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## Mathematical Modelling of Green Microalgae Invasion and Rehabilitation of the Taganrog Bay: Ecological-Hygienic and Medical Consequences

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### Abstract

***Introduction.*** The Taganrog Bay of the Azov Sea is one of the most eutrophic and ecologically vulnerable water areas in Russia, where massive blooms of toxic cyanobacteria (*Microcystis, Aphanizomenon, Anabaena, Nodularia*) regularly occur during summer. Their proliferation is accompanied by the accumulation of cyanotoxins (microcystin, anatoxin, cylindrospermopsin, saxitoxin), posing a serious threat to public health. This paper considers an approach to the biological rehabilitation of the bay based on the controlled introduction of the freshwater green microalgae *Chlorella vulgaris*, which competes with cyanobacteria for nutrients. The aim of the study is to develop and apply a comprehensive mathematical model describing phytoplankton kinetics and substance transport processes under conditions of increasing bay salinity, as well as to assess the ecological-hygienic and medical consequences of the proposed method.

***Materials and Methods.*** The research object is the Taganrog Bay of the Azov Sea. The modelling is based on the three-dimensional hydrodynamic model "Azov3D", previously used to calculate currents and vertical mixing under conditions of changing salinity. Water environment parameters (salinity, temperature, current velocities) were used as input data for solving the linearized hydrobiological problem. The source of bathymetric data was digitized nautical charts processed using automated depth recognition algorithms. The model grid was generated considering the actual coastline configuration and bottom topography. Calculations were performed on the computing cluster of the Southern Federal University. The numerical method is based on finite-difference schemes previously applied for hydrobiological calculations in the Azov Sea.

***Results.*** It is shown that a 30% increase in salinity leads to a shift in the cyanobacteria habitat from the Azov Sea water area to the eastern part of the Taganrog Bay, which is consistent with hydrological observations. Model calculations demonstrate an increase in the proportion of green algae with the controlled introduction of *Chlorella vulgaris* cultures, reflecting the potential for biomelioration. The forecast of the spatial distribution of populations shows stable dominance of green and blue-green algae, constituting 60‒70% of the bay's phytoplankton biomass, under various impact scenarios.

***Discussion.*** The results indicate that mathematical modelling is an effective tool for predicting the dynamics of phytoplankton populations under changing hydrological conditions. The model allows for assessing the influence of biological regulation and salinization scenarios, providing a basis for management decisions in the field of ecological rehabilitation of water bodies.

***Conclusion.*** The application of *Chlorella vulgaris* may be a promising biomelioration method but requires further verification based on field observations and controlled field experiments. The modelling results indicate the possibility of adaptive ecological management of the Taganrog Bay and minimizing the risk of toxic blooms.

**Keywords:** phytoplankton dynamics, *Chlorella vulgaris*, eutrophication modelling, hydrodynamic model, convection-diffusion equations, substance transport, cyanobacterial bloom, numerical modelling, biological regulation, Taganrog Bay

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*Оригинальное эмпирическое исследование*

# Математическое моделирование инвазии зеленых микроводорослей и оздоровления Таганрогского залива: эколого-гигиенические и медицинские последствия

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**Аннотация**

***Введение.*** Таганрогский залив Азовского моря является одной из наиболее эвтрофных и экологически уязвимых акваторий России, где в летний период регулярно формируются массовые цветения токсичных цианобактерий (*Microcystis, Aphanizomenon, Anabaena, Nodularia*). Их развитие сопровождается накоплением цианотоксинов (микроцистин, анатоксин, цилиндроспермопсин, сакситоксин), представляющих серьёзную угрозу для здоровья населения. В работе рассматривается подход к биологической реабилитации залива на основе контролируемого внесения пресноводных зелёных микроводорослей *Chlorella vulgaris*, конкурирующих с цианобактериями за биогенные элементы. Цель исследования заключается в разработке и применении комплексной математической модели, описывающей кинетику фитопланктона и процессы переноса веществ в условиях осолонения залива, а также в оценке эколого-гигиенических и медицинских последствий предложенного метода.

***Материалы и методы.*** Объектом исследования является Таганрогский залив Азовского моря. Моделирование выполнено на основе трёхмерной гидродинамической модели «Azov3D», ранее применённой для расчётов течений и вертикального перемешивания в условиях изменяющейся солёности. Параметры водной среды (солёность, температура, скорости течений) использовались как входные данные для решения линеаризованной гидробиологической задачи. Источник батиметрических данных — оцифрованные лоцманские карты, обработанные с применением автоматизированных алгоритмов распознавания глубин. Сеточная основа модели формировалась с учётом реальной конфигурации береговой линии и рельефа дна. Расчёты выполнялись на вычислительном кластере Южного федерального университета. Численный метод основан на разностных схемах, применяемых ранее для гидробиологических расчётов в Азовском море.

***Результаты исследования.*** Показано, что увеличение солёности на 30 % приводит к смещению ареала цианобактерий из акватории Азовского моря в восточную часть Таганрогского залива, что согласуется с гидрологическими наблюдениями. Модельные расчёты демонстрируют усиление доли зелёных водорослей при контролируемом внесении культур *Chlorella vulgaris*, что отражает потенциал биомелиорации. Прогноз пространственного распределения популяций показывает устойчивое доминирование зеленых и синезеленых водорослей, составляющих 60–70 % биомассы фитопланктона залива, при различных сценариях воздействия.

***Обсуждение.*** Результаты показывают, что математическое моделирование является эффективным инструментом для прогнозирования динамики фитопланктонных популяций в условиях изменяющейся гидрологии. Модель позволяет оценить влияние биологической регуляции и сценариев осолонения, предоставляя основу для принятия управленческих решений в сфере экологического оздоровления водоёмов.

***Заключение.*** Применение *Chlorella vulgaris* может быть перспективным методом биомелиорации, однако требует дальнейшей проверки с опорой на натурные наблюдения и контролируемые полевые эксперименты. Модельные результаты указывают на возможность адаптивного экологического управления Таганрогским заливом и минимизации риска токсичных цветений.

