UDC 519.6 10.23947/2587-8999-2021-1-2-80-87

VARIOUS APPROXIMATIONS OF VERTICAL TURBULENT EXCHANGE PARAMETERIZATION FOR THE ANALYSIS OF THE WAVES HYDRODYNAMIC IMPACT ON THE BOTTOM OF THE RESERVOIR*

V.V. Sidoryakina, S.V. Protsenko

Chekhov Taganrog Institute Taganrog branch of Rostov State University of Economics, Taganrog, Russia

✉ cvv9@mail.ru

The article discusses the possibilities of using various types of approximations for parametrization of vertical turbulent exchange for calculating and evaluating the hydrophysical characteristics of the wave regime in the accumulative coastal zone of the southwestern corner of the Tsimlyansk reservoir. It is impossible to carry out these studies without using various types and classes of approximations for parametrization of vertical turbulent mixing. Algebraic models for calculating the coefficient of vertical turbulent exchange and semi-empirical turbulence models are compared. Using ADCP data on velocity pulsations for several stations to measure hydrological characteristics, the results of parameterization of the vertical turbulent exchange coefficient were analyzed. The developed numerical algorithms and the software package implementing them are used to study the pressure field, the velocity vector field of the aquatic environment and the prediction of the baric field for this section of the reservoir water area.

Keywords: mathematical model, change of the bottom relief, ADCP data, vertical turbulent mixing, numerical experiments.

Introduction. Russian hydrographic network is not only a significant natural life support factor, but also extremely sensitive and vulnerable natural environment element, constantly changing under the influence of large-scale economic activities. Therefore, the problem of studying and rational use of water bodies and water resources, their monitoring, protection and restoration are becoming one of the most important state development tasks. Special attention is paid to studies of the coastal zone of seas, lakes and reservoirs, which are historically the most attractive places for the settlements emergence. One of the characteristic geographical features of the population distribution over the Russian territory is that the most inhabited are not the sea coasts, but the coasts of inland water bodies, especially reservoirs. First of all, this is due to the fact that the reservoirs were created in areas with developed socio-economic infrastructure and were designed to meet the needs for fresh water, irrigation, energy resources, convenient transport routes, etc. On the territory of the southwestern part of the Tsimlyansk reservoir, powerful industrial facilities are concentrated, including the Tsimlyanskaya hydroelectric power station, the Volgodonsk shipping canal, the Volgodonsk port, the Donskoy main canal, etc. The presence of this large water reservoir predetermined the choice of the construction site for the Rostov nuclear power plant and the largest industrial association of nuclear

* The research was supported by the Russian Foundation for Basic Research (project 19-01-00701).

power engineering Atommash. The consequence of the intensive economic development of the water space is an increase in the negative impact on its ecosystem.

The use of three-dimensional statements of those problems that traditionally belonged to the field of problems of the shallow water theory is dictated by the need to improve hydrodynamic models of real phenomena and the growing requirements for initial information in ecology, hydro-engineering. The three-dimensional formulation concludes the fundamental possibility of increasing the accuracy of calculating the phenomenon. The use of a three-dimensional formulation also allows us to obtain a more general, meaningful and interrelated description of the response of the coastal zone to large-scale atmospheric disturbances.

1. Statement of 3D wave hydrodynamics problem. The developed model for calculating 3D velocity vector of the aquatic environment movement based on is hydrodynamics mathematical model of shallow water bodies [9, 10]:

$$u_t' + uu_x' + vu_y' + wu_z' = -\frac{1}{\rho}P_x' + \left(\mu u_x'\right)_x' + \left(\mu u_y'\right)_y' + \left(\nu u_z'\right)_z', \quad (1)$$

$$v_t' + uv_x' + vv_y' + wv_z' = -\frac{1}{\rho}P_y' + \left(\mu v_x'\right)_x' + \left(\mu v_y'\right)_y' + \left(\nu v_z'\right)_z', \quad (2)$$

