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Correspondence to biophysical criteria of nonlinear effects in the occurrence of Feigenbaum bifurcation cascade in models of invasive processes

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Abstract

Introduction. The problem of creating a set of criteria for practically substantiated computational modeling of a number of complex staged biophysical processes with pronounced stages and critical transformations, for example, aggressive invasions, is discussed. Known models have a variety of behavior with the occurrence of bifurcations according to the same scenarios, the appearance of cycles, the coexistence of which is determined by Sharkovskii's theorem. In the limit of complication of cyclic behavior in such models, they often encounter chaotization of the trajectory, but with the existence of an infinite number of periodicity windows. The conditions for an infinite cascade of bifurcations for iterations are determined by the fulfillment of the conditions of Singer's theorem. The purpose of this work is to show that most of the nonlinear effects associated with chaotization scenarios do not have an ecological interpretation, but we will propose ways to exclude non-interpretable parametric ranges.

Materials and methods. Using methods for estimating the stability of stationary states and cyclic trajectories using Singer's theorem on the criterion for the occurrence of bifurcations for iterative models, we analyze interconnected nonlinear effects. The phenomena are considered on the example of cascades of the appearance of cycles of the period $p = 2^i + 1$, $i\to\infty$ and a cascade of cycles $p = 2^i - 1$, $i\to 0$ of "doubling" or "halfing" the period, which occur in ecological models often used to optimize fishing.

Results. It is confirmed that the coexistence of nonlinear effects turns out to be contradictory if the simulation results are interpreted in the field of biocybernetics, on the basis of model and real examples. Iterative models generate unnecessary non-linear modes of behavior, when predicting the dynamics of invasions or harvesting bioresources, taking into account the regulatory impact, for example, in the case of the well-known Feigenbaum scenario. It has been established that bifurcations connected in one scenario have no explanation in ecological reality and are not reflected in the observed biophysical systems. These mathematical artifacts are common to several biophysical models that are very different in their theoretical foundations. Chaotization in real population dynamics has somewhat different properties than can be obtained in a cascade of period doubling bifurcations. The formation of a non-attractive chaotic set in the form of a strange repeller is more consistent with the dynamics of the development of fast invasions.

Discussion and conclusions. It is shown that to describe the transformations of biosystemic processes with external influence, as the collapse of a commercial population, it is adequate to use models with the emergence of alternative attractors. These models correspond better to the transitions between the states of populations under the influence of fishing than models with the implementation of cascades of bifurcations of cycles, strange Cantor attractors and chaos regimes in the form of a continuum of unstable trajectories of all periods. The most promising are hybrid models of the life cycle with developmental stages for essential interpretation in ecology and forecasting of biosystems, as they allow to determine the parametric ranges of functioning and exclude unacceptable ranges of parameters where excessive non-linear effects occur, which have no justification for population processes. The analysis of the adequacy criteria is based on degradation scenarios for a complexly structured sturgeon population in the Volga basin, cod off the coast of Canada, outbreaks of invasive insects, and the spread of the invasive ctenophore Mnemiopsis leidy in the Caspian Sea.

Keywords: Dynamic models invasive processes; Feigenbaum bifurcation cascade; alternative attractors; complex dynamic processes, impact regulation for biosystems, hybrid computing systems, parametric ranges, theory of essential interpretation.

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Научная статья

Соответствие биофизическим критериям нелинейных эффектов при возникновении каскада бифуркаций Фейгенбаума в моделях инвазионных процессов

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Аннотация

Введение. Обсуждается проблема создания комплекса критериев для практически обоснованного вычислительного моделирования ряда сложных стадийных биофизических процессов с выраженной стадийностью и критическими трансформациями, например, агрессивных инвазий. Известные модели обладают разнообразным поведением с возникновением бифуркаций по одинаковым сценариям, появлением циклов, сосуществование которых определяется теоремой Шарковского. В пределе усложнения циклического поведения в таких моделях часто сталкиваются с хаотизацией траектории, но при существовании бесконечного числа окон периодичности. Условия бесконечного каскада бифуркаций для итераций определены выполнением условий теоремы Сингера. Цель работы — показать, что большинство связанных сценариями хаотизации нелинейных эффектов не имеют экологической интерпретации, но предполагаются способы исключения неинтерпретируемых параметрических диапазонов.