**Ключевые слова:** динамика фитопланктона, *Chlorella vulgaris*, моделирование эвтрофикации, гидродинамическая модель, уравнения конвекции-диффузии, перенос веществ, цветение цианобактерий, численное моделирование, биологическая регуляция, Таганрогский залив

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**Introduction.** The Azov Sea, and particularly the Taganrog Bay, is among the most eutrophic and ecologically vulnerable water areas in Russia. The influx of nutrients from the Don River basin, high water temperatures in summer, and limited water exchange lead to the massive proliferation of cyanobacteria (*Microcystis, Aphanizomenon, Anabaena, Nodularia*) [1–3]. A number of cyanobacteria produce toxic metabolites—microcystins, nodularin, cylindrospermopsin, saxitoxin—which pose a threat to human health [4–8]. The blue-green (cyanobacterial) algae *Microcystis aeruginosa, Aphanizomenon flos-aquae*, and representatives of the genus Anabaena are the primary species forming massive "blooms" in the freshened zone of Taganrog Bay [9–12]. Upon the decay of these organisms, anatoxins a and a(s), which affect the nervous system, appear in the water. It was previously hypothesized that poisoning by decomposing cells of blue-green algae causes the so-called Haff disease [2].

Monitoring by the Southern Scientific Centre of the Russian Academy of Sciences indicates that peak phytoplankton concentrations, primarily cyanobacteria, in the Taganrog Bay reach levels classified by the WHO as posing a high risk to the population during swimming and water contact [2–4]. The concentration of toxic algae, primarily cyanotoxin producers, can be reduced through the spatially distributed introduction of biologically significant quantities of the green microalgae *Chlorella vulgaris*, which provides effective competition for nutrients. This paper presents a hydrobiological model and the results of numerical modelling of various scenarios for the spatial distribution of green algae to achieve acceptable ecological and hygienic outcomes through biological regulation of cyanobacterial abundance.

We present initial data illustrating the biological and ecological-hygienic characteristics of toxic microalgae species (hydrobiota) in the Taganrog Bay. According to long-term observations by the SSC RAS [2, 3], the background summer phytoplankton abundance ranges from 7.5–53 thousand cells/ml, reaching up to 152 thousand cells/ml during peak blooms (2015). The corresponding biomass is 23.8 g/m³ [2]; the maximum biomass over the long-term period is 70–80 g/m³ [3]; and the proportion of cyanobacteria in the biomass structure reaches 90% [2, 3]. Such concentrations correspond to harmful algal blooms (HABs) and, according to the WHO classification presented in Table 1, belong to levels at which adverse health effects for the population are possible [4].

Table 1

Cyanobacterial Concentrations and WHO Risk Classification

| Indicator | Concentration | Conditions | Risk (WHO) | Sources |
|---|---|---|---|---|
| Background Abundance | 7.500–53.000 cells/ml | Summer | Low | [2, 3] |
| Peak Bloom Abundance | ≈152.000 cells/ml | Taganrog Bay | High (> 100.000) | [2] |
| Biomass (background) | 0.9–5.5 g/m³ | Summer | Low | [2] |
| Biomass (bloom) | 23.8 g/m³ | Bloom | Moderate–High | [2] |
| Long-term Maximum Biomass | 70–80 g/m³ | Azov Sea | HAB (Harmful Algal Bloom) | [3] |
| Microcystin-LR (Drinking Water MPC) | 1 µg/L | Water | Acceptable | [4, 5] |
| Microcystin-LR (Recreational Water) | > 20 µg/L | Bathing | Hazardous | [4] |

*(MPC – Maximum Permissible Concentration)*

Currently, the following pathways of cyanotoxin impact on humans are known: dermal contact (cutaneous pathway), inhalation, hemodialysis, and ingestion (oral pathway). When distinguishing these pathways, it should be noted that several routes of exposure can act simultaneously on a person. Cases of skin irritation and allergic reactions following contact with cyanobacteria in marine coastal waters have been recorded for at least 30 years. Symptoms have included rashes, blisters, allergic reactions resembling hay fever, asthma, conjunctivitis, and irritation of the ears and eyes. In eighty-five percent of patients, following initial neurotoxicosis, toxic symptoms developed, including painful hepatomegaly, as well as biochemical and histological signs of liver damage. Sixty fatalities were reported, caused either directly by hepatotoxicity or indirectly by complications including gastrointestinal bleeding, sepsis, and cardiovascular problems [8].

A study by A.Yu. Zhidkova et al. showed that an increase in the level of eutrophication in the Taganrog Bay is accompanied by a rise in gastrointestinal diseases, skin conditions, and allergic reactions among the population of coastal areas [1]. The authors note a direct link between the deterioration of water quality and the dynamics of visits to healthcare institutions.

An increase in acute allergic and toxic reactions during swimming should also be noted. At cyanobacterial concentrations exceeding 20–100 thousand cells/ml (levels typical for the Taganrog Bay in summer), the following are possible: skin itching, dermatitis, rashes, conjunctivitis, rhinitis, cough, throat irritation, asthma exacerbation, nausea, vomiting, and diarrhea from accidental water ingestion.

These effects are described in reports by the WHO and EPA [4, 7, 8] and are supported by statistically robust data for the region adjacent to the Azov Sea coast [1, 2]. Particular attention is drawn to cases of severe and acute intoxications, including those leading to hepatotoxic effects (microcystin, nodularin). Confirmed cases of consequences, including acute toxic hepatitis, a sharp increase in transaminase, damage to liver vessels, hemorrhagic necrosis, and others, are detailed in works [4, 6, 8]. To ensure the integrity of the analysis, both acute neurotoxic effects (saxitoxin, anatoxin) and hepato- and nephrotoxic effects, as well as long-term chronic consequences of cyanotoxin exposure, including potential carcinogenic risks, have been considered. Summary data on cyanobacterial and cyanotoxin concentrations, the nature of toxic action, and possible clinical manifestations are presented in Table 2.

Thus, harmful cyanobacterial blooms pose a significant threat to the health of the Azov region›s population and require systematic monitoring and prevention. Mathematical modelling is a relatively inexpensive, rapid, and accessible method for forecasting adverse situations associated with abundant cyanobacterial blooms in summer.

A number of domestic and international publications are devoted to modelling blooms of potentially harmful cyanobacteria. In [13], the influence of phosphorus on stimulating the development of blue-green algae is investigated. The article [14] presented a non-stationary three-component mathematical model of competition between two types of phytoplankton, including toxic ones, and their grazing by zooplankton.