$$w_t' + uw_x' + vw_y' + ww_z' = -\frac{1}{\rho}P_z' + \left(\mu w_x'\right)_x' + \left(\mu w_y'\right)_y' + \left(\nu w_z'\right)_z' + g, \quad (3)$$

$$\rho_t' + (\rho u)_x' + (\rho v)_y' + (\rho w)_z' = 0, \quad (4)$$

where $\mathbf{V} = \{u, v, w\}$ is the water flow of shallow water body velocity vector; P is the hydrodynamic pressure; ρ is the aquatic environment density; μ, ν are turbulent exchange coefficients in the horizontal and vertical directions; g is the gravity acceleration.

The system of equations for the movement of the aquatic environment in mouth areas (1)-(4) is considered under the following initial condition and boundary conditions: $\mathbf{V} = \mathbf{V}_0$, $\mathbf{V} = \mathbf{V}_0$, $P_{\mathbf{n}}' = 0$, $\rho_v \mu (\mathbf{V}_\tau)_{\mathbf{n}}' = -\boldsymbol{\tau}$, $\mathbf{V}_{\mathbf{n}} = 0$, $P_{\mathbf{n}}' = 0$, $(\mathbf{V}_\tau)_{\mathbf{n}}' = 0$, $\mathbf{V}_{\mathbf{n}}' = 0$, $P_{\mathbf{n}}' = 0$, $\rho_v \mu (\mathbf{V}_\tau)_{\mathbf{n}}' = -\boldsymbol{\tau}$, $w = -P_t'/\rho g$, $P_{\mathbf{n}}' = 0$, where $\mathbf{V}_{\mathbf{n}}, \mathbf{V}_\tau$ are the velocity vector normal and tangential component; $\mathbf{n}$ is the normal vector; $\boldsymbol{\tau} = \{\tau_x, \tau_y, \tau_z\}$ is the tangential stress vector; ρ_v is suspension density.

On the free surface of a water body, the tangential stress is calculated as follows $\boldsymbol{\tau} = \rho_a C d_s |\mathbf{w}| \mathbf{w}$, where $\mathbf{w}$ is the wind velocity relative to water; ρ_a is the atmosphere density; $Cd_s = 0.0026$ is the dimensionless surface resistance coefficient, which depends on wind speed [11].

At the water body's bottom, the tangential stress has the form $\boldsymbol{\tau} = \rho C d_b |\mathbf{V}| \mathbf{V}$, where $Cd_b = gk^2 / h^{1/3}$, $k = 0.025$ is the group roughness coefficient in Manning's formula; h is the distance from free surface to bottom.

2. Semiempirical turbulence models. The semi-empirical theory is based on the hypothesis of locality of the mechanism of turbulent transport, according to which it is assumed that turbulent stresses depend only on the local structure of the averaged flow. This hypothesis proved to be effective for describing equilibrium flows, however, for substantially nonequilibrium flows in which the structure of the averaged flow does not correspond to the internal structure of turbulence, the application of the locality hypothesis is less justified the greater the degree of this discrepancy. This obligation dictates the need to establish connections between the components of the turbulent stress tensor and the local turbulence parameters, since the equilibrium of the internal structure of turbulence is established faster than the equilibrium between turbulence and the average flow.

One of the directions of turbulence modeling is associated with the use of the Prandtl-Kolmogorov approximate similarity hypotheses, according to which the kinetic energy of turbulence q, turbulence coefficient ν and the rate of dissipation of turbulent energy into heat ε' related to the scale of turbulence l with relations $\nu = l\sqrt{q};\ \ \varepsilon = c_\varepsilon q^2/\nu$, where $\nu_b = \alpha_b \nu,\ \alpha_b,\ c_\varepsilon - const$. The equation of the balance of the kinetic energy of turbulence has the form $\frac{\partial q}{\partial t} - \nu\left|\frac{\partial \mathbf{\overline{v}}}{\partial z}\right|^2 + c_\varepsilon \frac{q^2}{\nu} = \alpha_b \frac{\partial}{\partial z}\nu\frac{\partial q}{\partial z}$.

