r} \frac{\partial}{\partial y}\left(\frac{\partial H_4(x,\xi_5)}{\partial t} \right) \right| \right\},$$

$$M_{25,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{y\in AD} \left| V_r \frac{\tau_{bc,r}^2}{\sin^2\varphi_0} \frac{A\tilde{\omega} d_r}{\left((\rho_r - \rho_0) g d_r\right)^3} \left[\frac{\partial}{\partial x}\left(\frac{\partial H_1(y,\xi_3)}{\partial t} \right) \left(\frac{\tau_{bc,r}}{\sin\varphi_0} M_{15,r}^2 - 2\tau_{bx} \right) + \right.\right.\right.$$

$$\left.\left.\left. + \frac{\partial}{\partial y}\left(\frac{\partial H_1(y,\xi_3)}{\partial t} \right) \left(\frac{\tau_{bc,r}}{\sin\varphi_0} M_{16,r}^2 - 2\tau_{by} \right) \right] \frac{\partial H_1(y,t_1)}{\partial x} \right| \right\},$$

$$M_{26,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{y\in AD} \left| k_{np,r} \frac{\partial}{\partial x}\left(\frac{\partial H_1(y,\xi_3)}{\partial t} \right) \right| \right\}.$$

$$M_{31,r}^2 \equiv \max_{t_1 \le t \le t_2} \left\{ \max_{y\in BC} \left| V_r \frac{\tau_{bc,r}}{\sin\varphi_0} \frac{A\tilde{\omega} d_r}{\left((\rho_r - \rho_0) g d_r\right)^3} \left[\frac{\partial}{\partial x}\left(\frac{\partial H_2(y,\xi_4)}{\partial t} \right) \left(2\tau_{bx} - \frac{\tau_{bc,r}}{\sin\varphi_0} M_{11,r}^2 \right) + \right.\right.\right.$$

$$\left.\left.\left. + \frac{\partial}{\partial y}\left(\frac{\partial H_2(y,\xi_4)}{\partial t} \right) \left(2\tau_{by} - \frac{\tau_{bc,r}}{\sin\varphi_0} M_{12,r}^2 \right) \right] \tau_{bx} \right| \right\},$$

$$M_{32,r}^2 \equiv \max_{t_1\le t\le t_2}\left\{\max_{x\in CD}\left|V_r\frac{\tau_{bc,r}}{\sin\varphi_0}\frac{A\tilde{\omega}d_r}{\left(\left(\rho_r-\rho_0\right)gd_r\right)^3}\left[\frac{\partial}{\partial x}\left(\frac{\partial H_4\left(x,\xi_5\right)}{\partial t}\right)\left(2\tau_{bx}-\frac{\tau_{bc,r}}{\sin\varphi_0}M_{13,r}^2\right)+\right.\right.\right.$$

$$\left.\left.\left.+\frac{\partial}{\partial y}\left(\frac{\partial H_4\left(x,\xi_5\right)}{\partial t}\right)\left(2\tau_{by}-\frac{\tau_{bc,r}}{\sin\varphi_0}M_{14,r}^2\right)\right]\tau_{by}\right|\right\},$$

$$M_{33,r}^2 \equiv \max_{t_1\le t\le t_2}\left\{\max_{y\in AD}\left|V_r\frac{\tau_{bc,r}}{\sin\varphi_0}\frac{A\tilde{\omega}d_r}{\left(\left(\rho_r-\rho_0\right)gd_r\right)^3}\left[\frac{\partial}{\partial x}\left(\frac{\partial H_1\left(y,\xi_3\right)}{\partial t}\right)\left(2\tau_{bx}-\frac{\tau_{bc,r}}{\sin\varphi_0}M_{15,r}^2\right)+\right.\right.\right.$$

$$\left.\left.\left.+\frac{\partial}{\partial y}\left(\frac{\partial H_1\left(y,\xi_3\right)}{\partial t}\right)\left(2\tau_{by}-\frac{\tau_{bc,r}}{\sin\varphi_0}M_{16,r}^2\right)\right]\tau_{by}\right|\right\},$$

$$t_1<\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \xi_5,\le t_2,\ t_1<t\le t_2.$$

Thus, using the estimates (38), we obtain:

$$\iint_G\left|H_{np}^2\left(x,y,t_2\right)-H_{lp}^{(2)2}\left(x,y,t_2\right)\right|dxdy\le\tau^2L_xL_y\sum_{r=1}^{R}\left(M_{01,r}^2+\tau M_{02,r}^2\right), \tag{39}$$

where

$$M_{r,01}^2=\frac{1}{1-\tilde{\varepsilon}}\left(M_{r,2}^2+M_{r,6}^2+M_{r,9}^2+M_3^2\left(M_{r,4}^2+M_{r,7}^2\right)\right),$$

$$M_{r,02}^2=\frac{1}{\left(1-\tilde{\varepsilon}\right)c_0}M_{r,1}^*\left(M_{r,1}^2+M_{r,5}^2+M_{r,8}^2\right).$$