Table 2

Cyanobacteria and Cyanotoxin Concentrations, Types of Exposure, and Possible Clinical Manifestations in Humans

| Cyanobacteria Concentration / Toxin Level | Exposure Type | Clinical Manifestations | Sources |
|---|---|---|---|
| 7.5–53 thousand cells/ml | Contact | Mild skin reactions | [2, 3] |
| ~152 thousand cells/ml | Bathing | Rash, itching, gastrointestinal disorders | [1, 2, 4] |
| 20–80 g/m³ biomass | Repeated contact | Diarrhea, vomiting, dermatitis | [3, 4] |
| Microcystin-LR ≥ 1 μg/L | Drinking water | Hepatotoxicity | [4, 5, 6] |
| Microcystin-LR ≥ 20 μg/L | Bathing | Acute intoxication | [4] |
| Saxitoxin > 3 μg/L | Ingestion via water/fish | Neurotoxic symptoms, paralysis | [7] |
| Cylindrospermopsin ~1 μg/L | Contact, water | Hepato- and nephrotoxicity | [9] |
| Chronic low doses | Long-term residence | Increased risk of oncological and chronic diseases | [1, 8] |

One of the methods for limiting mass cyanobacterial blooms is the biological regulation (biomelioration) of water bodies through the controlled introduction of cultures of the green microalgae *Chlorella vulgaris* [15]. The essence of the method is that green algae are introduced into the water body before the beginning of the blue-green algae growing season, where they absorb most of the nutrients, thereby limiting or even stopping the reproduction and growth of harmful cyanobacteria. In turn, green microalgae serve as a food base for zooplankton and juvenile fish, contributing to the stabilization of the water body›s trophic structure [16]. At typical concentrations, no negative impact of green algae on the aquatic ecosystem or harmful effects on human health have been detected. Furthermore, green algae have found application in agriculture as fertilizers, feed additives for livestock, and for wastewater treatment [17].

However, it is important to distinguish between the controlled introduction of *Chlorella vulgaris* cultures for biomelioration and the uncontrolled mass development of green algae. The latter can deteriorate the organoleptic properties of water, increase the concentration of dissolved organic matter, and promote bacterial contamination, which in turn leads to an increase in the formation of disinfection by-products during chlorination [5–7, 9]. These effects are not attributable to biomelioration biotechnology but to spontaneous green algal blooms under conditions of nutrient excess.

Given the pronounced eutrophication of the Azov Sea coastal waters, the use of green microalgae *Chlorella vulgaris* as a biological regulator of cyanobacterial abundance is of particular interest. Results from laboratory and semi-field experiments show that during co-cultivation of *Chlorella vulgaris* and toxic species (*Microcystis aeruginosa, Aphanizomenon flos-aquae, Anabaena spp.*), pronounced competition for available forms of nitrogen and phosphorus is observed, leading to a reduction in cyanobacterial growth rates and partial cell death within several weeks of the growing season. These data allow the consideration of controlled green microalgae introduction as a potential tool for biological regulation, the effectiveness of which is largely determined by the spatial distribution of biomass, initial phytoplankton concentrations, and the level of nutrient loading [18, 19].

In view of the above, the mathematical modelling of biological water body rehabilitation is a relevant task and is of interest from the perspective of regulating blue-green algae abundance under conditions of their geographically distributed introduction into the aquatic environment of the Taganrog Bay. The aim of this work is to conduct mathematical modelling of the remediation of the Taganrog Bay through the introduction of green microalgae and to assess the ecological-hygienic and medical consequences.

To achieve this aim, the authors of this study propose using a complex of mathematical models of phytoplankton population dynamics and hydrodynamics, accounting for advective and diffusive transport, weather conditions, geometry of the computational domain, growth limitation of microalgae by nutrient availability, and salinity and temperature regimes [20]. Modern finite-difference schemes and numerical methods were used to solve the stated problem.

**Materials and Methods.** The mathematical model of biological kinetics is based on the works of A.I. Sukhinov and E.V. Yakushev [21, 22]. The mathematical model, the nonlinear right-hand sides of the equations, and the formulation of the initial boundary value problem are described in detail in [22]. A brief description of the mathematical model and its linearization are provided below.

This model is based on a system of unsteady convection-diffusion-reaction equations of parabolic type with nonlinear source functions and first-order derivatives. Advective terms are presented in symmetric form, which guarantees the skew-symmetry of the transport operator and enables a correct problem formulation. For each substance $F_i$, included in the model, the equation has the form:

$$\frac{\partial q_i}{\partial t}+\frac{1}{2}\left(\nabla\cdot\left(\mathbf{V}q_i\right)+\left(\mathbf{V}\cdot\nabla\right)q_i\right)=\operatorname{div}\left(k\cdot\nabla q_i\right)+R_{q_i}, \tag{1}$$

where $q_i$ is the concentration of the $i$-th ($i=\overline{1,8}$) component, mg/L; $\mathbf{V}=\{u, v, w\}$ is the water flow velocity vector, m/s; $k=(k_h, k_h, k_v)$ is the turbulent exchange coefficient, m²/s; $\nabla$ denotes the gradient operator; $(x, y, z)\in G$; $0<t\le T$; $R_{qi}$ is the source function of biogenic substances, mg/(L·s); $i\in M$, $M=\{F_1, F_2, DOP, POP, PO_4, NO_3, NO_2, NH_4\}$; $F_1$ denotes the concentration of green algae, $F_2$ — blue-green algae. The following biogenic components are specified: $DOP$ refers to dissolved organic phosphorus, $POP$ — suspended organic phosphorus, $PO_4$ — phosphates, $NO_3$ — nitrates, $NO_2$ — nitrites, $NH_4$ — ammonium (ammonium nitrogen).

The biochemical interactions between the components of system (1), i. e., the right-hand side functions $R_{qi}=R_{qi}(x, y, z, t)$, are, in general, nonlinear dependencies that may depend on the temperature and salinity of the aquatic environment. They have the following form:

$$\begin{gathered}
R_{F_i}=C_{F_i}(1-K_{F_iR})q_{F_i}-K_{F_iD}q_{F_i}-K_{F_iE}q_{F_i},\ i=1,2,\\
R_{DOP}=\sum_{i=1}^{3}s_P K_{F_iE}q_{F_i}+K_{PD}q_{POP}-K_{DN}q_{DOP},\\
R_{POP}=\sum_{i=1}^{3}s_P K_{F_iD}q_{F_i}-K_{PD}q_{POP}-K_{PN}q_{POP},\\
R_{PO_4}=\sum_{i=1}^{3}s_P C_{F_i}\left(K_{F_iR}-1\right)q_{F_i}+K_{PN}q_{POP}+K_{DN}q_{DOP},\\
R_{NO_3}=\sum_{i=1}^{3}s_N C_{F_i}\left(K_{F_iR}-1\right)\frac{f_N^{(1)}\left(q_{NO_3},q_{NO_2},q_{NH_4}\right)}{f_N\left(q_{NO_3},q_{NO_2},q_{NH_4}\right)}\cdot\frac{q_{NO_3}}{q_{NO_2}+q_{NO_3}}q_{F_i}+K_{23}q_{NO_2},\\
R_{NO_2}=\sum_{i=1}^{3}s_N C_{F_i}(K_{F_iR}-1)\frac{f_N^{(1)}\left(q_{NO_3},q_{NO_2},q_{NH_4}\right)}{f_N\left(q_{NO_3},q_{NO_2},q_{NH_4}\right)}\cdot\frac{q_{NO_2}}{q_{NO_2}+q_{NO_3}}q_{F_i}+K_{42}q_{NH_4}-K_{23}q_{NO_2},\\
R_{NH_4}=\sum_{i=1}^{3}s_N C_{F_i}\left(K_{F_iR}-1\right)\frac{f_N^{(2)}\left(q_{NH_4}\right)}{f_N\left(q_{NO_3},q_{NO_2},q_{NH_4}\right)}q_{F_i}+\sum_{i=1}^{3}s_N\left(K_{F_iD}+K_{F_iE}\right)q_{F_i}-K_{42}q_{NH_4},
\end{gathered} \tag{2}$$

where $K_{F_iR}$ is the specific respiration rate of phytoplankton; $K_{F_iD}$ is the specific mortality rate of phytoplankton; $K_{F_iE}$ is the specific excretion rate of phytoplankton; $K_{PD}$ is the specific rate of POP autolysis; $K_{PN}$ is the specific rate of POP phosphatefication; $K_{DN}$ is the specific rate of DOP phosphatefication; $K_{42}$ is the specific rate of ammonium oxidation to nitrites during nitrification; $K_{23}$ is the specific rate of nitrite oxidation to nitrates during nitrification; $S_P$, $S_N$ are normalization coefficients for the content of $N$ and $P$ in organic matter. The growth rate of phytoplankton populations is expressed as a function dependent on salinity $S$ and temperature $T$:

$$C_{F_{1,2}} = K_{NF_{1,2}} f_T\left(T\right) f_S\left(S\right) min\left\{ f_P\left(q_{PO_4}\right), f_N\left(q_{NO_3}, q_{NO_2}, q_{NH_4}\right)\right\},$$

where $K_{NF}$ is the maximum specific growth rate of phytoplankton.

The growth of microalgae also depends on the concentration of main nutrients — nitrogen compounds (nitrates, nitrites, ammonia) and phosphorus (phosphates, dissolved organic phosphorus, suspended organic phosphorus). The functional dependencies for these are written in the Michaelis-Menten form. All these factors are limiting, and their influence reflects Liebig's law.

The functional dependencies on abiotic factors are expressed by the following formulas:

$$f_T\left(T\right) = \exp\left(-a_i\left\{\left(T - T_{opt}\right)/T_{opt}\right\}^2\right),$$

$$f_S\left(S\right) = \exp\left(-b_2\left\{\left(S - S_{opt}\right)/S_{opt}\right\}^2\right),$$

$$f_S\left(S\right) = \begin{cases} k_s, \text{ for } S \le S_{opt}, \\ \exp\left(-b_1\left\{\left(S - S_{opt}\right)/S_{opt}\right\}^2\right), \text{ for } S > S_{opt}, \end{cases}$$

where $k_s = 1$; $T_{opt}$, $S_{opt}$ are the optimal salinity and temperature for the given aquatic species; $a_i > 0$, $b_i > 0$; $i = 1{,}2$ are coefficients characterizing the width of the tolerance range of the aquatic organisms to salinity and temperature, respectively.

For system (1), an initial boundary value problem is formulated with the addition of appropriate initial and boundary conditions. The initial conditions for system (1) have the form:

$$\begin{gathered} q_i\left(x,y,z,0\right) = q_{0i}\left(x,y,z\right),\ i \in M,\ t = 0,\ \left(x,y,z\right) \in \overline{G}, \\ \mathbf{V}\left(x,y,z,0\right) = \mathbf{V}_0\left(x,y,z\right),\ T\left(x,y,z,0\right) = T_0\left(x,y,z\right),\ S\left(x,y,z,0\right) = S_0\left(x,y,z\right), \end{gathered} \tag{3}$$

where $G$ is the computational domain of the enclosed water body, bounded by the lateral surface (cylindrical surface) $\sigma$, the bottom $\partial\Sigma_H = \partial\Sigma_H\left(x,y\right)$ and $\Sigma_0$ — the undisturbed free water surface; $\Sigma$ is the piecewise smooth boundary of $G$, defined for $0 < t \le T$ at $\Sigma = \Sigma_0 \cup \Sigma_H \cup \sigma$.

Taking into account the introduced notations, the boundary conditions for equation (1) are formulated as follows on $\sigma$:

$$\begin{gathered} q_i = 0, \text{ if } u_n < 0, \\ \frac{\partial q_i}{\partial n} = 0, \text{ if } u_n \ge 0, \\ \frac{\partial q_i}{\partial z} = 0 \text{ on } \Sigma_o, \\ \frac{\partial q_i}{\partial z} = -\varepsilon_i q_i \text{ on the bottom } \Sigma_H, \end{gathered} \tag{4}$$

where $\varepsilon_i$ are non-negative constants, $i \in M$; $\varepsilon_i$ account for the sedimentation of algae to the bottom, their sinking, and the uptake of nutrients by bottom sediments for $i \in \{F_1, F_2\}$.

On a uniform temporal grid $\omega_\tau = \{t_n = n\tau,\ n = 0{,}1...,\ N;\ N\tau = T\}$ within the interval $0 < t \le T$ the nonlinearity of the right-hand side functions of the initial boundary value problem system (1)–(4) was linearized for the continuous model. Solutions of the linearized problem will be denoted as functions of the form $\tilde{q}_i^n$, $n = 1{,}2...,\ N$ taking into account the initial and boundary conditions. The linearization involves specifying the concentration functions of the substances appearing in the right-hand sides of the equations from the previous time layer $t_{n-1}$. If $n = 1$, the known initial conditions (3) are used.