In the simplest models, the assumption of the dimension of the mixing length determined by formulas such as Prandtl formulas was used to determine the turbulence scale. In more complex models, differential equations were used instead of algebraic expressions for the turbulence scale. However, two-parameter models have become the most widespread, in which there is also a differential equation for the turbulence scale.

3. The results of parameterization of the vertical turbulent exchange coefficient. Currently, one of the urgent problems in modeling complex hydrodynamic systems is the need to calculate the vertical structure of the flow to solve a number of applied problems, primarily in the case of anthropogenic pollution of water areas. The transition to a system of Navier-Stoke equations for three-dimensional motion with a model for closing the coefficient of vertical turbulent exchange is dictated by the need to improve hydrodynamic models of real phenomena and the growing requirements for initial information in ecology. The inclusion of the parametrization of the vertical turbulent exchange coefficient in the system of hydrodynamic equations makes it possible in principle to increase the accuracy of calculating the phenomenon.

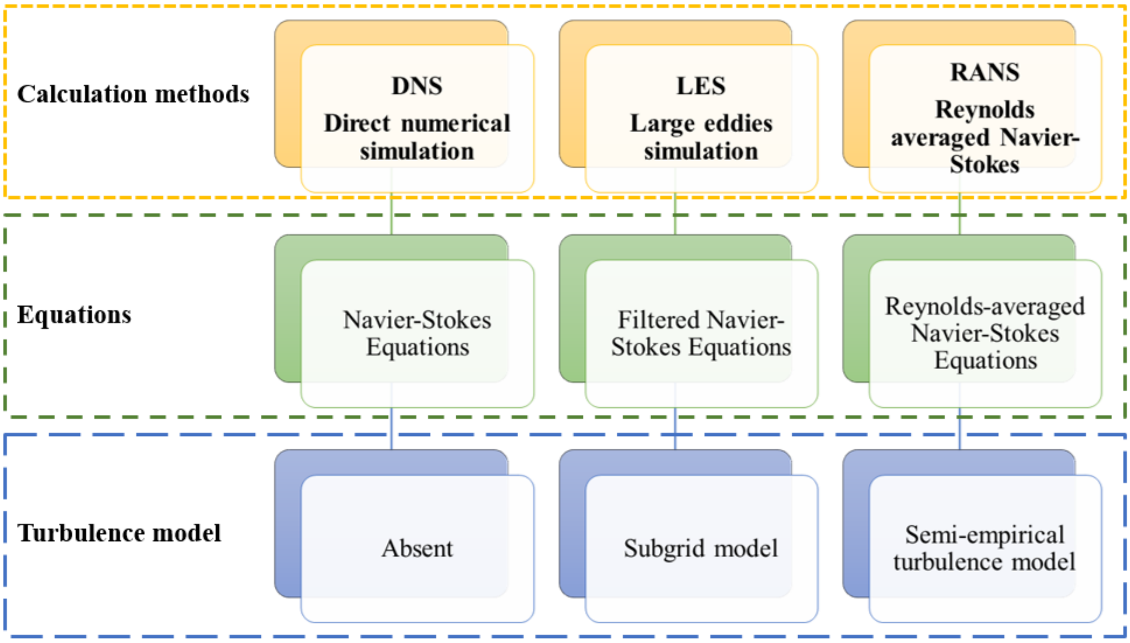

Figure 1. Methods for calculating turbulent flows

Consider the approaches to numerical simulation of turbulence. DNS-direct numerical simulation, model-free numerical solution of the Navier-Stokes equations, free from assumptions, but satisfying the physical criteria for the suitability of computational grids and the requirements for the quality of approximation schemes; RANS-Reynolds averaged Navier-Stokes; LES-Large eddies simulation (versions: VLES, DES etc.) – if the Reynolds averaging is carried out taking into account the distribution of ripples over frequencies, not including those that are resolved by grid partitioning, then you will get a blank for the LES model.

