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About the analytical solution for the advertising model of two competing firms

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Abstract

Introduction. The criterion for the success of an advertising campaign is the maximum profit from sales, taking into account the costs of its implementation, while the sale of the same type of goods is sales occur in a competitive environment. The article examines a model for predicting mass sales of two similar products depending on the tactics of an advertising campaign. First of all, the distribution of funds between its separate types is considered: expenses for advertising paper products, banners and advertising in electronic media (EMM).

Materials and methods. The model is formulated in the form of a Cauchy problem for a system of two ordinary differential equations with nonlinear right-hand sides, taking into account: the total number of potential solvent buyers of the first and second goods; the intensity of the advertising campaign, mainly through EMM, the positive impact on sales of the interaction of those who have already bought the first or second type of goods with potential buyers, as well as informal (at the level of buyers) anti-advertising.

The results of the study. A solution is given for the case of constant coefficients determined by the above factors for the corresponding Cauchy problem in closed form.

Discussion and conclusions. The results obtained can be used to replay model situations of advertising organization in order to determine the conditions for extracting the greatest profit from sales minus advertising costs.

Keywords: sales forecasting model, Cauchy problem, advertising costs, advertising campaign, advertising main types, competing products sales.

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Научная статья

Об аналитическом решении для модели рекламы двух конкурирующих фирм

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Аннотация

Введение. Критерием успешности рекламной кампании является максимальное извлечение прибыли от продаж с учетом затрат на ее проведение, при этом реализации однотипных товаров является продажи происходят в конкурентной обстановке. В статье исследуется модель прогнозирования массовых продаж двух однотипных товаров в зависимости от тактики рекламной кампании. Рассматривается, в первую очередь, распределение средств между ее отдельными видами: расходы на рекламную бумажную продукцию, баннеры и рекламу в электронных средствах массовой информации (ЭСМИ).

Материалы и методы. Сформулирована модель в виде задачи Коши для системы из двух обыкновенных дифференциальных уравнений с нелинейными правыми частями, учитывающими: общее число потенциальных платежеспособных покупателей первого и второго товаров; интенсивность рекламной кампании, в основном, посредством ЭСМИ; положительное влияние на продажи взаимодействия уже купивших первый или второй вид товара с потенциальными покупателями, а также неформальную (на уровне покупателей) антирекламу.

Результаты исследования. Приведено решение для случая постоянных коэффициентов, определяемых указанными выше факторами для соответствующей задачи Коши в замкнутом виде.
Обсуждение и заключения. Полученные результаты могут быть использованы для «проигрывания» модельных ситуаций организации рекламы с целью определения условий извлечения наибольшей прибыли от продаж за вычетом расходов на рекламу.

Ключевые слова: модель прогнозирования продаж, задача Коши, расходы на рекламу, рекламная кампания, основные виды рекламы, продажи конкурирующих товаров.

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Introduction. The slogan "advertising is the engine of trade" is becoming more and more relevant with the development of new electronic communication technologies. The main source of advertising was printed products at the beginning of the development of advertising technologies, such as booklets and brochures, articles in newspapers and magazines, posters, etc. The possibilities of advertising have increased significantly with the advent of radio and television, and the costs of it have also increased significantly. Advertising has become an integral part of the Internet space at the present stage. It has penetrated into all social networks and, thanks to computer and multimedia technologies, has turned into an all-pervading electronic digital environment (electronic mass media). Overexpenditures for advertising could be a small fraction of the cost of goods or services before, they reach 30 percent or more of the cost of goods and services sold now. Sales forecasting is a very important issue, which is due to the tactics of the advertising campaign and the distribution of costs between its individual types. It is important to make a timely decision about the moment of termination of advertising, especially the most expensive types of advertising, for example, the demonstration of commercials on central television channels in prime-time mode. The main criterion for the success of an advertising campaign is the extraction of maximum profit from sales, taking into account the costs of its implementation. An important factor in the subsequent sale of similar, long-term used goods (for example, iPhone) is the formation of a high consumer reputation on sales of previous generations of goods. Their popularity is determined by a higher number of products sold and an informal assessment of the product or service among buyers. Sales take place in a competitive environment and this fact is also subject to accounting. A dynamic model of an advertising campaign for sales of two competing products of the same type is proposed in the form of a Cauchy problem (with initial data) for first-order differential equations with nonlinear right-hand sides that take into account the main types of advertising and the potential market capacity. The solution of this problem is obtained in a closed form with some simplifying assumptions, which was previously repeatedly considered for the sale of one type of product [1, 2] and led to solutions in the form of logistic functions. This model can be developed and applied in other fields of activity where there is a competitive information struggle, for example, elections, public opinion formation, etc. However, this aspect is not discussed here [3, 4].