Материалы и методы. Методами оценки устойчивости стационарных состояний и циклических траекторий с применением теоремы Сингера о критерии возникновения бифуркаций для итерационных моделей анализируются связанные между собой нелинейные эффекты. Явления рассмотрены на примере каскадов появления циклов периода $p = 2^i + 1$, $i \to \infty$ и каскада циклов $p = 2^i - 1$, $i \to 0$ «удвоения» или «ополовинивания» периода, возникающих в часто применявшихся для оптимизации промысла экологических моделях.

Результаты исследования. На основе модельных и реальных примеров подтверждается, что сосуществование нелинейных эффектов оказывается противоречиво, если результаты моделирования интерпретируются в области биокибернетики. При прогнозировании динамики инвазий или промысла биоресурсов с учетом регулирующего воздействия итерационные модели генерируют ненужные нелинейные режимы поведения, например, в случае известного сценария Фейгенбаума. Установлено, что связанные в один сценарий бифуркации не имеют объяснений в экологической реальности и не отображаются в наблюдаемых биофизических системах. Данные математические артефакты общие для нескольких, очень разных по своим теоретическим основам, биофизическим моделям. Хаотизация в реальной популяционной динамике имеет несколько иные свойства, чем можно получить в каскаде бифуркаций удвоения периода. Более соответствует динамике развития быстрых инвазий образование непритягивающего хаотического множества в форме странного репеллера.

Обсуждение и заключения. Показано, что для описания трансформаций биосистемных процессов с внешним воздействием, как коллапса промысловой популяции, адекватно использовать модели с возникновением альтернативных аттракторов. Данные модели лучше соответствуют переходам между состояниями популяций под действием промысла, чем модели с реализацией каскадов бифуркаций циклов, странных канторовских аттракторов и режимов хаоса в форме континуума неустойчивых траекторий всех периодов. Наиболее перспективны гибридные модели жизненного цикла со стадиями развития для сущностной интерпретации в экологии и прогнозирования биосистем, так как позволяют определять параметрические диапазоны функционирования, и исключать неприемлемые диапазоны параметров, где возникают избыточные нелинейные эффекты, которые не имеют обоснования для популяционных процессов. Анализ критериев адекватности базируется на сценариях деградации сложно структурированной популяции осетровых рыб бассейна Волги, трески у берегов Канады, вспышек численности инвазионных насекомых и распространению инвазивного гребневика *Mnemiopsis leidy* в Каспийском море.

Ключевые слова: динамические модели инвазий; каскад бифуркации Фейгенбаума; альтернативные аттракторы; сложные динамические процессы, регуляции воздействия для биосистем, гибридные вычислительные системы, параметрические диапазоны, теория сущностной интерпретации

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Introduction. The method of organizing a basic model for analyzing the variability of scenarios, rapidly changing processes with threshold effects is consistently being developed for biological cybernetics problems [1]. Nonlinear effects can occur in a purely applied problem of forecasting the expected value of annual replenishment of populations considered from the point of view of the exploitation of biological resources. The basic computational models developed were aimed at analyzing the reproduction of the Caspian sevryuga after the overlap of spawning grounds. The models were implemented in the form of a system of equations describing the interrelated rates of population decline of the initial generation of individuals and the average growth rates of groups that form a new generation. Using auxiliary equations as a superstructure over the hybrid structure, the model managed to take into account the effect of a rapid increase in the growth rate of individuals and its further stop during the transition to maturation.

Forecasting the entry into the fishing stock of a new sequence of adjacent generations, which, according to various independent factors, may turn out to be significantly different in number, is the key task of conducting a careful fishing. The paper used a method for calculating the stages of development and adjusting the loss coefficients, which are used in the equation from the very first stage of life, where the number of the initial generation is assumed to be $N(0)$. This moment is interpreted as an event of the release of larvae of marine fish or crabs from eggs. The dynamics of a sequential decrease in the initial generation is described by a first-order differential equation, but with an overridable structure of the right part. To fix the redefinition of the calculation scheme, the event space is set on a closed time interval framed by event numbers $[0,T]$.