Currently, there is no universal and accurate turbulence model for all cases. Direct numerical simulation (DNS) involves solving the Navier-Stokes equations on a grid with steps (including the time step) small enough to accurately resolve all turbulent vortices. The grid step must be of the order of the Kolmogorov scale. The computational cost is proportional to the number of nodes in each direction and the number of time steps. Today, DNS is only applicable for very limited Reynolds numbers.

To parametrize the coefficient of vertical turbulent exchange, algebraic subgrid models based on finding turbulent flows as products of deviations of the components of the flow velocity and the transferred physical quantity averaged over space or time are considered. Studies were carried out on the basis of several approaches for calculating the coefficient of turbulent exchange along the vertical: Belotserkovsky parametrization, Boussinesq parametrization, Smagorinsky parametrization.

All methods of parametrization of the vertical turbulent exchange coefficient allow in most cases to obtain similar distributions of the vertical turbulent exchange coefficient in the order of magnitude and localization of maxima-minima.

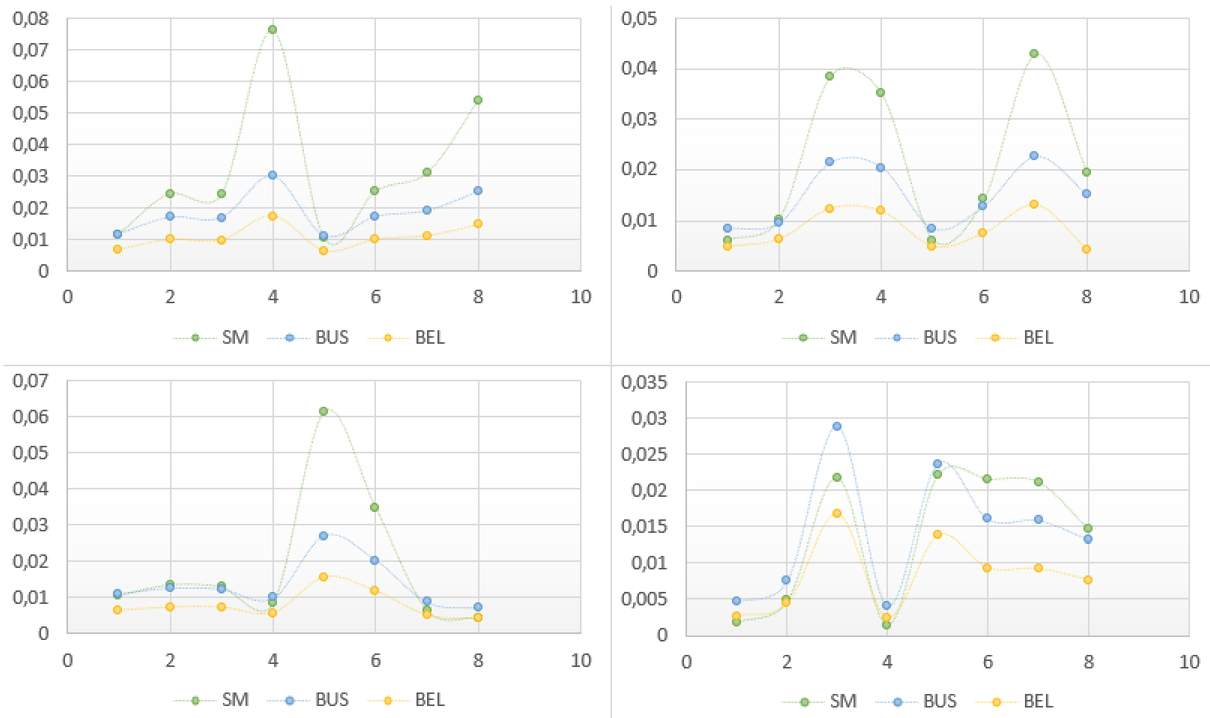

Figure 2. The coefficient of vertical turbulent exchange calculated on the basis of various types of approximations for parametrization of vertical turbulent mixing (horizontal values in m^2/s)

