

## Spatial-three-dimensional wave processes' modeling in shallow water bodies taking into account the vertical turbulent exchange features

E. A. Protsenko<sup>1</sup>  , N. D. Panasenko<sup>1</sup>  , A. V. Strazhko<sup>1</sup> 

<sup>1</sup>Taganrog Institute named after A. P. Chekhov (branch) of RSUE, 48, Initiative St., Taganrog, Rostov region, Russian Federation

 [eapros@rambler.ru](mailto:eapros@rambler.ru)

### Abstract

**Introduction.** Reliable prediction of indicators of turbulent flows is a very difficult task, which is explained by the exceptional physical complexity of turbulence, in particular its probabilistic nature, a wide space-time spectrum and a fundamentally three-dimensional non-stationary nature. Despite conducting a wide range of studies focused on the problem under consideration, they did not fully reflect the totality of various factors and processes affecting the structure and parameters of vertical turbulent mixing. This indicates the need for a systematic analysis of the problem and modeling of such complex formalized systems. The aim of the work is to construct a scenario of changes in hydrodynamic wave processes of the coastal zone, based on an improved mathematical model of wave processes.

**Materials and methods.** The article is devoted to the study of spatial-three-dimensional wave processes in shallow water bodies, taking into account the features of turbulent exchange depending on the source and localization in the column of liquid, as well as the study of the influence of regular wave processes on turbulent exchange and vertically using a mathematical model of wave processes based on the system of Navier-Stokes equations, including three equations of motion in the with dynamically changing geometry of the computational domain.

**The results of the study.** Based on the developed software package, a scenario of changes in hydrodynamic wave processes of the coastal zone is constructed, the formation of vortex structures is predicted.

**Discussion and conclusions.** The separation of the wave flow into a near-surface macroturbulent layer caused by wave motion and a lower layer with background hydrodynamic turbulence is proved, the strength and intensity of turbulence changed synchronously with wave oscillations, demonstrating a pronounced asymmetry of turbulence generation throughout the water column.

**Keywords:** three-dimensional model of hydrodynamics, vertical turbulent exchange, numerical methods, wave processes, data filtering.

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Научная статья

## Моделирование пространственно-трехмерных волновых процессов в мелководных водоемах с учетом особенностей вертикального турбулентного обмена

E. A. Проценко<sup>1</sup>  , Н. Д. Панасенко<sup>1</sup>  , А. В. Стражко<sup>1</sup> 

<sup>1</sup>Таганрогский институт имени А. П. Чехова (филиал) РГЭУ (РИНХ), Российская Федерация, Ростовская область, г. Таганрог, ул. Инициативная, 48

 [eapros@rambler.ru](mailto:eapros@rambler.ru)

## Аннотация

**Введение.** Достоверное предсказание показателей турбулентных потоков является весьма сложной задачей, что объясняется исключительной физической сложностью турбулентности, в частности ее вероятностной природой, широким пространственно-временным спектром и принципиально трехмерным нестационарным характером. Несмотря на проведение широкого круга исследований, ориентированных на рассматриваемую проблему, в них не была достаточно полно отражена вся совокупность разнообразных факторов и процессов, влияющих на структуру и параметры вертикального турбулентного перемешивания. Это указывает на необходимость проведения системного анализа проблемы и моделирования подобных сложно формализуемых систем. Целью работы является построение сценария изменения гидродинамических волновых процессов береговой зоны на основе усовершенствованной математической модели волновых процессов.

**Материалы и методы.** Исследуются пространственно-трехмерные волновые процессы в мелководных водоемах с учетом особенностей турбулентного обмена в зависимости от источника и локализации в столбе жидкости. Рассматривается влияние регулярных волновых процессов на турбулентный обмен по вертикали с помощью математической модели волновых процессов, базирующейся на системе уравнений Навье-Стокса. Модель включает в себя три уравнения движения в областях с динамически изменяющейся геометрией расчетной области.

**Результаты исследования.** На основе разработанного комплекса программ построен сценарий изменения гидродинамических волновых процессов береговой зоны, предсказано формирование вихревых структур.