The phenomenon of reduction of daily loss occurs consistently at the stages of development. The structure of the basic model takes into account various key factors of mortality and a decrease in the rate of attrition during adulthood. The predicative-redefined hybrid structure of the model is written as follows:

$$\frac{dN}{dt} = \begin{cases} -(\alpha w(t)N(t) + U[x]\beta\ )N(t), & t < \tau \\ -(\alpha_1 N(\tau)/w(\tau) + \beta\ )N(t), & t > \tau, \quad w(t) < w_{D2}, \\ -\alpha_2 w(t)N^2(t)\ , & w(t) < w_{D3}, \end{cases} \tag{1}$$

where, adjusted by stages of development, α is the density-determined mortality rate from depletion of vital resources; β is the coefficient of the ever-present loss from a variety of natural factors that are not related to density.

The concept of “reproductive potential” is considered abstract for ecological models of real fishing and artificial replenishment of stocks. There is reason to believe that it is reasonable to switch to a natural indicator of average fertility λ. This indicator can be estimated based on monitoring data, which was conducted on the basis of studies of fish spawning in the Volga and Don basin. Fertility will set the initial conditions for calculating the first form of the right part $N(0)=\lambda S$. The interval τ specified in (1) is the duration of the first stage with endogenous nutrition. This is an important interval for all anadromous fish. The model requires conditions for stopping calculations. An indicator is used as a conditional level of development. In the calculations of the model, when w_d is reached, the severity of mortality factors changes. The ecologically determined adjustment of parameters is interpreted by the change of habitats of juveniles in the river and the avoidance of predators during the already independent migration to the marine habitat.

The Sevryuga of the Caspian Sea was maintained artificially. When growing sturgeon fish in crowded ponds with high density (this is called the term “stocking”), instead of the value $w(\tau)$ in the denominator for the redefined form, the effect of a delay at the final stage of development was established $N(t-\zeta)$.

A dynamically redefined coefficient $U[x]$ plays an important role. The trigger, dynamically adjusted function is enabled in (1), but with a limited scope for its values. The idea of correcting the function in adjacent generations is a way to reflect the influence of extreme conditions. Often, in order to predict the success of reproduction or special states of biosystems, they encounter threshold transformations, as in the degradation scenario of a large predator population structured by spawning groups, the Kamchatka crab off the coast of the Kodiak Archipelago in the waters of Alaska.

The purpose of this article is to determine which nonlinear effects from all their diversity in the dynamics of the trajectories of biological models should be ignored when discussing the results of calculations, and which exclude descriptive possibilities when discussing environmental results. Based on the results obtained, it is possible to solve problems of adequate interpretation of the results of computational modeling of situations that arise during the development of invasive processes and collapses of biological resources. The task of modeling is to support the adoption of control decisions when regulating the impact on biosystems, for which an assessment of the state of biosystems is carried out, but where not all parameters are direct characteristics of species. The task of practical application of models is to find some narrow ranges of parameters that are not suitable for justifying decision-making.

Materials and methods. Based on the fact that the life cycle of the standard length of the organism of both fish and insects is accompanied by structural transformations, the hybrid structure presents a method for diversifying the life cycle according to a fixed set of stages and model equations for each stage.

In this study, an original version of a continuous-discrete model with a special time organization is used. The time of ontogenesis of a species is divided into frames and a hierarchy of nested continuous time segments is created, the ends of which will be discrete events of various types, which is important for analyzing the stages of species invasions.

A list of numbered events has been created within the general interval for the life cycle, where, depending on the type, an upper or lower index is used in the designation. The interval event-hybrid time divided into frames is formalized with a multiset of ordered elements combined into tuples, the number of which corresponds to generations:

$$\bigcup_n \left\{ \partial L_n, \left\{ \bigcup_i [t_0, t^i, t^{i+1}, T] \right\}_n, \partial R_n \right\},$$

where the lower indexes are the event numbers in a fixed interval of the total time interval, and the upper ones are the initial events of each frame.

B hybrid-event format, y model time with events, the number n indicates the frame number in the list of all generations. Recording time with event components leaves boundary slits excluded from the sequence of model frames, which have a service purpose in numerical calculations. In modifications for invasive species in a new environment, it is advisable to specify time frames with floating boundaries that are set by growth functions.