Let us formulate the non-linearized (original) system (1) as a chain of coupled initial-boundary value problems of the form:

$$\frac{\partial q_i^n}{\partial t} + \frac{1}{2}\left(\operatorname{div}\left(\mathbf{V} \cdot q_i^n\right) + q_i^n \cdot \operatorname{div}\mathbf{V}\right) = \operatorname{div}\left(\mathbf{k} \cdot \operatorname{grad}\ q_i^n\right) + R_{q_i}^n, \tag{5}$$

where $i \in M$, $(x, y, z) \in G$, $n = 1{,}2...,\ N$, $t_{n-1} < t \le t_n$, $t \in \omega_\tau = \{t_n = n\tau$, n = ,2..., $N\}$ with initial and boundary conditions considered on the interval $t_{n-1} < t \le t_n$ for each of the equations.

Linearization involves specifying the concentration functions of the substances appearing in the right-hand sides of the equations from the previous, relative to the current, time layer:

$$\frac{\partial \tilde{q}_i^n}{\partial t} + \frac{1}{2}\left(\operatorname{div}\left(\mathbf{V}\cdot\tilde{q}_i^n\right) + \tilde{q}_i^n \cdot \operatorname{div}\mathbf{V}\right) = \operatorname{div}\left(\mathbf{k}\cdot\operatorname{grad}\,\tilde{q}_i^n\right) + \tilde{R}_{q_i}^n,$$
$$\tilde{R}_{q_i}^{n-1} = R_{q_i}\left(x,y,z,t_{n-1},\tilde{q}_i^{n-1}\right),\ i\in M. \tag{6}$$

It has been proven that the norm of the error $\left\|z_i^n\right\|_{L_2(G)}$ tends to zero for any $n$ and $i$ under conditions motivated by hydrophysical and biogeochemical constraints. Inequalities have been obtained that guarantee the closeness of the solutions of the linearized and nonlinear problems for each substance $F_i$ in $L_2(G)$ on a sequence of grids $\omega_\tau$ at $\tau \to 0$:

$$\underset{C_1=const>0}{\left\|z_1^n\left(x,y,z,t_n\right)\right\|_{L_2(G)}} \le \underset{n=1,2,\ldots,N}{C_1\tau}.$$

The presented mathematical model requires input data with initial values for the concentrations of the studied substances, salinity, temperature, water flow velocities, etc. In 2022–2024, researchers from the Azov-Black Sea Branch of FSBSI “VNIRO” — “AzNIIRKH” investigated the hydrobiological characteristics of the Azov Sea, particularly water salinity and temperature. Values of salinity at the points of the hydrobiological survey grid are presented in [11]. Field measurement data are consistent with the assumption of the authors of this article regarding a 30% increase in the salinity of the Azov Sea, specifically in the Taganrog Bay, relative to normal values for the water body, as reflected in [10]. A forecast of the development of the main phytoplankton population species during summer under various salinization scenarios for the Azov Sea was also made.

As a result of the salinization of the Azov Sea, the habitat of blue-green algae has shifted to the eastern part of the Taganrog Bay, while they are almost absent in the main part of the sea, which is confirmed by data from “AzNIIRKH” [10].

Taking into account the above, it can be assumed that the obtained phytoplankton population habitats under salinity values increased by 30% from normal can be used as initial distributions of phytoplankton population concentrations for conducting a computational experiment on the biological rehabilitation of the water body. The forecast of the geographical position of phytoplankton populations shown in Fig. 1 reflects the ratio of green and blue-green algae, whose biomass in the Taganrog Bay constitutes 60–70% of the total phytoplankton biomass [3].

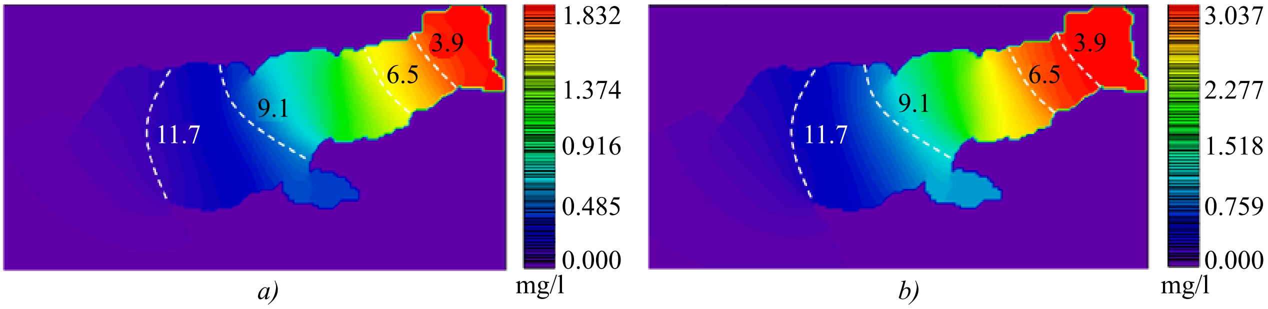
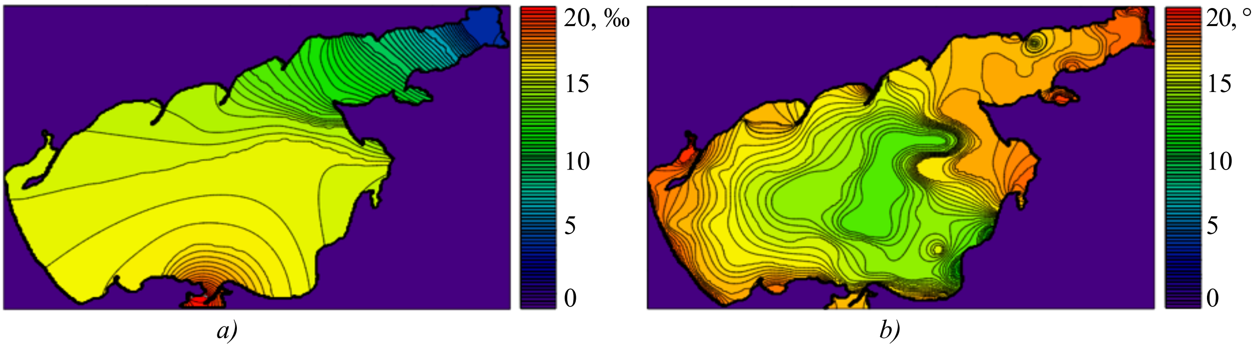


Fig. 1. Phytoplankton population habitats in summer:
*a* — green algae; *b* — blue-green algae

At the beginning of the growing season, nutrients are abundant, entering the Taganrog Bay with the runoff of the Don River during winter. At the start of the experiment, the distributions of major nutrients are set as uniform. The phosphate concentration is 0.04 mg/L, and the nitrate concentration is 0.204 mg/L. According to data from “AzNIIRKH” [10], the average phytoplankton biomass concentration in the Taganrog Bay is 1 mg/L, with cyanobacteria accounting for 70% of the biomass. The habitats of the initial phytoplankton population distributions are shown in Fig. 1, with the maximum concentration of green algae being 0.1 mg/L and that of cyanobacteria 0.7 mg/L. For the experiment, the optimal temperature for green algae is set at 25 °C, and for blue-green algae at 28 °C. The distributions of salinity and temperature values input into the software module for modelling the biological rehabilitation of the water body are shown in Fig. 2 [23].