The phenomenon of sharp jumps in the coefficient on all graphs is associated with errors in the measurements of the pulsations of the vertical velocity component, which is included in the calculation formula of the method. The presence of errors in the measurements of the pulsations of the vertical velocity component is associated with many phenomena occurring at the time of measurement, such as ship deviation, fluctuations in the free surface, changes in depth, stability, wind and waves.

The profiles of the coefficient of vertical turbulent exchange at a time show that the parametrization of Belotserkovsky and Boussinesq most adequately reflects the processes of turbulent exchange for shallow water bodies, but to assess the quality of parametrization, a more in-depth analysis using mathematical statistics methods is necessary.

Thus, on the basis of statistical analysis, it was revealed that the coefficients of vertical turbulent exchange obtained using the Smagorinsky parameterization have the smallest standard deviation. With this parameterization, when testing the hypothesis about the normality of the distribution, in most cases the hypothesis was accepted.

4. Results of numerical experiments. The designated area for carrying out numerical experiments is located in the extreme southwestern corner of the Tsimlyansk reservoir in the area of the Volgodonsk port. The «local» modeling area has dimensions of $5{\cdot}10^3 \times 5{\cdot}10^3$ m and a maximum depth of 8 m, the peak point rises 2 m above sea level. The source of disturbances is set at some distance from the coastline. At the initial moment of time, the liquid is at rest.

In order to prepare the input data, a digital model area was built, which displays the depth map of the computational area and the isolines of the depth function of the bottom surface and the coastline (Fig. 3). The model area is characterized by a geometrically complex coastline configuration.

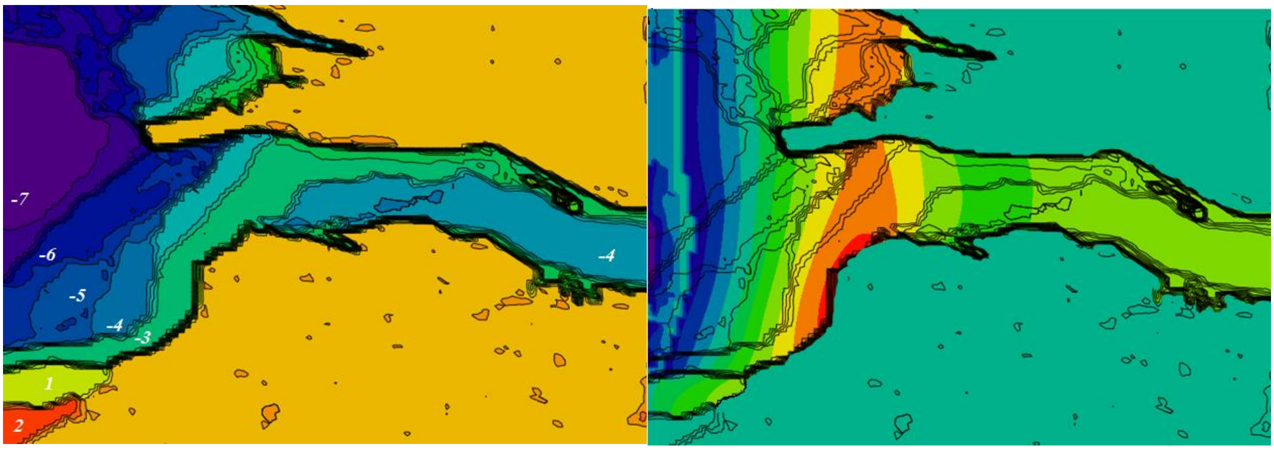

Figure 3. Bottom surface and coastline depth's function isolines and the pressure field [kPa]