Materials and methods. Differential equations describing the rate of change over time in the number of buyers who have learned about the goods and bought them have the form:

$$\frac{dN_1}{dt}=[\alpha_{11}(t)+\alpha_{12}(t)\times N_1(t)-\alpha_{22}(t)\times N_2(t)]\times(N_0-N_1-N_2), \qquad (1)$$

$$\frac{dN_2}{dt}=\left[\alpha_{21}(t)+\alpha_{22}(t)\times N_2(t)-\alpha_{12}(t)\times N_1(t)\right]\times\left(N_0-N_1-N_2\right), \qquad (2)$$

$$t>0,$$

$$N_1(0)=N_{10},\ N_2(0)=N_{20},\ t=0, \qquad (3)$$

where t is the time elapsed since the beginning of the advertising campaign; $N_1(t)$ and $N_2(t)$ are the numbers of buyers of the first and second goods, respectively; N_0 is the total number of potential solvent buyers of the first and second goods;

$\alpha_{11}(t)$ and $\alpha_{21}(t)$ characterize the intensity of the advertising campaign (mainly through the media) of the first and second goods; $\alpha_{12}(t)$ characterizes the positive result of the interaction of those who have already bought the first product with potential buyers; the coefficient has a similar meaning $\alpha_{22}(t)$ in the equation (2), (the word of mouth effect).

Terms $\alpha_{22}(t)\times N_2$ и $\alpha_{12}(t)\times N_1$ in the equations (1) and (2), respectively, show the influence of anti-advertising on the part of buyers: in the first case — those who preferred the second product, and in the second case — those who bought the first product, and not the second. To obtain a solution in a closed (analytical) form, all coefficients included in the system (1), (2) are considered constants, i. e. α_{ij} = const; i, j = 1,2. For forecasts close to real ones, these coefficients should be considered time–dependent, then it is necessary to apply numerical methods for integrating the problem (1)–(3).

As a result of the addition of equations (1) and (2), it turns out:

$$\frac{d\left(N_1+N_2\right)}{dt}=\left[\alpha_{11}+\alpha_{21}\right]\times\left(N_0-\left(N_1+N_2\right)\right). \tag{4}$$

The substitution of variables in equation (4) is introduced:

$$v(t)\equiv(N_0-(N_1t)+(N_1t)), \tag{5}$$

then an equation with separable variables of the form is derived

$$\frac{dv}{dt}=-[\alpha_{11}+\alpha_{21}]\times v,\ \ t>0, \tag{6}$$

with an initial condition

$$v(0)\equiv N_0-(N_{10}+N_{20}t),\ t=0, \tag{7}$$

the solution of which will be the function

$$v(t)\equiv\left(N_0-(N_{10}+N_{20})\right)\times\exp(-(\alpha_{11}+\alpha_{21})t)\ ,\quad t>0\ . \tag{8}$$

The resulting solution allows us to obtain an ordinary differential equation (ODE) of the first order with respect to each of the functions $N_1(t)$, $N_2(t)$. Let's express $N_2(t)$ from the ratio (8), taking into account the replacement (5):

$$N_2(t)=N_0-N_1(t)-\left(N_0-\left(N_{10}+N_{20}\right)\right)\times\exp\left(-\left(\alpha_{11}+\alpha_{21}\right)t\right). \tag{9}$$

This representation for $N_2(t)$ must be substituted into equation (1) and arrive at a first-order ODE with respect to the function $N_1(t)$:

$$\frac{dN_1}{dt}=\left[\alpha_{11}+\alpha_{12}N_1-\alpha_{22}\left(N_0-N_1-(N_0-(N_{10}+N_{20})\exp(-(\alpha_{11}+\alpha_{21})t)\right)\right]\times$$
$$\times\left(N_0-N_1-\left(N_0-N_1-(N_0-(N_{10}+N_{20})\right)\exp(-(\alpha_{11}+\alpha_{21})t))\right).$$