**Обсуждение и заключения.** Доказано разделение волнового потока на приповерхностный макротурбулентный слой, вызванный волновым движением, и нижерасположенный слой с фоновой гидродинамической турбулентностью, сила и интенсивность турбулентности изменялись синхронно с волновыми колебаниями, демонстрируя явно выраженную асимметрию генерации турбулентности по всей толще воды.

**Ключевые слова:** трехмерная модель гидродинамики, вертикальный турбулентный обмен, численные методы, волновые процессы, фильтрация данных.

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**Introduction.** Modern numerical models — SWAN, SWASH, FINLAB, H2Ocean and XBeach are constantly being improved due to new scientific discoveries as a result of research involving laboratory and field experiments. With the help of laboratory experiments, it is possible to obtain information about the details of the flow under controlled conditions. Flow velocities, turbulence properties and forces acting on objects can be determined and used to interpret observed phenomena, for example, erosion, and the data can be used to validate the model. At the same time, the complexity of obtaining full-scale data in the real area indicates the need to involve 3D models of hydrodynamics that take into account the specifics of coastal systems [1, 2].

Turbulence and further mixing of the aquatic environment are important mechanisms that determine the dynamics of the coastal zone, the transfer of momentum, mass and heat. Turbulence usually occurs as a result of shear or unstable stratification, while in the coastal zone, wind waves are an alternative source of turbulent mixing. In addition, turbulence can be generated as a result of bottom friction that occurs in the presence of tidal or wind currents, while baroclinic currents, nonlinear internal waves and inertial currents are important [3–5].

The task of monitoring the water surface involves the creation and verification of effective methods for clustering these objects on the surface of reservoirs, in particular, restoring the boundaries of the reservoir based on remote sensing data. Multispectral satellite images are used as sensing data. Based on the obtained images, the initial conditions for the mathematical model of hydrodynamics are determined, based on which prognostic calculations are performed.

Remote sensing data makes it possible to determine the dynamics of changes in the coastline due to a series of processed images of the same water area at different times.

Coastal areas require special attention, since interaction with bathymetry, currents, stratification, as well as vegetation, leads to complex nonlinear interactions affecting the evolution of waves.

## Materials and methods

### 1. Spatially inhomogeneous three-dimensional wave hydrodynamics mathematical model of shallow reservoir.

Mathematical model includes [6, 7]:

$$u'_t + uu'_x + vu'_y + wu'_z = -\frac{1}{\rho} p'_x + (\mu u'_x)_x + (\mu u'_y)_y + (vu'_z)_z, \quad (1)$$

$$v'_t + uv'_x + vv'_y + wv'_z = -\frac{1}{\rho} p'_y + (\mu v'_x)_x + (\mu v'_y)_y + (vv'_z)_z, \quad (1)$$

$$w'_t + uw'_x + vw'_y + ww'_z = -\frac{1}{\rho} p'_z + (\mu w'_x)_x + (\mu w'_y)_y + (vw'_z)_z + g; \quad (1)$$

$$\rho'_t + (\rho u)'_x + (\rho v)'_y + (\rho w)'_z = 0, \quad (2)$$

where  $V = \{u, v, w\}$  is the water flow of shallow water body velocity vector;  $P$  is the hydrodynamic pressure;  $\rho$  is the aquatic environment density;  $\mu, n$  are turbulent exchange coefficients in the horizontal and vertical directions;  $g$  is the gravity acceleration.



Fig. 1. Satellite images of the Azov Sea (low tide, November 22, 2019)

The raster model of the computational domain is constructed based on observations at individual points in space (Fig. 1). Discrete operational-territorial units correspond to cells of a regular grid. Figure 2 shows raster model of the computational domain.

**2. Processing and parameterization of in-situ ADCP sensing data.** The wave disturbance detected by the ADCP probe differs in frequency and intensity from the disturbance on the sea surface. This is due to the mutual movement of the vessel, the waves and the commensurability of their geometric dimensions. The movement of the vessel on the wave changes the frequency of the wave disturbance. If the wave scales are close to the dimensions (length, width, draft) of the vessel, then it cannot repeat its profile during its movement. The degree of impact of pitching is provided

by means of reduction coefficients, which have the form of amplitude-frequency characteristics of linear low-frequency filters [7, 8]. To represent the minimum solvable scales, it is necessary that the filter width does not exceed the step of the difference grid [9–11].