The uniform grid with dimensions of 100 × 100 × 40 was built, which corresponds to cell size about 50 m in horizontal directions and 0.2 m in vertical direction. The input data on the readings of the depths of the water body were interpolated to this grid. In order to be able to speak about the adequacy of the hydrodynamic wave model, an initial set of information is required, including information about the fields of the wind speed and the water environment. These circumstances led to a surge of microalgae in the coastal zone, clogging of water intake facilities and the cessation of water supply to Volgodonsk.

Conclusion. The possibilities of using different types of approximations for parametrization of vertical turbulent exchange are considered. Optimal parametrization is included in a three-dimensional hydrophysical model of wave processes, which takes into account wind stress on the free surface, friction on the bottom, evaporation and precipitation not only in the continuity equation, but also in the equations of motion of the aquatic environment, which is essential for the waters of large reservoirs. Algebraic models for calculating the coefficient of vertical turbulent exchange and semi-empirical turbulence models are compared. Based on ADCP data on velocity pulsations and hydrological characteristics, the results of parameterization of the vertical turbulent exchange coefficient are analyzed. Based on the analysis, it was found that the coefficients of vertical turbulent exchange obtained using the Smagorinsky parametrization have the smallest standard deviation. The presented model and calculation methods can be used to solve practical problems of calculating the force effect of waves on the geometry of coastal territories and coastlines, forecasting changes in water ecology processes as a result of natural and man-made hazards.

References

1. Monin AS 1973 Turbulence and microstructure in the ocean (Physics-Uspekhi, 16(1)) p 121-131.
2. Shokin YuI, Chubarov LB, Marchuk AnG, Simonov KV 1989 Vychislitelnyi eksperiment v probleme tsunami (Novosibirsk: Nauka. Sib. otd.) p 164.
3. Belotserkovskiy OM 2003 Turbulentnost: novye podkhody (M.: Nauka) p 286.
4. Belotserkovskii OM, Gushchin VA, Shchennikov VV 1975 Use of the splitting method to solve problems of the dynamics of a viscous incompressible fluid USSR (Computational Mathematics and Mathematical Physics, 15(1)) p. 190-200. DOI:10.1016/ 0041-5553(75)90146-9.
5. Belotserkovskiy OM, Gushchin VA, Kon'shin VN 1987 The splitting method for investigating flows of a stratified liquid with a free surface (USSR Computational Mathematics and Mathematical Physics, 27(2)) p 181-191.
6. Sukhinov AI, Chistyakov AE, Protsenko EA, Sidoryakina VV, Protsenko SV 2019 Accounting method of filling cells for the hydrodynamics problems solution with complex geometry of the computational domain (Matem. Mod., 31(8)) pp 79–100. DOI: https://doi.org/10.1134/S0234087919080057.
7. Sukhinov AI, Chistyakov AE, Protsenko EA, Sidoryakina VV, Protsenko SV 2020 Set of coupled suspended matter transport models including three-dimensional hydrodynamic processes in the coastal zone (Matem. Mod., 32(2)) pp 3–23. DOI: https://doi.org/10.20948/mm-2020-02-01.
8. Sukhinov AI, Chistyakov AE, Protsenko EA, Sidoryakina VV, Protsenko SV 2020 Parallel algorithms for solving the problem of coastal bottom relief dynamics (Num. Meth. Prog., 21(3)) p 196–206. DOI: https://doi.org/10.26089/NumMet.v21r318.
9. Sukhinov AI, Chistyakov AYe, Fomenko NA 2013 Metodika postroyeniia raznostnykh skhem dlia resheniia zadach diffuzii-konvektsii- reaktsii, uchityvaiushchikh stepen zapolnennosti kontrolnykh yacheek (Izvestiia YUFU. Tekhnicheskie nauki, 4(141)) p 87-98.
10. Sidoryakina V.V., Sukhinov A.I. 2017 Well-posedness analysis and numerical implementation of a linearized two-dimensional bottom sediment transport problem (Comput. Math. Math. Phys.,57(6)) pp 978–994. DOI:https://doi.org/10.1134/S0965542517060124.
11. Sukhinov A.I., Sukhinov A.A. 2005 3D Model of Diffusion-Advection-Aggregation Suspensions in Water Basins and Its Parallel Realization (Parallel Computational Fluid Dynamics, Multidisciplinary Applications, Proceedings of Parallel CFD 2004 Conference, Las Palmas de Gran Canaria, Spain, ELSEVIER, Amsterdam-Berlin-London-New York-Tokyo) pp 223-230.
12. Sukhinov AI, Chistyakov AE 2012 Adaptive Modified Alternating Triangular Iterative Method for Solving Grid Equations with a Non-Self-Adjoint Operator (Mathematical Models and Computer Simulations, 4(4)) p 398-409.

