MATHEMATICAL MODELLING
МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ

UDC 004.942:532.1

Original Empirical Research

https://doi.org/10.23947/2587-8999-2025-9-4-46-55

Mathematical Modelling of the Bioproductivity of a Shallow Water Body under Sudden Depression Caused by Scyphozoan Jellyfish

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Abstract

Introduction. The relevance of this study is determined by the need for a quantitative assessment of the negative impact of mass outbreaks of scyphozoan jellyfish (*Aurelia aurita* and *Rhizostomeae*) on the bioresources of the Azov Sea, which is subjected to a complex combination of anthropogenic pressures. The theoretical framework of the research is based on the concept of trophic interactions and biological invasions in marine ecosystems. The aim of this study is to develop a mathematical model of the dynamics of the fish community in the Azov Sea that accounts for both competitive and predatory pressure exerted by jellyfish, in order to identify critical biomass thresholds leading to the depression of commercial fish stocks.

Materials and Methods. To investigate the influence of scyphozoan jellyfish on the bioresources of the Azov Sea, a mathematical model of biological kinetics was employed as the primary research tool. The model describes the dynamics of three key ecosystem components (zooplankton, fish, and jellyfish), incorporating mechanisms of competition and predation. The research material consists of a system of theoretical equations with appropriate interaction parameters and initial and boundary conditions.

Results. Numerical simulations demonstrated that under environmental conditions typical of the summer period in the Azov Sea (elevated water temperature and eutrophication), an increase in scyphozoan jellyfish biomass by more than threefold during July–August leads to an abrupt shift of the ecosystem to an alternative stable state dominated by jellyfish. This transition is driven by the combined effects of intense competition for zooplankton and direct predation by jellyfish on the early life stages of fish, and is accompanied by a critical reduction in food availability, which suppresses the recovery of commercial fish populations.

Discussion. The results confirm the high ecological significance of mass aggregations of scyphozoan jellyfish and provide a quantitative justification for the risk of a regime shift in the Azov Sea ecosystem toward an alternative, less productive state dominated by jellyfish. From a theoretical perspective, the study contributes to the development of trophic interaction models that incorporate multiple impact mechanisms of invasive species.

Conclusion. The practical significance of this work lies in the fact that the proposed model serves as a tool for predictive assessment of bioresource status and for substantiating management decisions aimed at mitigating the consequences of eutrophication and biological invasions. Future research will focus on further refinement of the model, including the incorporation of seasonal and climatic factors to improve the accuracy of long-term forecasts.

Keywords: mathematical modelling, bioproductivity, Azov Sea, scyphozoan jellyfish, trophic interactions, alternative stable states, ecological forecasting

Funding. The study was supported by grant No. 22–11–00295–П from the Russian Science Foundation, https://rscf.ru/project/22-11-00295/

For Citation. Bondarenko D.V., Nikitina A.V. Mathematical modelling of the bioproductivity of a shallow water body under sudden depression caused by scyphozoan jellyfish. *Computational Mathematics and Information Technologies*. 2025;9(4):46–55. https://doi.org/10.23947/2587-8999-2025-9-4-46-55

Оригинальное эмпирическое исследование

Математическое моделирование биопродуктивности мелководного водоема при внезапной депрессии сцифоидными медузами

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Аннотация

Введение. Актуальность исследования обусловлена необходимостью количественной оценки негативного влияния массового развития сцифоидных медуз (*Aurelia aurita* и *Rhizostomeae*) на биоресурсы Азовского моря, испытывающего комплекс антропогенных нагрузок. Теоретической основой для решения данной проблемы выступает концепция трофических взаимодействий и инвазий в морских экосистемах. Целью настоящей работы является разработка математической модели динамики рыбного сообщества Азовского моря, учитывающей конкурентное и хищническое давление со стороны медуз, для оценки критических порогов его биомассы, приводящих к депрессии промысловых запасов.

Материалы и методы. Для исследования влияния сцифомедуз на биоресурсы Азовского моря в качестве основного инструмента использована математическая модель биологической кинетики, описывающая динамику трёх ключевых компонентов (зоопланктон, рыбы, медузы) с учётом конкуренции и хищничества. Материалом исследования выступали теоретические уравнения системы с соответствующими параметрами взаимодействий и начально-краевыми условиями.

Результаты исследования. Результаты численного моделирования показали, что при характерных для летнего сезона условиях в Азовском море (высокая температура, эвтрофикация) прирост биомассы сцифомедуз более чем в три раза за период июль-август приводит к резкому переходу экосистемы в альтернативное устойчивое состояние с их доминированием. Этот переход обусловлен комбинированным эффектом интенсивной конкуренции за зоопланктон и прямого хищничества медуз на ранних стадиях развития рыб и сопровождается критическим снижением доступности кормовой базы, что подавляет восстановление промысловых рыбных популяций.