When solving the linearized problem (1)–(10), the input data include the com-ponents of the water flow velocity vector at the nodes of the hydrodynamic com-putational grid, calculated based on the 3D hydrodynamic model implemented in the “Azov3D” software suite [24], as well as the values of salinity $S_0$, temperature $T_0$ and calculated concentrations $q_{0i}$ at the time $t_0$. The boundaries of the computational domain were determined using depth values obtained from pro-cessing navigational charts [25].



*a)* *b)*

Fig. 2. Initial data. Distributions of values:
*a* — salinity; *b* — temperature

The numerical solution of the problem involves constructing a discrete model (finite-difference scheme) using the input data and applying a numerical method for solving the grid equations. The modelling domain is assumed to be inscribed in a three-dimensional stepped region and is covered by a computational grid $\omega_\tau \times \omega_h$, uniform in time and the three spatial directions:

$$\omega_\tau = \{t_n = n\tau, n = 0,1..., N, N\tau = T\},$$

$$\omega_h = \{x_j = j \cdot h_x, y_k = k \cdot h_y, z_l = l \cdot h_z; \mathrm{j} = 0,1..., N_x, k = 0,1..., N_y, 1 = 0,1..., N_z\},$$

where $\tau$ is the time step; $0 \le t \le T$ is the time interval; $h_x$, $h_y$, $h_z$ are the steps in the spatial directions *Ox*, *Oy*, and *Oz*, respectively; $N_x$, $N_y$, $N_z$ are the maximum number of grid nodes in each spatial direction; $L_x$, $L_y$, $L_z$ are the maximum dimensions of the computational domain in space.

The linearization discussed above allows obtaining a system of linear grid equations. The discretization of problem (1), based on the system of convection-diffusion-reaction equations, is carried out using implicit monotonic schemes constructed on hydrodynamic grids.

The biological rehabilitation experiment proceeds as follows: a suspension of green algae is introduced at the beginning of their growing season, i. e., in March–April. By the start of the blue-green algae growing season (in May–June), the green algae have consumed most of the nutrients, leaving insufficient amounts for a massive bloom of blue-green algae.

The chlorella suspension is best introduced into areas of the water body with the highest convection, such as river channels, tips of spits, etc. The water flow velocity values were obtained from the "Azov3D" software suite, which implements a three-dimensional unsteady mathematical model of hydrodynamics. In the Azov Sea, easterly and northeasterly winds prevail from October to April. Such directions are formed under the influence of a spur of the Siberian anticyclone [26]. Therefore, the flow pattern obtained under an easterly wind direction was chosen as input data for conducting a computational experiment on the biological rehabilitation of the Taganrog Bay under conditions of increased salinity. The flow pattern in the Azov Sea under an easterly wind speed of 5 m/s is shown in Fig. 3. Red dots mark the locations of suspension introduction. The selection of points considered the flow velocities, the fact that *Chlorella vulgaris* is a freshwater alga, and the accessibility for introducing the suspension from the shore. The concentration of Chlorella vulgaris in the suspension is 1167 mg/L, the release rate is 5 L/s, with a total of 25 tons released, 2.5 tons at each of the 10 release points.

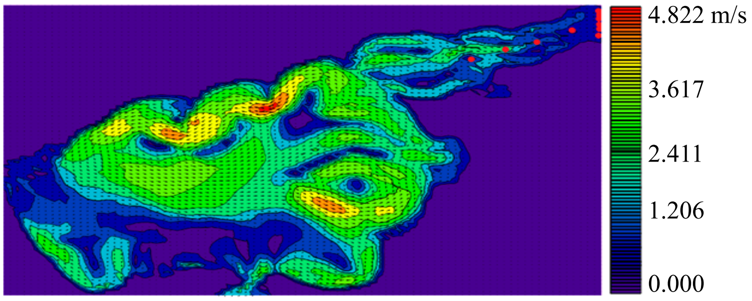


Fig. 3. Flow pattern in the Azov Sea under an easterly wind of 5 m/s

**Results.** As part of this study, modelling of the biological rehabilitation of the Taganrog Bay under conditions of salinization, based on the introduction of green microalgae, was conducted. As a result of the computational experiment, the authors obtained distributions of green algae and blue-green algae concentrations at time intervals of 15 days (Fig. 4) and 30 days (Fig. 5) for a *Chlorella vulgaris* suspension concentration of 1167 mg/L and a total volume of 25 tons.

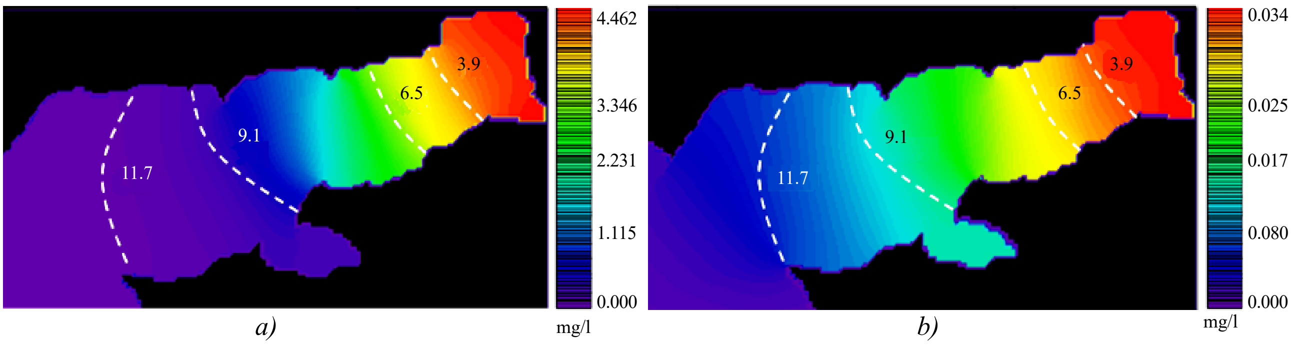


Fig. 4. Concentration distributions 15 days
after introducing the *Chlorella vulgaris* suspension (concentration 1167 mg/L):
*a* — green algae; *b* — blue-green algae