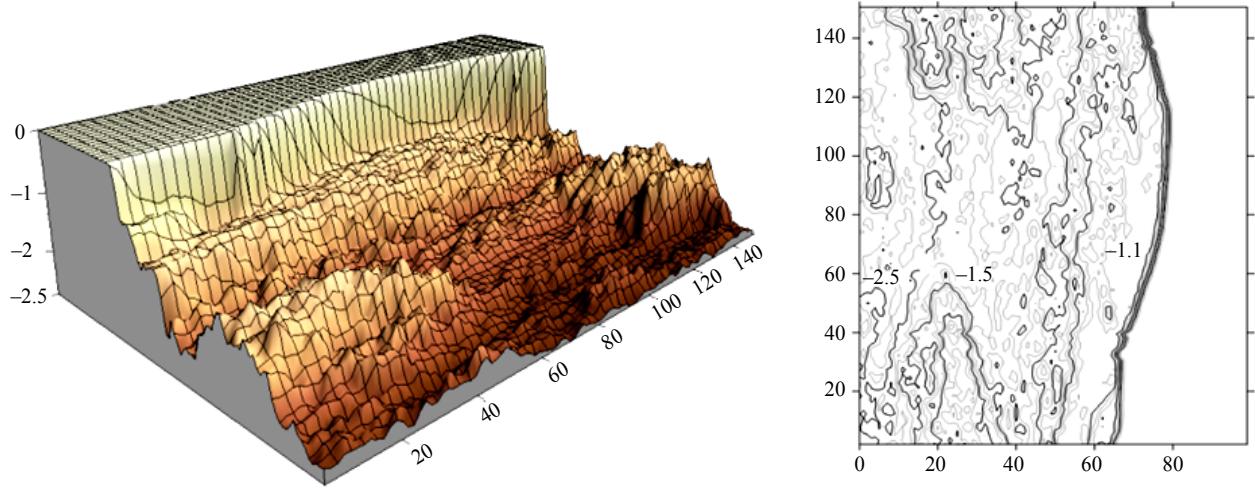


Fig. 2. Raster model of the computational domain

The initial data were obtained during the expedition in the Central-Eastern part of the Azov Sea and in the Taganrog Bay. The hydrophysical ADCP probe Workhorse Sentinel 600 was used to measure the three-dimensional velocity vector of the water medium. To process the instantaneous velocities of the water flow obtained during measurements, Gauss and Fourier filters were used at different filter widths. In these calculations, the filter scale was set based on the dimension of the hydrodynamics problem to be solved and the corresponding grid scale to this dimension.