Обсуждение. Проведённое исследование подтверждает высокую экологическую значимость массовых скоплений сцифомедуз и количественно обосновывает риск перехода экосистемы Азовского моря в альтернативное, менее продуктивное состояние, доминируемое медузами. С теоретической точки зрения работа вносит вклад в развитие моделей трофических взаимодействий с учётом множественных механизмов воздействия инвазионных видов.

Заключение. Практическая значимость работы заключается в том, что разработанная модель представляет собой инструмент для прогнозной оценки состояния биоресурсов и обоснования управленческих решений, направленных на смягчение последствий эвтрофикации и биологических инвазий. Перспективы исследования связаны с дальнейшей детализацией модели и включением в неё сезонных и климатических факторов для повышения точности долгосрочных прогнозов.

Ключевые слова: математическое моделирование, биопродуктивность, Азовское море, сцифоидные медузы, трофические взаимодействия, альтернативные устойчивые состояния, экологический прогноз

Финансирование. Исследование выполнено за счет гранта Российского научного фонда № 22–11–00295–П, https://rscf.ru/project/22-11-00295/

Для цитирования. Бондаренко Д.В., Никитина А.В. Математическое моделирование биопродуктивности мелководного водоема при внезапной депрессии сцифоидными медузами. *Computational Mathematics and Information Technologies*. 2025;9(4):46–55. https://doi.org/10.23947/2587-8999-2025-9-4-46-55

Introduction. The Azov Sea is one of the shallowest seas in the world and plays a key role in the fisheries and ecological system of southern Russia. Over recent decades, its ecosystem has been subjected to substantial pressures, including eutrophication, changes in river runoff, pollution, and invasions of alien species [1]. One of the most pronounced recent trends is the mass development of scyphozoan jellyfish, primarily Aurelia aurita and representatives of the order Rhizostomeae. Owing to their high reproductive potential and ecological plasticity, these jellyfish form extensive aggregations in coastal waters during the spring–summer period. In certain years, their total biomass reaches thousands of tons, exerting significant pressure on trophic networks [2].

Under such conditions, it is reasonable to speak of a sudden depression of the water body caused by scyphozoan jellyfish and their impact on the bioproductivity of the aquatic ecosystem. Fig. 1 illustrates aggregations of scyphozoan jellyfish observed in the Azov Sea.

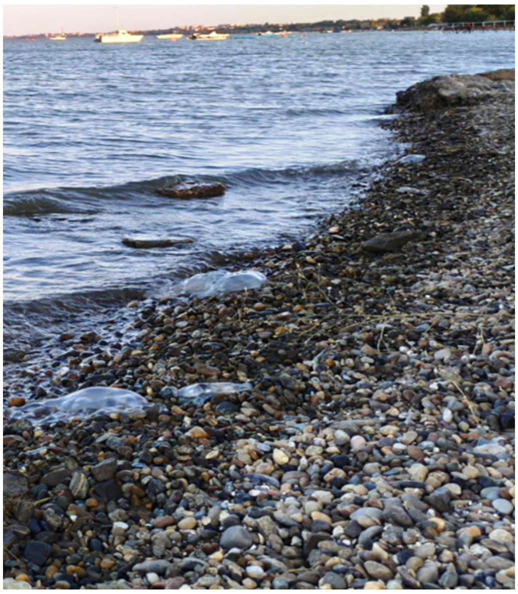

Fig. 1. Coast of the Yeysk Estuary with jellyfish, July 2025

Scyphozoan jellyfish may pose a danger to humans, as contact with their tentacles can cause skin burns, itching, or irritation. Scyphozoan jellyfish (class *Scyphozoa*, phylum *Cnidaria*) are marine organisms characterized by the presence of stinging cells (cnidocytes), which they use for predation and defense. The class comprises a relatively small number of species, approximately 200 in total. Their life cycle is characterized by metagenesis, including an asexual (polypoid) stage and a sexual (medusoid) stage. The medusae of some species reach large sizes and are capable of forming massive aggregations, whereas scyphozoan polyps (scyphistomae) are extremely small, typically only a few millimeters in size.

Common scyphozoan jellyfish species found in the seas of Russia include the moon jellyfish (*Aurelia aurita*), the lion's mane jellyfish (*Cyanea capillata*), and the barrel jellyfish (*Rhizostoma pulmo*). Some scyphozoan species, such as the so-called "sea wasp", are particularly dangerous: contact causes intense pain and burning sensations comparable to a whip strike. Severe pain shock may lead to loss of consciousness, followed by symptoms of intoxication such as dry mouth and breathing difficulties; in rare cases, stings may be fatal.