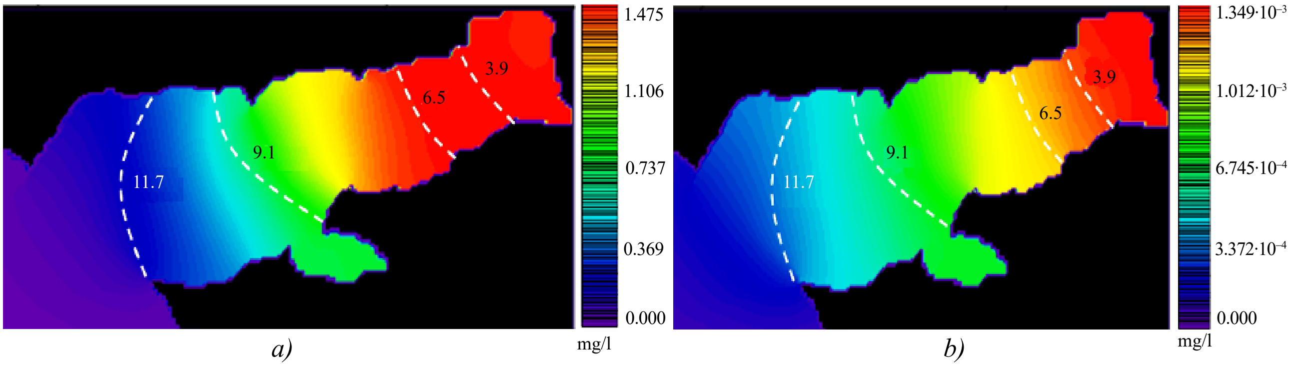


Fig. 5. Concentration distributions 30 days
after introducing the *Chlorella vulgaris* suspension (concentration 1167 mg/L):
*a* — green algae; *b* — blue-green algae

Fig. 6 shows the distributions of green algae and blue-green algae (surface layer) over a 30-day time interval for a *Chlorella vulgaris* suspension concentration of 2333 mg/L and a total volume of 25 tons.

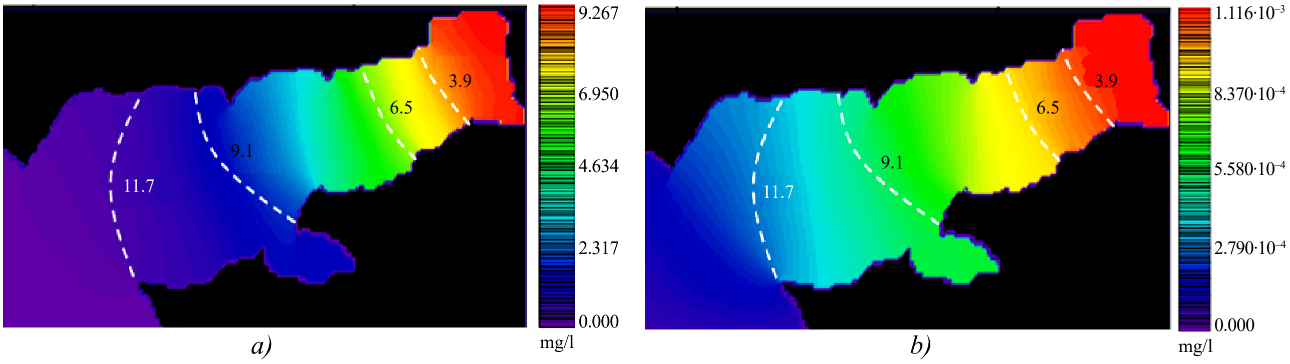


Fig. 6. Concentration distributions 30 days
after introducing the *Chlorella vulgaris* suspension (concentration 2333 mg/L):
*a* — green algae; *b* — blue-green algae

Fig. 4–6 depict the concentration values of the two microalgae species on the water surface.

**Discussion.** The concentration distributions of green and blue-green algae obtained from the modelling indicate the success of the computational experiment on the biological rehabilitation of the Taganrog Bay for the given concentration and volume of the introduced suspension. The experiment simulated the introduction of a *Chlorella vulgaris* phytoplankton suspension into the water body during the spring period, prior to the growing season of the potentially toxic blue-green algae *Aphanizomenon flos-aquae*. The introduction points were selected in the freshened zone (salinity values up to 7–8‰), which allowed the freshwater green algae to survive and grow successfully. The green microalgae consumed phosphates ($PO_4$) and nitrates ($NH_4$), leading to a nutrient deficiency by the beginning of the blue-green algae growing season.

At the start of the experiment, the concentration of blue-green algae exceeded that of green algae (0.7 mg/L and 0.1 mg/L, respectively). After 15 days, the concentration of blue-green algae was 131 times lower than that of green algae (0.034 mg/L and 4.462 mg/L, respectively). After 30 days, the difference in concentrations increased further ($1.349 \times 10^{-3}$ mg/L and 1.475 mg/L, respectively). Additionally, as shown in Fig. 6, doubling the concentration of the introduced green algae (to 2333 mg/L while keeping the volume constant) resulted in a potentially hazardous *Chlorella vulgaris* concentration of 9.267 mg/L after 30 days. Such a concentration of green algae, combined with other phytoplankton species, could lead to eutrophication of the water body and fish kills.

Furthermore, increasing the quantity of introduced green algae is costly and, therefore, economically unviable. The computational experiment empirically determined the optimal concentration and volume of the *Chlorella vulgaris* suspension to be introduced. It is important to note that the results of the computational experiment were obtained using reliable data on salinity, temperature, and the distributions of modeled substances, confirmed by field studies and long-term observations.

From the perspective of assessing ecological-hygienic and medical consequences, the scenario where green algae concentration exceeds that of blue-green algae (Fig. 5) is favorable, and no significant negative impacts from blue-green algae on recreational conditions in the Taganrog Bay are expected. Moreover, the concentration of green algae does not exceed permissible limits and is considered acceptable.

**Conclusion.** The modelling results were obtained using modern and high-precision mathematical modelling methods. The study's findings demonstrate the advantage of employing an integrated approach in mathematically modelling processes occurring in complex natural systems. These methods can be successfully used to simulate various scenarios for the development and rehabilitation of water bodies.