The hybrid time model is designed to use an instrumental modeling environment with a library of numerical methods with varying integration steps. The principles of the model analysis will be the theory of the dynamics of iterations having extremes of functions with a Schwarzian of variable sign (according to the works of Singer and Sharkovsky).

In addition to using the hybrid model (1) to describe the situation of collapse of Kamchatka crab stocks, the hybrid structure was able to predict other complex stage biophysical processes — outbreaks of invasive species and the spread of new infections. A computational model has been constructed for a specific situation in the dynamics of dangerous invasive insects causing sawtooth outbreaks.

In the scenario under consideration, the rate of weight gain is indicated in inverse dependence on the average number of new generation individuals. However, it is not possible to use the inversely proportional method. For this purpose, a form of fractional dependence was chosen. This function is active until switching to active power. The increased decrease in this period of time appears due to an increase in the calorie requirement for larvae with low mobility. It should be noted that invasive species differ in their development features.

The model dynamics of the generation number for an invasive species $N(t)$ is calculated by equations combined into a system with an explicit trigger function in the interval of the event model time:

$$\begin{cases} \dfrac{dN}{dt} = -(\alpha w(t)N(t) + \Theta(S)\beta)N(t), \\ \dfrac{dw}{dt} = \dfrac{g}{\sqrt{N^k(t)} + \zeta}, \end{cases} \tag{2}$$

where S is the value of the spawning part of the fishing stock; $w(t)$ is the fixed value for the dimensional development of generation; g is the temporarily constant parameter that takes into account the limited number of available calories. ξ is the parameter that limits the rate of development regardless of $N(t)$; λ is the average fertility of the spawning part of the fishing stock, which determines the initial calculation conditions (1) as $w(0) = w_0$, $N(0) = \lambda S$; α and β are the instantaneous loss coefficients. Calculations are carried out for the time of ontogenesis, defined as the "vulnerability interval". This is a specific period of time for each species.

For aggressive invasive species, this interval depends on the environmental resistance conditions and the adaptation time of the biotic environment.

Numerically from (2) is the value of the spawning part of the commercial stock $S = N(T)$ with a small number of re-breeding individuals. Taking into account additional reproduction will lead to the formation of a vector from the components of spawning generations. Then you need to calculate the initial generation like this: $N(0) = \lambda_1 S_1 + \ldots + \lambda_i S_i$. For the task of modeling the invasion of insect pests, we will choose an alternative situational trigger action function: $\Theta(S) = 1 + \exp(-cS^2), \lim_{S\to\infty}\Theta(S) \to 1$. The purpose of this function is to reflect the effect of the known effect of the aggregated group, which is important for invasive processes. Alien dangerous pests that have penetrated into a new area generate a local outbreak when they pass the critical threshold of their abundance. Then high activity manifests itself in the form of repeated peaks [2]. A computational system is proposed for the model — a predicatively redefined hybrid structure of equations with a delay.

Biocybernetics develops methods of active intervention and suppression of invasive processes. Regulated resistance to an aggressively reproducing species in a biological community is produced with a delay. The situation leads to a sharp transition into the depression phase of the universe population. To stop the spread of a harmful invasive species, a special introduction of an antagonist species is carried out, but the effectiveness of this method of suppression in practice is unstable.

The model is investigated by presenting a computational scenario with a set of parameters, initial values and an algorithm for making decisions about the impact change for discrete time. Using computational experiments, it is possible to describe a real outcome scenario for a situation that leads to the collapse of the biophysical system at a controlled level of exposure. The model scenario in [3] sets the logic of managerial decision-making to change the level of external pressure on the natural population. The simulation showed that the transition of the process to an oscillatory mode leads to the choice of a risky control mode. It was also found that the dynamics of real aquatic populations has a point of threshold reduction in the efficiency of replenishment of biological resources, which cannot be predicted based on statistical data.

Research results. The model scenario previously developed by the authors for the collapse of the Kamchatka crab of Alaska uses transformations of the phase portrait of iterations, and all of them were justified on graphs with data. In the presence of disconnected boundaries of the regions of attraction, alternative attractors and a strange chaotic repeller lead to the fact that due to chaotic regimes, uncertainty effects arise in the deterministic model.