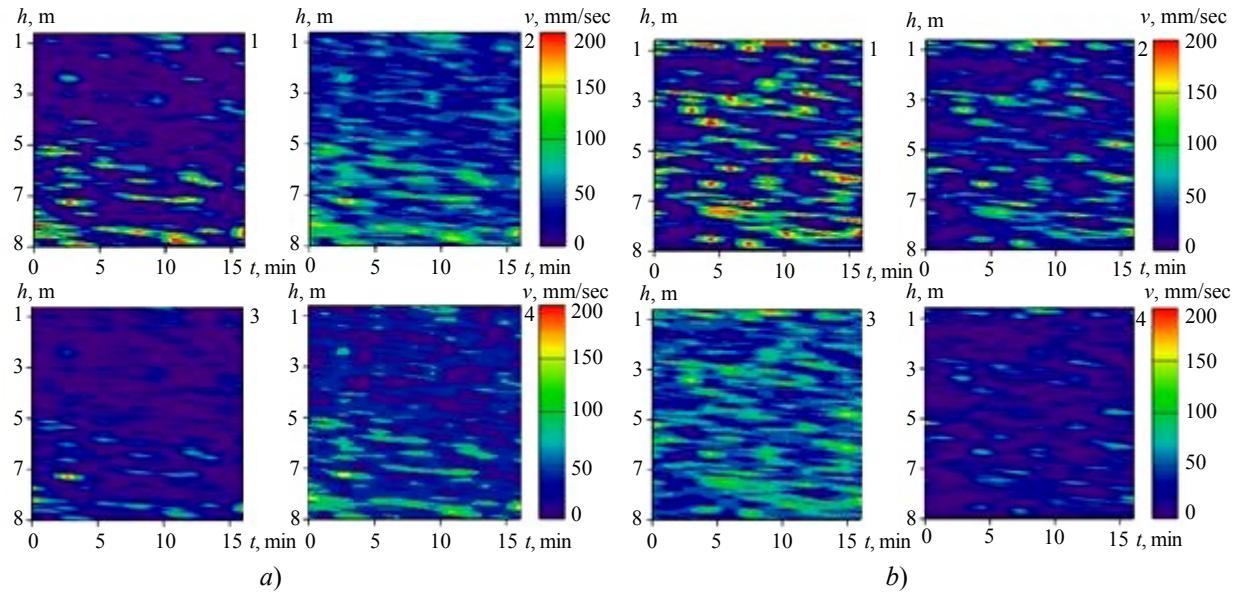


Fig. 3. Application of Gauss (a) and Fourier (b) filters: 1 — initial data, 2, 3, 4 — data obtained by filtering,

with different filter widths:  $\Delta_4 < \Delta_3 < \Delta_2$

Fig. 3 demonstrates the result of the software designed to eliminate the noise of expedition measurements, using the example of one of the components of the velocity vector of the water flow in the two-dimensional case. The color indicates the velocity of the water flow in mm/s in accordance with the given color scale.

**Research results** On the basis of numerical experiments, the distributions of the coefficients of vertical turbulent exchange are analyzed taking into account the influence of regular waves, as well as in their absence (Fig. 4); the separation of the wave flow into a near-surface macroturbulent layer caused by wave motion and a lower layer with background hydrodynamic turbulence is proved. A specific feature of the effect of regular waves on the turbulent exchange along the vertical was the increase in the coefficient of turbulent exchange in the near-surface layer and its decrease in the bottom layer compared with the distribution of coefficients obtained using the Smagorinsky parametrization.