In the Azov Sea, *Aurelia* aurita and *Rhizostomeae* actively consume zooplankton, including copepods and larvae of crustaceans and mollusks, which constitute the primary food source for juvenile and planktivorous fish species such as Baltic herring, roach, and juvenile pikeperch. This results in intense competition for food resources. In addition, jellyfish exhibit direct predation on fish eggs and larvae. Field observations indicate that at high abundances, jellyfish may consume up to 10–30% of the daily ration of fish larvae [3]. Despite the availability of biological observations, a quantitative assessment of the impact of jellyfish on fish stocks remains insufficient [4–7].

This study presents a mathematical model describing the dynamics of a fish community while explicitly accounting for competitive and predatory pressure exerted by jellyfish. The model enables the estimation of critical jellyfish biomass thresholds at which the productivity of valuable and commercially important fish species in the Azov Sea becomes suppressed [8]. Therefore, the emergence of scyphozoan jellyfish as invasive species may lead to a sudden depression of the state of the main bioresources of the Azov Sea.

Materials and Methods. To quantitatively assess the impact of scyphozoan jellyfish (*Aurelia aurita* and *Rhizostomeae*) on fish communities of the Azov Sea, a dynamic model was developed describing the interaction of three key ecosystem components: $Z(t)$ — zooplankton concentration (resource), $F(t)$ — biomass of the fish community, $J(t)$ — biomass of scyphozoan jellyfish. The model incorporates two primary mechanisms by which jellyfish affect fish populations: competition for a shared food resource—zooplankton, and direct predation by jellyfish on fish eggs and larvae. The structure of these interactions is illustrated in Fig. 2.

In Fig. 2, the following notations are used: (1) consumption of zooplankton by scyphozoan jellyfish; (2) consumption of zooplankton by fish communities; (3) predation by scyphozoan jellyfish on fish eggs and larvae; (4) influence of changes in external environmental factors (temperature, salinity, etc.); (5) influence of terrestrial-driven environmental changes (anthropogenic pressure, river runoff, eutrophication).

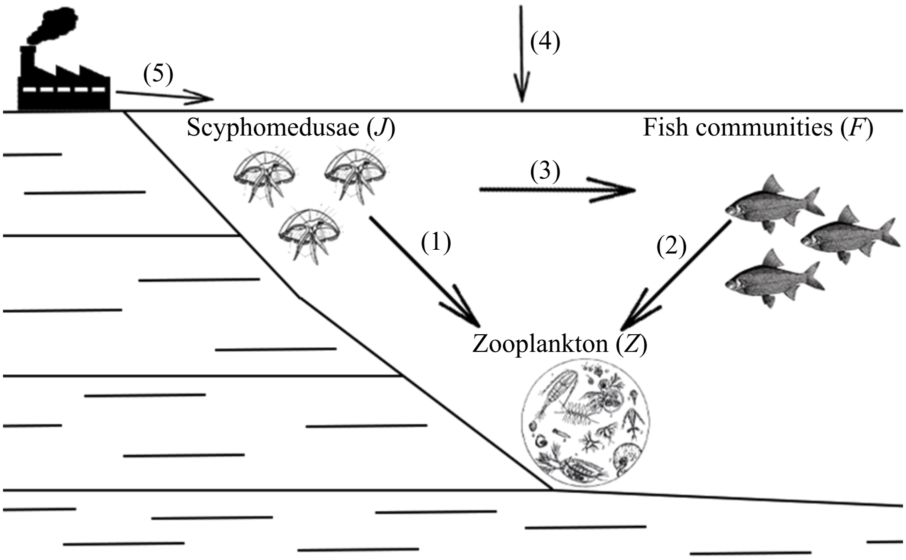

Fig. 2. Structural scheme of trophic interactions between jellyfish, fish, and zooplankton

The biological kinetics model is based on well-established approaches [9, 10] and has the following form:

$$\begin{cases} \dfrac{\partial Z}{\partial t}+u\dfrac{\partial Z}{\partial x}+v\dfrac{\partial Z}{\partial y}+(w+w_{gZ})\dfrac{\partial Z}{\partial z}=\dfrac{\partial}{\partial x}\left(\mu_Z\dfrac{\partial Z}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(\mu_Z\dfrac{\partial Z}{\partial y}\right)+\dfrac{\partial}{\partial z}\left(\nu_Z\dfrac{\partial Z}{\partial z}\right)+\psi_Z, \\ \dfrac{\partial F}{\partial t}+u\dfrac{\partial F}{\partial x}+v\dfrac{\partial F}{\partial y}+(w+w_{gF})\dfrac{\partial F}{\partial z}=\dfrac{\partial}{\partial x}\left(\mu_F\dfrac{\partial F}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(\mu_F\dfrac{\partial F}{\partial y}\right)+\psi_F, \\ \dfrac{\partial J}{\partial t}+u\dfrac{\partial J}{\partial x}+v\dfrac{\partial J}{\partial y}+(w+w_{gJ})\dfrac{\partial J}{\partial z}=\dfrac{\partial}{\partial x}\left(\mu_J\dfrac{\partial J}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(\mu_J\dfrac{\partial J}{\partial y}\right)+\psi_J, \end{cases} \tag{1}$$

$$\psi_Z=rZ\left(1-\frac{Z}{K}\right)-\frac{a_Z FZ}{1+h_a a_Z Z}-\frac{b_Z JZ}{1+h_b b_Z Z},$$

$$\psi_F=\alpha\left(\frac{a_Z FZ}{1+h_a a_Z Z}\right)-d_F F-\varepsilon JF,\quad \psi_J=\beta\left(\frac{b_Z FZ}{1+h_b b_Z Z}\right)+\delta JF-d_J J.$$