As a result of the sequential numerical solution of the equations, a dynamic structure is determined, where the discrete component of the trajectory of a "hybrid" continuous-discrete model is studied in a computational environment as an iteration of a mapping with several extremes. Hybrid time models are designed for scenario research, taking into account the logic of making decisions regulating the impact on biosystems, which is used by experts. For the previously described behavior of the hybrid model trajectory in the form of transient randomization and changes in the boundaries of the attractor attraction areas, it was possible to choose an ecological interpretation using the example of collapses of three populations of Caspian sturgeon fish.

The properties of the described model scenario in [3] for exploited aquatic biological resources with a chaotic dynamic regime are confirmed by the author on the example of catching oceanic crustacean species. Iterative models obey the fundamental theorems of nonlinear dynamics, which is the essence of the problem of their application in the management of biosystems. It can be assumed that the nonlinear effect (bifurcation, attractor crisis or stochastic blurring for the separatrix) is hypothetically interesting for describing population processes. However, it cannot be excluded that the effect is accompanied by another metamorphosis of the phase portrait, for which it is impossible to find any biological explanation.

The methods of forecasting and assessing the state of the biosystem used by experts in the formation of the control effect require a separate analysis. In expert methods of ecology, the construction of regression models and the search for correlational relationships takes place on monitoring data. To construct dependence curves in the reproductive process of invasive species, which include values of R depending on the spawning stock S, transformations of the initial monitoring data and the construction of regression curves were proposed. In [5] to predict the dynamics of populations, the author proposed a function for evaluating the efficiency of reproduction:

$$f(S) = aS\exp(-bS). \tag{3}$$

(3) then logarithmed as follows:

$$\ln R - \ln S = \ln a - bS. \tag{4}$$

and built a curve using regression $\ln R / S$ on S for the geometric and arithmetic mean. The method cannot predict oscillations with a large amplitude, since the points would have to be grouped in a certain radius from the intersection with the bisector of the coordinate angle [6].

The motion of the trajectory points in time for a dynamic system in the dissipative case is represented by a motion in phase space to an attractor, a subset of the phase space $A \subseteq M$, invariant with respect to evolution: $\psi^{(t)}(A) = A$ for all $t \in T$. There is also a neighborhood U of the set A, in which for all $y \in U$ $\lim_{t\to\infty} \psi^{(t)}(y) = A$ is true. In the case of dynamic systems used, three topological varieties of attractors are distinguished [7].

A regular attractor for displaying the interval Ψ: $I \to I$ the state of equilibrium with the fixed point is considered x^*: $\lim_{t\to\infty} \psi^{(t)}(y) = x^*$ and a steady cycle. This mode of periodic self-oscillations can be considered expressed approximately by periodic fluctuations for biology, when the period can "float" in a certain range.

In discrete-continuous hybrid-type models, a series of tangent bifurcations is observed with the appearance of stable cycles of periods [8] $p \neq 2^i$ with sequential increase, starting from $a_1 = e^2$ In the event that the one-dimensional mapping $R_{j+1} = \psi(R_j)$ at the value of the control parameter $a = \tilde{a}$ has a period cycle $p = 3$, then it is at the sam value of the control parameter $a = \grave{a}$ has an infinite set of cycles of all other periods. A. N. Sharkovsky [9] proved that if the mapping $\Psi: I \to I$ has a cycle $p = n$, then $\Psi: I \to I$ also has cycles with all possible periods with the same value of the control parameters preceding the number $p = n \in \check{y}^{+}$ among the integers written out in a special order that completes the numbe 3 [10]. The cycle "three" found in the calculations means a periodic window among chaotic variants of dynamics. Hence, the appearance of a period 3 cycle is strange for a biological model.