Let's give similar terms and transfer the terms containing the function $N_1(t)$ to the left side, linear with respect to the desired function, an ODE of the form will turn out:

$$\frac{dN_1}{dt}+(\alpha_{12}+\alpha_{22})\left(N_0-(N_{10}+N_{20})\right)\exp(-(\alpha_{11}+\alpha_{21})t))N_1=$$
$$=-\left(\alpha_{11}++\alpha_{22}N_0+\alpha_{22}\left(N_0-(N_{10}+N_{20})\right)\exp(-(\alpha_{11}+\alpha_{21})t)\right)\left(N_0-(N_{10}+N_{20})\right)\exp(-(\alpha_{11}+\alpha_{21})t). \tag{10}$$

The notation is introduced:

$$N_{30}=N_0-(N_{10}+N_{20}),\ \alpha_3=\alpha_{11}+\alpha_{21},\ \alpha_4=\alpha_{12}+\alpha_{22}.$$

The equation (10) takes the form:

$$\frac{dN_1}{dt}+\alpha_4N_{30}\exp(-\alpha_3t)\,N_1=\ -\left(\alpha_{11}-\alpha_{22}N_0++\alpha_{22}N_{30}\exp(-\alpha_3t)\right)N_{30}\exp(-\alpha_3t)\ . \tag{11}$$

This equation is solved by the method of variation of an arbitrary constant. It is not difficult to find a solution to a homogeneous ODE function $N_{01}(t)$:

$$\frac{dN_{01}}{dt}+\alpha_4N_{30}\exp(-\alpha_3t)\,N_{01}=0,\quad N_{01}(t)=C\ \exp\left(\frac{\alpha_4}{\alpha_3}N_{30}\exp(-\alpha_3t)\right), \tag{12}$$

where C is the time-dependent function, $C = C(t)$:

$$N_1(t) = C(t)\, N_{01}(t). \tag{13}$$

A new independent variable has been introduced for the convenience of further transformations, the new independent variable has been introduced:

$$u = \exp(-\alpha_3 t). \tag{14}$$

Then:

$$\frac{dN_1}{dt} = \frac{dN_1}{du}\frac{du}{dt} = -\alpha_3 \frac{dN_1}{du}\exp(-\alpha_3 t) = -\alpha_3 \frac{dN_1}{du} u. \tag{15}$$

The equality (12) is written taking into account the relation (14) in the form:

$$N_1(u) = C(u)\exp\left(\frac{\alpha_4}{\alpha_3} N_{30} u\right) \tag{16}$$

and the equation (11) taking into account the equalities (14)–(16) takes the form:

$$-\alpha_3 \frac{dN_1}{du} + \alpha_4 N_{30} N_1 u = [\alpha_{11} - \alpha_{22} N_0 + \alpha_{22} N_{30} u] N_{30} u.$$

If both parts of the last equality are reduced by a function $u \neq 0$ from (14), and put (16) in the resulting equation, then the ODE for $C(u)$ takes the form:

$$\frac{dC}{du} = [\alpha_{11} - \alpha_{22} N_0 + \alpha_{22} N_{30} u] N_{30} \exp\left(-\frac{\alpha_4}{\alpha_3} N_{30} u\right). \tag{17}$$

The solution of this equation with separable variables is not difficult, because $C(u)$ can be represented as a sum of easily taken integrals

$$I_1 = \int [\alpha_{11} - \alpha_{22} N_0] N_{30} \exp\left(-\frac{\alpha_4}{\alpha_3} N_{30} u\right) du = (\alpha_{22} N_0 - \alpha_{11}) \frac{\alpha_3}{\alpha_4} \exp\left(-\frac{\alpha_4}{\alpha_3} N_{30} u\right) \tag{18}$$

and the integral I_2, which is calculated by a single application of the integration formula in parts

$$I_2 = \int \alpha_{22} (N_{30})^2 u \exp\left(-\frac{\alpha_4}{\alpha_3} N_{30} u\right) du = -\frac{\alpha_3 \alpha_{22} N_{30} u}{\alpha_4} \exp\left(-\frac{\alpha_4}{\alpha_3} N_{30} u\right) - \frac{\alpha_3 \alpha_{22}}{\alpha_4^2} \exp\left(-\frac{\alpha_4}{\alpha_3} N_{30} u\right). \tag{19}$$

The equation is formed by adding (18) and (19) to calculate the coefficient $C(u)$:

$$C(u) = ((\alpha_{22} N_0 - \alpha_{11}) \frac{\alpha_3}{\alpha_4} - \frac{\alpha_3 \alpha_{22} N_{30} u}{\alpha_4} - \frac{\alpha_3{}^2 \alpha_{22}}{\alpha_4{}^2}) \exp\left(-\frac{\alpha_4}{\alpha_3} N_{30} u\right). \tag{20}$$