In system (1), the following notations are introduced: $\mathbf{u}=(u,v,w)$ — velocity field of the water flow; $w_{g\varphi}$ — settling (sedimentation) velocity of substance φ, $\varphi\in\{Z,F,G\}$; μ_φ, v_φ — diffusion coefficients of substance φ, $\varphi\in\{Z,F,G\}$ in the horizontal and vertical directions, respectively; r — intrinsic growth rate of zooplankton; K — environmental carrying capacity; a_z — grazing (consumption) rate coefficient of zooplankton by fish; b_z — grazing (consumption) rate coefficient of zooplankton by jellyfish; h_a — food handling time for fish; h_b — food handling time for jellyfish; α — fish biomass growth coefficient (zooplankton-to-fish conversion efficiency); d_F — natural mortality coefficient; ε — fish biomass loss coefficient accounting for jellyfish predation on fish eggs and larvae; β — jellyfish growth coefficient; δ — nonlinear predation coefficient; d_J — jellyfish mortality coefficient.

Let Γ denote the boundary of the spatial domain G, $\overline{G}=G\cup\Gamma$; where σ is the lateral boundary surface, $\Sigma_{пов}$ is a part of the free water surface, and $\Sigma_{дно}$ — is the bottom surface. $\Gamma=\sigma\cup\Sigma_{пов}\cup\Sigma_{дно}$.

We specify:

– initial conditions at $t=0$

$$\varphi(x,y,z,0)\equiv\varphi_0(x,y,z); \tag{2}$$

– boundary conditions on the lateral surface σ at any time $\sigma\times(0,T]$

$$\frac{\partial\varphi}{\partial\mathbf{n}}=0,\ \text{if}\ (\mathbf{u}_\Gamma,\mathbf{n})<0, \tag{3}$$

$$\frac{\partial\varphi}{\partial\mathbf{n}}=-\frac{u_\Gamma}{\mu_\varphi}\varphi,\ \text{if}\ (\mathbf{u}_\Gamma,\mathbf{n})\geq 0, \tag{4}$$

where $\mathbf{n}$ is the outward unit normal vector to the boundary of the domain σ; $\mathbf{u}_\Gamma$ is the fluid velocity vector on the boundary S; u_Γ — is the normal component of the flow velocity $\mathbf{n}$ at the domain boundary G;

– boundary conditions at the water surface

$$\frac{\partial\varphi}{\partial\mathbf{n}} = 0; \tag{5}$$

– boundary conditions at the bottom surface $\Sigma_{дно} \times \left(0 < t \le T\right]$

$$\frac{\partial\varphi}{\partial\mathbf{n}} = -\frac{w_{g\varphi}}{\nu_i}\varphi,\ \varphi \in \{Z, F, J\}. \tag{6}$$

To investigate the dynamics of the considered hydrobionts of the Azov Sea ecosystem and to identify the key patterns governing the interactions between scyphozoan jellyfish and fish communities, a qualitative analysis of the proposed nonlinear system of ordinary differential equations is performed. The main focus is placed on the identification of equilibrium states (steady states) and on the analysis of their stability based on the Jacobian matrix. This approach makes it possible to determine the conditions under which coexistence of the selected ecosystem components is feasible, as well as to identify threshold parameter values beyond which a transition to an alternative stable state occurs, such as jellyfish dominance.

Let us consider system (1). The equilibrium states of the system are determined by setting all right-hand sides equal to zero:

$$\frac{dZ}{dt} = 0, \frac{dF}{dt} = 0, \frac{dJ}{dt} = 0.$$

We examine four biologically relevant cases.

1. Trivial equilibrium $E_0 = (K,0,0)$.

This equilibrium corresponds to the absence of both fish and jellyfish populations, while zooplankton reaches the carrying capacity of the environment. Substituting $F = 0$, $J = 0$ into system (1) yields:

$$rZ\left(1 - \frac{Z}{K}\right) = 0 \Rightarrow Z = K.$$

Hence, $E_0 = (K,0,0)$ is an equilibrium point. To analyze its stability, we compute the coefficients of the Jacobian matrix in a neighborhood of E_0:

$$J\left(E_0\right) = \left.\begin{pmatrix} \frac{\partial Z}{\partial Z} & \frac{\partial Z}{\partial F} & \frac{\partial Z}{\partial J} \\ \frac{\partial F}{\partial Z} & \frac{\partial F}{\partial F} & \frac{\partial F}{\partial J} \\ \frac{\partial J}{\partial Z} & \frac{\partial J}{\partial F} & \frac{\partial J}{\partial J} \end{pmatrix}\right|_{(K,0,0)}.$$