Authors:

Sidoryakina Valentina Vladimirovna, Taganrog Institute of A.P. Chekhov (branch) RSUE (Initiative Street, Taganrog, Russian Federation), Candidate of Science in Physics and Maths, Associate professor
e-mail: cvv9@mail.ru.

Protsenko Sofya Vladimirovna, Taganrog Institute of A.P. Chekhov (branch) RSUE (Initiative Street, Taganrog, Russian Federation)
e-mail: rab55555@rambler.ru.

УДК 519.6 10.23947/2587-8999-2021-1-2-80-87

ПРИМЕНЕНИЕ РАЗЛИЧНЫХ ТИПОВ АППРОКСИМАЦИЙ ВЕРТИКАЛЬНОГО ТУРБУЛЕНТНОГО ОБМЕНА ДЛЯ АНАЛИЗА ГИДРОДИНАМИЧЕСКОГО ВОЗДЕЙСТВИЯ ВОЛН НА ДНО ВОДОХРАНИЛИЩА*

В.В. Сидорякина, С.В. Проценко

Таганрогский институт имени А.П. Чехова (филиал) Ростовского государственного экономического университета (РИНХ), Таганрог, Российская Федерация

✉ cvv9@mail.ru

В статье рассматриваются возможности использования различных типов приближений для параметризации вертикального турбулентного обмена для расчета и оценки гидрофизических характеристик волнового режима в аккумулятивной прибрежной зоне юго-западного угла Цимлянского водохранилища. Проведение этих исследований невозможно без использования различных типов и классов приближений для параметризации вертикального турбулентного перемешивания. Проведено сравнение алгебраических моделей расчета коэффициента вертикального турбулентного обмена и полуэмпирических моделей турбулентности. Используя данные ADCP о пульсациях скорости для нескольких станций для измерения гидрологических характеристик, были проанализированы результаты параметризации коэффициента вертикального турбулентного обмена. Разработанные численные алгоритмы и реализующий их программный комплекс используются для исследования поля давления, векторного поля скорости водной среды и прогнозирования барического поля для данного участка акватории водохранилища.

Ключевые слова: математическая модель, изменение рельефа дна, данные ADCP, вертикальное турбулентное перемешивание, численные эксперименты.

Авторы:

Сидорякина Валентина Владимировна, Таганрогский институт им. А.П. Чехова (филиал) РГЭУ (РИНХ) (347936 Таганрог, улица Инициативная, д. 48), кандидат физико-математических наук, доцент.
e-mail: cvv9@mail.ru.

Проценко Софья Владимировна, Таганрогский институт им. А.П. Чехова (филиал) РГЭУ (РИНХ) (347936 Таганрог, улица Инициативная, д. 48).
e-mail: rab55555@rambler.ru.

* Исследование выполнено при финансовой поддержке РФФИ в рамках научного проекта № 19-01-00701.