From inequality (39), we can move to the following estimate:

$$\left\|H_{lp}^{(2)}\left(x,y,t_2\right)-H_{np}\left(x,y,t_2\right)\right\|_{L_1\left(G\times\left[t_1,t_2\right]\right)}\le\frac{1}{2c_0}\tau^2L_xL_y\left(1+\tau\right)\sum_{r=1}^{R}M_{2,r}^*, \tag{40}$$

where

$$M_{2,r}^*=\max_{(x,y)\in\bar{G}}\left\{M_{01,r}^2;\ M_{02,r}^2\right\},$$

$$\left\|Q\left(x,y,t\right)\right\|_{L_1\left(G\times\left[t_1,t_2\right]\right)}\equiv\int_{t_1}^{t_2}\left(\iint_G\left|Q\left(x,y,t\right)\right|dxdy\right)dt.$$

The required estimate for $n = 2$ is obtained. The first step of the induction is completed.

Next, we assume that for $n = s$ the estimate holds:

$$\left\|H_{lp}^{(s)}\left(x,y,t_s\right)-H_{np}\left(x,y,t_s\right)\right\|_{L_1\left(G\times\left[t_{s-1},t_s\right]\right)}\le\frac{1}{2c_0}\tau^2L_xL_y\frac{1-\tau^s}{1-\tau}\sum_{r=1}^{R}M_{s,r}^*, \tag{41}$$

where $M_{s,r}^*$ is some constant function.

For $n = s + 1$ in equations (23)–(27), we consider integrals over the time interval $t_s < t \le t_{s+1}$. By estimating these integrals with the consideration of (41) on the previous time step, we obtain the inequality:

$$\iint_G\left|H_{lp}^2\left(x,y,t_{s+1}\right)-H_{np}^2\left(x,y,t_{s+1}\right)\right|dxdy\le\tau^2L_xL_y\sum_{r=1}^{R}\left(M_{01,r}^{s+1}+\tau M_{02,r}^{s+1}+\ldots+\tau^sM_{0s+1,r}^{s+1}\right) \tag{42}$$

with constants $M_{01,r}^{s+1},M_{02,r}^{s+1},\ldots,M_{0s+1,r}^{s+1}$, that depend on the magnitudes of the derivatives.

Using inequality (42), we arrive at the estimate:

$$\left\|H_{lp}\left(x,y,t_{s+1}\right)-H_{np}\left(x,y,t_{s+1}\right)\right\|_{L_1\left(G\times\left[t_s,t_{s+1}\right]\right)}\le\frac{1}{2c_0}\tau^2L_xL_y\frac{1-\tau^{s+1}}{1-\tau}\sum_{r=1}^{R}M_{s+1,r}^*, \tag{43}$$

where

$$M_{s+1,r}^*=\max_{(x,y)\in\bar{G}}\left\{M_{01,r}^{s+1};M_{02,r}^{s+1};\ldots;M_{0s+1,r}^{s+1}\right\},$$

$$\left\|Q(x,y,t)\right\|_{L_1\left(G\times\left[t_s,t_{s+1}\right]\right)} \equiv \int_{t_s}^{t_{s+1}}\left(\iint_G \left|Q(x,y,t)\right| dxdy\right)dt.$$

The inductive step has been completed, allowing us to state the validity of the assertion for any s, $1 < s \le N$.

Using the estimates (37), (40), and (43) from formula (22), we obtain:

$$\left\|H_{lp}(T)-H_{np}(T)\right\|_{L_1(G\times[0,T])} \le \frac{1}{2c_0(1-\tau)}\tau^2 L_x L_y C\left((1-\tau)+\left(1-\tau^2\right)+\ldots+\left(1-\tau^{N+1}\right)\right) \le$$
$$\le \frac{1}{2c_0(1-\tau)}\tau^2 L_x L_y C\left[N+1-\frac{\tau-\tau^{N+2}}{1-\tau}\right], \tag{44}$$

where

$$C \equiv \max_{(x,y)\in\bar{G}}\left\{\sum_{r=1}^{R} M_{1,r}^{*}, \sum_{r=1}^{R} M_{2,r}^{*}, \ldots, \sum_{r=1}^{R} M_{s+1,r}^{*}\right\},$$

$$\left\|Q(x,y,t)\right\|_{L_1(G\times[0,T])} \equiv \int_0^T\left(\iint_G\left|Q(x,y,t)\right|dxdy\right)dt = \sum_{n=1}^{N}\left[\int_{t_{n-1}}^{t_n}\left(\iint_G\left|Q(x,y,t)\right|dxdy\right)dt\right].$$

Since $N\tau \equiv T \equiv const$, inequality (44) leads to

$$\left\|H_{lp}(T)-H_{np}(T)\right\|_{L_1(G\times[0,T])} = O(\tau), \tag{45}$$

which completes the study of the convergence of the linearized problem to the solution of the original nonlinear problem.

Discussion and Conclusion. The conditions for the convergence of the solutions of the linearized sediment transport problem with a multicomponent composition to the solution of the nonlinear problem in the Banach space norm L_1 with a rate $O(\tau)$ of convergence have been determined. The obtained research results can be used in the forecasting of nonlinear hydrophysical processes, improving their accuracy and reliability due to the availability of new functional capabilities for accounting for physically significant factors.

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