After computing the partial derivatives and substituting the equilibrium values, we obtain:

$$J\left(E_0\right) = \begin{pmatrix} -r & -\frac{a_Z K}{1 + h_a a_Z K} & -\frac{b_Z K}{1 + h_b b_Z K} \\ 0 & \alpha\frac{a_Z K}{1 + h_a a_Z K} - d_F & 0 \\ 0 & 0 & \beta\frac{b_Z K}{1 + h_b b_Z K} - d_J \end{pmatrix}.$$

Since the Jacobian matrix is upper triangular, its eigenvalues are given by the diagonal elements:

$$\lambda_1 = -r < 0, \lambda_2 = \alpha\frac{a_Z K}{1 + h_a a_Z K} - d_F, \lambda_3 = \beta\frac{b_Z K}{1 + h_b b_Z K} - d_J.$$

The equilibrium E_0 is asymptotically stable if $\lambda_2 < 0$ and $\lambda_3 < 0$, i. e.

$$\alpha\frac{a_Z K}{1 + h_a a_Z K} < d_F,\ \beta\frac{b_Z K}{1 + h_b b_Z K} < d_J, \text{ at } \alpha > 0, \beta > 0.$$

These inequalities define threshold values of the carrying capacity K, below which neither fish nor jellyfish are able to colonize the ecosystem. When these thresholds are exceeded, the equilibrium E_0 becomes unstable, and growth of one or both populations is initiated.

2. Fish-only equilibrium $E_F = \left(Z_F^*, F^*, 0\right)$.

Next, we consider the case where jellyfish are absent, i. e., $J = 0$, $F > 0$. This corresponds to a state in which the fish community is established and maintained through interaction with zooplankton. From system (1), we obtain:

$$\frac{dZ}{dt} = 0 \Rightarrow rZ\left(1 - \frac{Z}{K}\right) = \frac{a_Z F Z}{1 + h_a a_Z Z},$$

$$\frac{dF}{dt}=0\Rightarrow\alpha\frac{a_ZFK}{1+h_aa_ZK}=d_FF\Rightarrow\alpha\frac{a_ZK}{1+h_aa_ZK}=d_F\Rightarrow\frac{a_ZZ}{1+h_aa_ZZ}=\frac{d_F}{\alpha}.$$

Solving the corresponding equations yields:

$$Z_F^*=\frac{d_F}{a_Z\left(\alpha-h_ad_F\right)}\ (\mathit{upon\ condition}\,\alpha>h_ad_F).$$

Substituting Z_J^* into the first equation allows us to determine the equilibrium fish biomass F^*. Thus, a nontrivial equilibrium $E_F=\left(Z_F^*,F,0\right)$, exists provided that $d_F<\alpha\frac{a_Z}{1+h_aa_ZK}$. The stability of E_F depends critically on the ability of jellyfish to invade this equilibrium. If $\beta\frac{b_ZZ_F^*}{1+h_bb_ZZ_F^*}>d_J$, then jellyfish can successfully invade the system, implying that the equilibrium is unstable.

3. Jellyfish-only equilibrium $E_J=\left(Z_F^*,0,J^*\right)$.

We now consider the case opposite to the fish-only equilibrium, namely $F = 0$, $J > 0$. This state corresponds to domination of jellyfish communities. Substituting $F = 0$ into system (1) yields:

$$\frac{dZ}{dt}=0\Rightarrow rZ\left(1-\frac{Z}{K}\right)=\frac{JZ}{1+h_bZ},$$

$$\frac{dJ}{dt}=0\Rightarrow\beta\frac{JZ}{1+h_bZ}=d_JJ\Rightarrow\beta\frac{Z}{1+h_bZ}=d_J\Rightarrow\frac{Z}{1+h_bZ}=\frac{d_J}{\beta}.$$

From the remaining equations, we obtain:

$$Z_J^*=\frac{d_J}{\left(\beta-h_bd_J\right)}\ (\mathit{upon\ condition}\ \beta>h_bd_J).$$

Substituting Z_J^* into the first equation, we obtain the equilibrium jellyfish biomass J^*. Thus, a jellyfish-only equilibrium $E_J=\left(Z_F^*,0,J^*\right)$, exists, corresponding to the exclusion of fish by jellyfish. The stability of E_J in this case depends on the impact of fish; specifically, if $\alpha\frac{a_ZZ_J^*}{1+h_aa_ZZ_J^*}>d_F$, then fish are able to invade the system and begin displacing jellyfish, which implies that the equilibrium is unstable.