Despite the obtained results, the invasion of *Chlorella* into the ecosystem of the Taganrog Bay cannot be considered as the sole method for improving the ecological state of the water body. However, it can be an effective tool for water body rehabilitation when combined with other methods.

## References

1. Zhidkova A.Y., Podberesnij V.V., Zarubina R.V., Kononova O.A. The effect of eutrophication on human health on the example of the Gulf of Taganrog of the Sea of Azov. *IOP Conference Series: Earth and Environmental Science.* 2020;548:052053. https://doi.org/10.1088/1755-1315/548/5/052053

2. Matishov G.G., Fushtei T.V. On the problem of harmful algal blooms in the Azov Sea. *Electronic Journal "Research in Russia"*. 2003;6:213–225. (In Russ.)

3. Kovaleva G.V. Issue VIII: Modelling and Analysis of Natural Hazards in the Sea of Azov Region. In book: *Studies of the Southern Scientific Centre of the Russian Academy of Sciences (Series)*. Rostov-on-Don: SSC RAS Publishers; 2020. 328 p. (In Russ.)

4. World Health Organization. Guidelines on recreational water quality: Volume 1 — Coastal and Fresh Waters. *Geneva: WHO*; 2021. ISBN: 978-92-4-003130-2. URL: https://www.who.int/publications/i/item/9789240031302 (дата обращения: 11.09.2025).

5. Hygienic standards and safety requirements for environmental factors. SanPiN 1.2.3685–21. No. 62296. Moscow: Ministry of Health of Russia; 2021. URL: https://docs.cntd.ru/document/573500115 (accessed: 09.09.2025).

6. Chernova E.N., Shevchenko A.N. Determination of cyanotoxins in drinking water by HPLC. *Hygiene and Sanitation.* 2018;97(8):692–697. https://doi.org/10.18821/0016-9900-2018-97-8-692-697

7. Hallegraeff G.M. Harmful algal blooms: A global overview. *Elsevier Oceanography Series*. 2003;65:25–50. https://doi.org/10.1016/S0422-9894(03)80050-9

8. Backer L.C., Carmichael W.W. Human health effects from exposure to cyanobacteria and their toxins. *Environmental Health Perspectives*. 2016;124(11):1333–1343. https://doi.org/10.1289/EHP.1510459

9. Griffiths D.J., Saker M.L. Cylindrospermopsis raciborskii and cylindrospermopsin. *Environmental Toxicology*. 2003;18(2):78–93. https://doi.org/10.1002/tox.10113

10. AzNIIRKH. Summer cyanobacterial blooms in the Taganrog Bay. 7 September 2018. (In Russ.) URL: https://azniirkh.vniro.ru/ (accessed: 11.09.2025).

11. Berdnikov S.V., Kulygin V.V., Dashkevich L.V. Causes of the rapid increase in salinity of the Sea of Azov in the 21st century. *Marine Hydrophysical Journal*. 2023;39(6):760–778. (In Russ.)

12. Chorus I., Welker M., editors. *Toxic cyanobacteria in water: A guide to their public health consequences, monitoring and management*. 2nd ed. Boca Raton: CRC Press; 2021.

13. Xiao M., Burford M.A., Wood S.A., et al. Schindler's legacy: From eutrophic lakes to phosphorus utilization strategies. *FEMS Microbiology Reviews*. 2022;46(6):fuac029. https://doi.org/10.1093/femsre/fuac029

14. Neverova G., Zhdanova O. Mathematical modeling of plankton evolutionary dynamics. *Mathematics*. 2023;11(22):4673. https://doi.org/10.3390/math11224673

15. Bogdanov N.I. *Biological rehabilitation of water bodies*. Penza: PGSKhA Publishing; 2008;18–26. (In Russ.)

16. Aly S.M., ElBanna N.I., Fathi M. Chlorella in aquaculture. *Aquaculture International*. 2024;32:1559–1586. https://doi.org/10.1007/s10499-023-01229-x

17. Bogdanov N.I. *Chlorella suspension in the diet of farm animals. Russian Academy of Agricultural Sciences, Research Institute of Agriculture*. 2nd ed. Penza; 2007;17. (In Russ.)

18. Pavlyuk T.E., Nikolaev A.D., Ivanova I.V., et al. Correction of algal communities with Chlorella introduction. *Water Resources*. 2023;50(3):324–333. https://doi.org/10.31857/S0321059623030106

19. Gao J., Shao N., Sun Y., et al. Impact of Chlorella vulgaris on pond microbiota. *Sustainability*. 2023;15(9):7362. https://doi.org/10.3390/su15097362

20. Kudinov N.V., Filina A.A., Nikitina A.V., Bondarenko D.V., Razveeva I.F. Simulation of vertical movements of seawater in stratified reservoirs. *Advanced Engineering Research (Rostov-on-Don).* 2023;23(2):212–224. https://doi.org/10.23947/2687-1653-2023-23-2-212-224

21. Yakushev E.V., Pollehne F., Jost G., et al. Analysis of oxic/anoxic interfaces. *Marine Chemistry*. 2007;107:388–410. https://doi.org/10.1016/j.marchem.2007.06.003

22. Sukhinov A., Belova Y., Chistyakov A., Beskopylny A., Meskhi B. Modeling phytoplankton population geographic dynamics. *Mathematics*. 2021;9:3025. https://doi.org/10.3390/math9233025

23. Belova Yu.V., Chistyakov A.E. Modeling the Dynamics of Harmful Phytoplankton Species Concentration in Taganrog Bay of the Azov Sea. *Safety of Technogenic and Natural Systems*. 2025;9(4):284–293. https://doi.org/10.23947/2541-9129-2025-9-4-284-293

24. Sukhinov A.I., Nikitina A.V., Atayan A.M., Litvinov V.N., Belova Yu.V., Chistyakov A.E. Supercomputer simulation of hydrobiological processes. *Mathematical Models and Computer Simulations.* 2022;14:677–690. (In Russ.) https://doi.org/10.20948/mm-2022-01-06

25. Rakhimbaeva E.O., Alyshov T.A., Belova Yu.V. Automatic depth value recognition. *Computational Mathematics and Information Technologies*. 2025;9(1):52–60. https://doi.org/10.23947/2587-8999-2025-9-1-52-60

26. Unified State System of Information on the World Ocean (ESIMO). *Oceanographic data portal*; 2025. URL: http://esimo.oceanography.ru/ (accessed: 11.09.2025).

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**M.I. Gabueva:** Preparation, creation of the published work, specifically writing the initial draft; formatting the research article; working with sources.

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