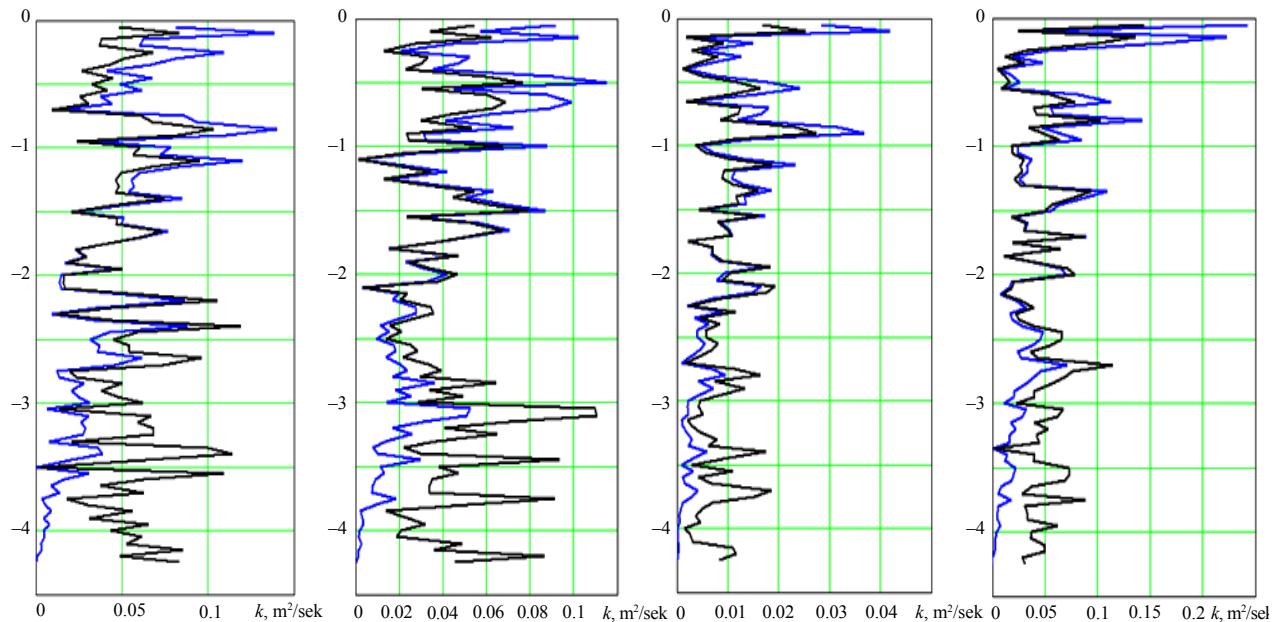


Fig. 4. Profiles of the vertical turbulent exchange coefficient (black line — excluding regular waves; blue — taking into account regular waves)

A wide range of variability of turbulent velocity pulsations is also demonstrated. The strength and intensity of turbulence changed synchronously with wave oscillations, showing a pronounced asymmetry of turbulence generation throughout the water column, including the near-surface layer, where waves amplified fluctuations in the flow velocity [12–14].

When waves break in the surf zone, turbulence can occur in several ways:

- due to fluid displacement;
- by separating the flow around the roughness elements;
- due to the injection of turbulent kinetic energy from breaking waves.

Turbulence begins to appear over a smooth seabed in the boundary layer flow when the Reynolds number ( $Re$ ) is greater than  $1.5 \times 10^5$  ( $Re = Au/\nu$ , where  $A$  is the orbital amplitude and  $\nu$  is the kinematic viscosity). The shift between the flow and the seabed creates microturbulent vortices such as vortex tubes and turbulent spots, which arise at the lower boundary and propagate upwards due to diffusion. The horizontal flow velocity and the turbulent kinetic energy ( $k$ ) are more or less the same in phase, with  $k$  scaling with  $u^2$ . When the incoming waves are distorted, the maximum formation of  $k$  occurs under the crest of the wave.

Bottom shapes, such as wave sediments or megaripples, can appear outside and inside the surf zone, when sediments have moderate or high steepness, they often create flow separation and vortex dispersion. Turbulent vortices are formed on the leeward slope when the horizontal velocity is zero. In these coherent vortices, turbulence spreads upward due to convection, not due to diffusion. Experimental data and the Reynolds averaging method (RANS) have shown that vortex ejection increases  $k$  near the layer. For oblique, shallow waves, the generation of  $k$  is maximal when the flow reverses from the shore to the shelf.