4. Coexistence conditions and bifurcation.

Coexistence of fish and jellyfish is possible provided that the following conditions are simultaneously satisfied:

$$\alpha\frac{a_ZZ}{1+h_aa_ZZ}>d_F+\varepsilon J,\ \beta\frac{b_ZZ}{1+h_bb_ZZ}+\delta F>d_J.$$

However, numerical analysis indicates that the coexistence region is narrow. As the parameters J and b_z increase, the system loses stability and competitive exclusion occurs, resulting in $F\to 0$. The critical jellyfish biomass threshold at which this transition takes place can be estimated from the condition:

$$\alpha\frac{a_ZZ}{1+h_aa_ZZ}=d_F+\varepsilon J.$$

For a fixed value of $Z\approx Z_J^*$, this equation yields a threshold value, exceeding this value renders the persistence of the fish community impossible.

After analyzing the stability of the equilibrium states, we proceed to a qualitative analysis of the system dynamics, which allows visualization of typical ecosystem trajectories and identification of key behavioral scenarios. To this end, phase portraits are constructed in the projection of fish biomass F and scyphozoan jellyfish biomass J at a fixed zooplankton level Z, This corresponds to a quasi-stationary approximation (dimension reduction), which is commonly employed in models of biological kinetics [11]. We consider a simplified system describing the dynamics of F and J at a constant zooplankton concentration $Z = Z^*$:

$$\begin{cases}\dfrac{dF}{dt}=\alpha\left(\dfrac{a_ZFZ^*}{1+h_aa_ZZ^*}\right)-d_FF-\varepsilon JF,\\[2ex]\dfrac{dJ}{dt}=\beta\left(\dfrac{b_ZFZ^*}{1+h_bb_ZZ^*}\right)+\delta JF-d_JJ.\end{cases}\tag{7}$$

This approach eliminates the fast zooplankton dynamics and focuses attention on the long-term interaction between jellyfish and fish under a given level of ecosystem productivity. Fig. 3 presents the phase portrait of system (7) for $Z^* = 1.5$ (dimensionless units).

Analysis of the phase portrait reveals the presence of two attractors:

1. A stable equilibrium characterized by fish dominance, which is observed at low initial jellyfish biomass.
2. A jellyfish-dominated state, which is reached when the jellyfish biomass exceeds a critical threshold.

Between these attractors lies the boundary of the basins of attraction, which determines which of the two scenarios is realized depending on the initial conditions. This behavior indicates the existence of alternative stable states in the ecosystem: under identical external parameters, two qualitatively different equilibrium regimes may occur.

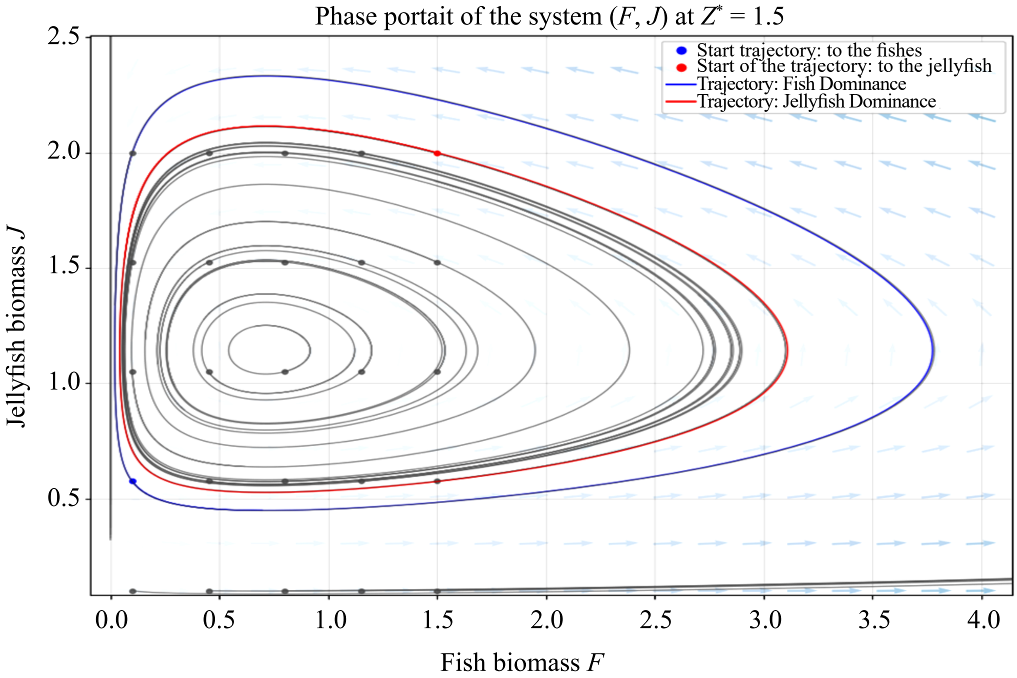

Fig. 3. Phase portrait of the system in the (F, J) plane at Z^* = 1.5 (dimensionless units)

To quantitatively assess the transition between these regimes, a bifurcation diagram was constructed, representing the dependence of the equilibrium fish biomass on jellyfish biomass (Fig. 4). Such behavior is typical of systems with positive feedback mechanisms: an increase in jellyfish abundance leads to a decline in zooplankton growth and elevated mortality of fish larvae, which in turn reduces competition for food resources and promotes further growth of the jellyfish population. As a result, the transition to the alternative state becomes weakly reversible in the absence of external intervention, such as mitigation of eutrophication in the aquatic ecosystem [12–14].