Let's substitute expression (20) for $C(u)$ into equality (16), a general solution for the function N_1 is obtained, which after multiplying by some constant K, defined in such a way as to satisfy the initial condition $N_1(0) = N_{10}$, gives a solution for the function N_1, who which is part of the Cauchy problem:

$$N_1(u) = C(u)\exp\left(\frac{\alpha_4}{\alpha_3} N_{30} u\right),$$

$$C\,(u) = K((\alpha_{22} N_0 - \alpha_{11}) \frac{\alpha_3}{\alpha_4} - \frac{\alpha_3 \alpha_{22} N_{30} u}{\alpha_4} - \frac{\alpha_3{}^2 \alpha_{22}}{\alpha_4{}^2}) \exp\left(-\frac{\alpha_4}{\alpha_3} N_{30} u\right).$$

As a result , the last two equalities will take the form:

$$N_1(u) = K((\alpha_{22} N_0 - \alpha_{11}) \frac{\alpha_3}{\alpha_4} - \frac{\alpha_3 \alpha_{22} N_{30} u}{\alpha_4} - \frac{\alpha_3{}^2 \alpha_{22}}{\alpha_4{}^2}), \tag{21}$$

$$u = \exp(-\alpha_3 t),\ N_{30} = N_0 - (N_{10} + N_{20}),\ \alpha_3 = \alpha_{11} + \alpha_{21},\ \alpha_4 = \alpha_{12} + \alpha_{22}. \tag{22}$$

To complete the formation of the solution for the function $N_1(t)$, it is necessary to determine the constant K, based on the initial condition (3) for $N_1(0)$. Simple calculations lead to the following equality:

$$K = N_{10} / ((\alpha_{22} N_0 - \alpha_{11}) \frac{\alpha_3}{\alpha_4} - \frac{\alpha_3 \alpha_{22} N_{30}}{\alpha_4} - \frac{\alpha_3{}^2 \alpha_{22}}{\alpha_4{}^2}). \tag{23}$$

Determining the function $N_2(t)$ from equality (5), in the case of the found function $N_1(t)$ is not difficult:

$$N_2(t) = N_0 - N_1(t) - (N_0 - (N_{10} + N_{20})) \times \exp(-(\alpha_{11} + \alpha_{21}) t).$$

Remark.

In the case of a non-simultaneous start of sales of goods (for example, sales of the second type do not start at the initial moment of time, but at $t = T$), it is necessary to determine $N_1(T)$, based on a known solution for the logistic model of an advertising campaign for the sale of one type of product [1], then change the right parts of the ODE (1) and (2) and initial conditions (3):

$$\frac{dN_1(t)}{dt}=[\alpha_{11}(t)+\alpha_{12}(t)\times N_1(t)-\alpha_{22}(t)\times N_2(t)]\times(N_0-N_1(t)-N_1(T)-N_2(t)),$$

$$\frac{dN_2}{dt}=[\alpha_{21}(t)+\alpha_{22}(t)\times N_2(t)-\alpha_{12}(t)\times N_1(t)]\times(N_0-N_1(t)-N_1(T)-N_2(t)),$$

$$t>T,$$

$$N_{10}=N_1(T),\ N_2(T)=N_{20},\ t=T.$$

Research results. The solution is given for the case of constant coefficients determined by the following factors: the total number of potential solvent buyers of the first and second goods; the intensity of the advertising campaign, mainly through EMM; the positive impact on sales of the interaction of those who have already bought the first or second type of goods with potential buyers, informal (at the level of buyers) anti-advertising, for the corresponding Cauchy problem in closed form.

Discussion and conclusions. The results obtained can be used to "replay" model situations of advertising organization in order to determine the conditions for extracting the greatest profit from sales minus advertising costs. For forecasts that are close to real, we should abandon the assumption α_{ij} = const; $i, j = 1, 2$ and consider these coefficients time-dependent. Then it is necessary to apply numerical methods of integration of the problem (1)–(3). In addition, the determination of these coefficients in predictive models is a separate task of mathematical statistics [3, 4], for which, for example, you can use survey data at the exit from places of mass sales of goods (shopping centers). In fact, this approach is close to the exit polls technology used in elections. These aspects of the study will be the subject of future work on this topic.

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