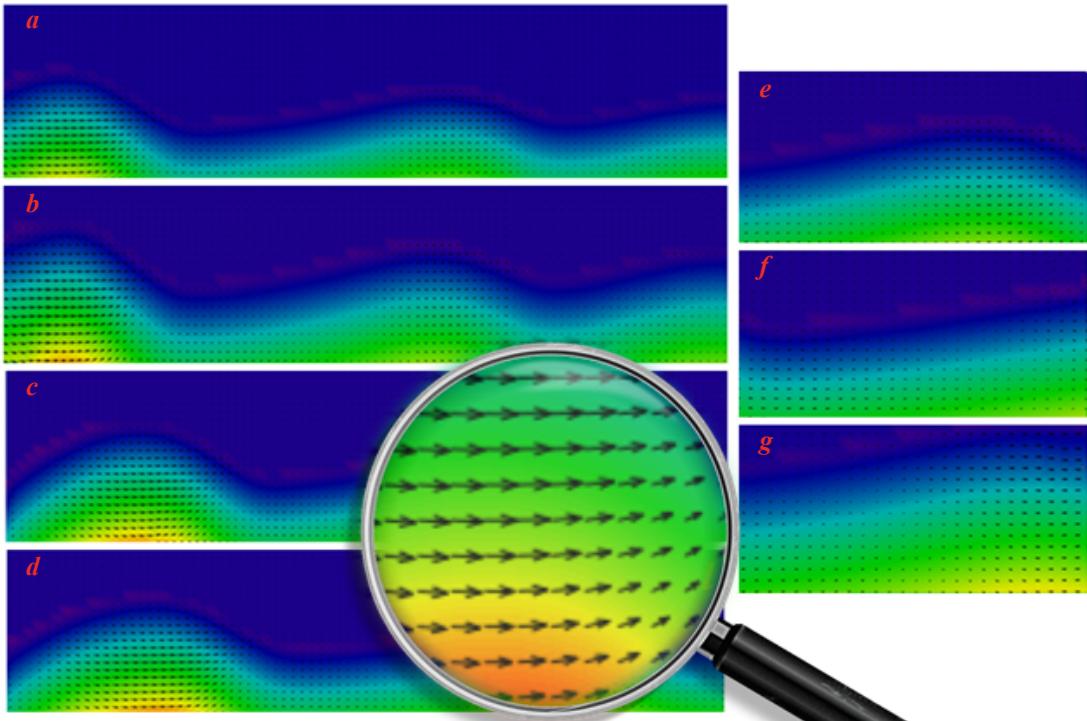


Fig. 5. Wave profiles and velocity vector fields at different time points, formation of vortex structures

The destruction of the wave is characterized by a sudden transition from a vortex-free flow to a rotational one, accompanied by a strong transformation of the wave energy into turbulence and, ultimately, into heat. Large-scale coherent vortices create strong vertical mixing, and turbulence created by surface destruction spreads downwards due to convection, however, a relatively small part of the wave energy is dissipated below the level of the trough, and most of it is between the crest of the wave and its trough. The relative height of the wave  $\gamma$  ( $\gamma = H/h$ , where  $H$  is the height of the wave,  $h$  is the depth of the water) Greek straight, Latin obliquely, higher too, did not notice is a useful parameter for characterizing or scaling the range of processes in the surf zone; for example,  $\gamma$  is used to predict the onset of wave destruction and the intensity of destruction. If the relative height of the wave is large enough ( $\gamma > 0.4$ ), the turbulence of the breakers can penetrate into the boundary layer of the wave and collapse on the seabed (Fig. 5).

Although wave disruption is the main source of turbulence in the surf zone, this process is best described as stochastic. In an irregular wave field, some waves break, and some do not, so the formation (and scattering) of turbulence occurs with great frequency, and the instantaneous levels of  $k$  can be several orders of magnitude greater than the phase-averaged waves. There are different types of wave destruction, so the scale and intensity of turbulent vortices depend on the type of destruction.

When the breakers spill, turbulence spreads down to the seabed behind the crest of the wave. This process causes the appearance of obliquely descending vortices (ODES), which are pulled behind the crest of the wave. In this case, turbulence is created at the wave front and slowly spreads down to the bottom through descending vortices. Coherent turbulent structures collapse to the bottom at some distance behind the crest of the wave.

Sinking breakers create large vortices or downdraft that rotate around a horizontal axis parallel to the crest of the wave and generate vertical velocity fluctuations.

**Discussion and conclusions.** The paper presents the results of mathematical modeling of spatial-three-dimensional wave processes in shallow water bodies, taking into account the features of turbulent exchange. The initial conditions for the simulation were set based on the processing of remote sensing data. The process of filtering in-situ data has significantly reduced the spread of data and the amplitude of fluctuations. The separation of the wave flow into a near-surface macroturbulent layer caused by wave motion and a lower layer with background hydrodynamic turbulence is proved. A distinctive feature of the effect of regular waves on the turbulent exchange along the vertical is revealed.

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### About the Authors:

**Protsenko, Elena A.**, PhD (Physical and Mathematical Sciences), Associate Professor of the Mathematics Department, Leading Researcher, Taganrog Institute named after A. P. Chekhov (branch) of RSUE (48, Initiative St., Taganrog, Rostov region, 347936, RF), [ORCID](#), [eapros@rambler.ru](mailto:eapros@rambler.ru)

**Panasenko, Natalia D.**, Researcher, Taganrog Institute named after A. P. Chekhov (branch) of RSUE (48, Initiative St., Taganrog, Rostov region, 347936, RF), [ORCID](#)

**Strashko, Alexander V.**, Researcher, Taganrog Institute named after A. P. Chekhov (branch) of RSUE, (48, Initiative St., Taganrog, Rostov region, 347936, RF), [ORCID](#)

### Conflict of interest statement

The authors declare that there is no conflict of interest.

*All authors have read and approved the final manuscript.*