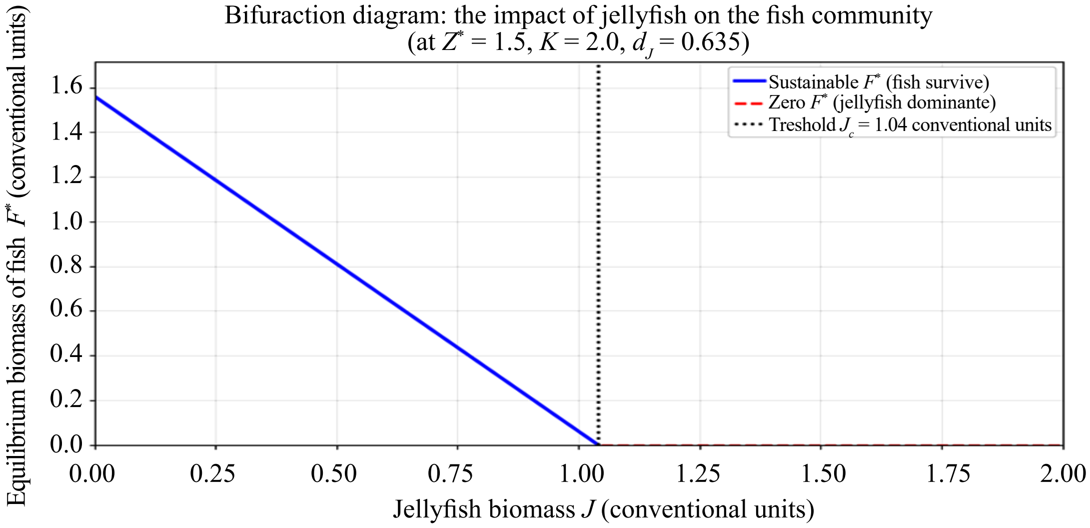

Fig. 4. Bifurcation diagram showing the equilibrium fish biomass as a function of jellyfish biomass

The obtained results confirm that the Azov Sea ecosystem can exist in two alternative stable states:

– a fish-centered state (at moderate jellyfish abundance);

– a jellyfish-centered state (when jellyfish abundance exceeds a critical threshold).

Thus, phase-plane analysis clearly demonstrates the risk of ecological collapse and highlights the necessity of continuous environmental monitoring and ecosystem management in the Azov Sea [9, 10].

Results. Numerical modelling of the Azov Sea ecosystem dynamics for the period from July 1 to August 31, 2025 revealed a substantial impact of scyphozoan jellyfish (*Rhizostomeae* and *Aurelia aurita*) on the state of the fish community through a combined effect of competition for zooplankton and direct predation on early life stages of fish. The initial conditions were chosen to represent a typical ecosystem state at the beginning of July, when jellyfish begin active reproduction, while fish populations exploit high plankton productivity to support juvenile growth [15].

The key model parameters were specified as follows:

- $r = 0.8$ day^{-1} — zooplankton growth rate, corresponding to elevated water temperatures (22–26 °C) and high nutrient availability due to eutrophication;;
- $K = 2.0$ dimensionless units — environmental carrying capacity, reflecting the maximum sustainable zooplankton biomass in the coastal zone;
- $a_z = 1.2$; $b_z = 2$ — zooplankton consumption rates by fish and jellyfish, respectively. The value accounts for the high filtration capacity of *Aurelia aurita*, which is capable of processing large volumes of water;
- $h_a = 0.9$; $h_b = 0.3$ — food handling times. The lower value indicates the higher efficiency of jellyfish as filter feeders compared to fish;
- $\alpha = 0.3$; $\beta = 0.4$ — food-to-biomass conversion efficiency coefficients. The coefficient is higher for jellyfish, reflecting their lower energetic costs for maintenance metabolism;
- $\delta = 0.03$ — additional jellyfish biomass gain due to consumption of fish eggs and larvae, characterizing their predatory activity;
- $d_F = 0.05$; $d_J = 0.635$ — natural mortality rates. Jellyfish mortality increases toward the end of August as a result of strobilation and post-reproductive senescence;
- $\varepsilon = 0.15$ — predation coefficient of jellyfish on fish larvae.

The temporal dynamics of all three system components — zooplankton, fish, and jellyfish — are illustrated in Fig. 5.

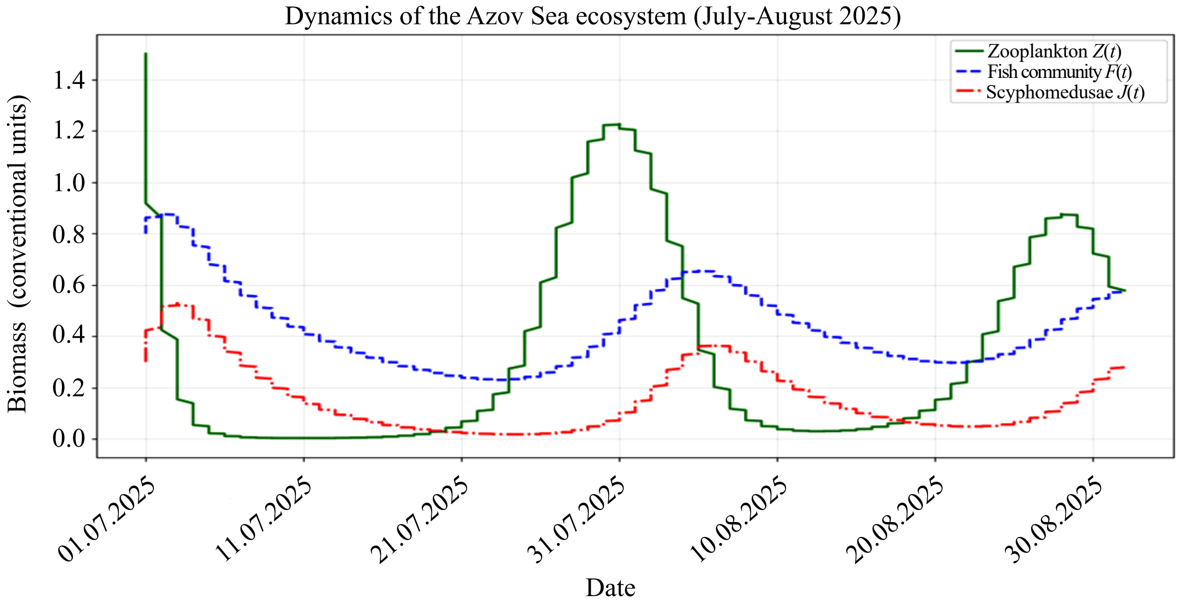

Fig. 5. Results of the numerical experiment illustrating the dynamics of the main hydrobionts and scyphozoan jellyfish in the Azov Sea ecosystem

The numerical simulations show that, for an initial jellyfish biomass of 0.3 (dimensionless units) and under favorable environmental conditions (high water temperature and eutrophication), jellyfish abundance increases by more than threefold by the end of August. This growth leads to a sharp reduction in zooplankton availability and a pronounced suppression of fish biomass growth. The obtained results demonstrate a transition of the ecosystem to a state in which jellyfish temporarily dominate the trophic structure, thereby limiting the recovery of fish populations.

Discussion. In this study, a mathematical model describing the complex trophic interactions between scyphozoan jellyfish and fish communities in the Azov Sea ecosystem was developed, analyzed, and numerically implemented. The proposed model explicitly accounts for both competition for a shared resource—zooplankton—and direct predation by jellyfish on the early life stages of fish, which makes it more realistic than classical “resource–consumer” systems.

The results of numerical experiments for the period July-August 2025 indicate that mass proliferation of jellyfish can lead to a substantial suppression of the productivity of valuable and commercially important fish species, especially under conditions of ongoing eutrophication and increasing water temperature. This points to a risk of a persistent shift of the ecosystem toward a jellyfish-dominated regime, which reduces both ecosystem resilience and fisheries value.

Conclusion. Mathematical modelling confirms the necessity of comprehensive monitoring of gelatinous invasive species and their integration into environmental management frameworks for marine resources in southern Russia. The proposed model can be used as a tool for forecasting ecosystem states, assessing the effectiveness of environmental protection measures, and substantiating the management of fishing pressure in the region. In future work, the model may be extended by incorporating seasonal variability of external factors and the influence of climate change [16].

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***Conflict of Interest Statement:* the authors declare no conflict of interest.**

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Заявленный вклад авторов:

Д.В. Бондаренко: проведение расчетов; подготовка текста; оформление графических материалов; работа с источниками.

А.В. Никитина: формирование основной концепции; цели и задачи исследования; анализ результатов исследований; формулирование выводов; корректировка текста.

Конфликт интересов: ***авторы заявляют об отсутствии конфликта интересов.***

Все авторы прочитали и одобрили окончательный вариант рукописи.

Received / Поступила в редакцию 27.10.2025
Reviewed / Поступила после рецензирования 12.11.2025
Accepted / Принята к публикации 10.12.2025