In order to establish the properties necessary for the biological model, a functional iteration of the smoothness class C^2 the straight-line segment R^1 is selected, which is given by the function $f(x)$. This ecological function will be interpreted as a link between the spawning herd and the resulting replenishment. Let the fixed point of the display depend on the coefficients used in predicting the state of biological resources:

$$x^* = x^*(a, b), \text{ но } f'_x(x^*) = p(a), f'x \neq 0, \text{ если } x \neq c,\ f''(c) \neq 0.$$

In the above example, a differential invariant is defined for the dependence $f(x; x \neq c)$ everywhere, the sign of which is preserved for all iterations:

$$S_f = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2.$$

In the case of function (1), the following properties exist:

$$\begin{aligned} f'(x) &= a\mathbf{e}^{-bx}(1-bx), \\ f''(x) &= ab\mathbf{e}^{-bx}(bx-2), \\ f'''(x) &= ab^2\mathbf{e}^{-bx}(3-bx). \end{aligned}$$

In general, the n-th derivative is expressed: $f^{(n)}(x) = a(-1)^n b^{n-1}\mathbf{e}^{-bx}(bx-n)$

The sign of the differential invariant for all iterations will look like this: $f(f(\ldots(x)\ldots)) \equiv f^n(x)$. Obviously, the property $S_f < 0$ is saved $x \in \Re$, since for $f(S) = aS\exp(-bS)$ the expression takes the form:

$$S_f = b^2\frac{-b^2x^2 + 4bx - 6}{2(1-bx)^2}$$

The position of the stationary point for function (1) depends on two parameters: $x^* = \ln a / b$, the stability criterion is a one–parameter function, and x^* loses stability:

$$f'(x^*) = -1, \text{ где критерии устойчивости}$$

$$f'(x^*) = a\mathbf{e}^{-b\frac{\ln a}{b}} - b\frac{\ln a}{b}a\mathbf{e}^{-b\frac{\ln a}{b}} = \frac{a(1-\ln a)}{\mathbf{e}^{\ln a}} = 1 - \ln a.$$

When $a = \mathbf{e}^2, f'(x^*) = -1$ for the second iteration $f^2(x)$ of the property at a stationary point losing stability x^*:

$$\frac{df^2(x^*)}{dx} = 1,$$

$$\frac{d^2f^2(x)}{dx^2} = \frac{df'(f(x))f'(x)}{dx} = f''(f(x))(f'(x))^2 + f'(f(x))f''(x),$$

$$\frac{d^2f^2(x^*)}{dx^2} = f'(x^*)f''(x^*)(f'(x^*)+1)) = 0.$$

In this case, the Schwarzman exponent y of the second iteration is always identical to the third derivative at this point:

$$S_{f^2(x^*)} = \frac{d^3f^2(x^*)}{dx^3}.$$

The cascade of bifurcations of doubling the cycle period with corresponding periodic windows is realized infinitely if the Schwarzian sign is $S_{f2} < 0$. Then $df^2(x)/dx$ at ae^2 has the local maximum in x^* and only then a bifurcation occurs with the appearance of two new intersection points at the highest iteration. criterion for the realization of an infinite cascade of bifurcations, indicated by Mitchell Feigenbaum doubling the cycle period.

Chaotization through the Feigenbaum cascade is a consequence of the fundamental theorem from [11], where it is established that the mapping of the unimodal function with $S_f < 0$ can have no more than one stable trajectory, and this trajectory is the ω-limit set for the extremum c: $f'(c) = 0$. The problem is that for biosystems this cascade often occurs in models, but according to real data it is not justified.

The biocybernetics model (3), from a mathematical point of view, is classified as a SU-mapping. Model (1) differs from the objects studied by M. Feigenbaum and in other works on universality and renormalization theory by the presence of an inflection point $f''(x_s) = 0, x_s = 2/b$ and points where all the higher derivatives vanish. The property $\lim_{x\to\infty} f(x) \to 0$ for (1) means that the chaotic attractor can increase indefinitely, since there will be no "boundary crisis of the attractor" phenomenon.

Often in practice, a different reproduction function was used as an alternative model of the theory of the formation of generations of bioresources with the marginal biomass of the commercial reserve K and the degree of b denominators:

$$f(x) = \frac{ax}{1+\left(\frac{x}{K}\right)^b}, \tag{5}$$

where $a > 1$ is interpreted as the reproductive potential, K is the value of the ecological niche and the limited limiting capacity of the medium. The degree of impact of environmental limitation on the part of the environment in (5) will be determined by b. The iteration of model (5) was analyzed from the point of view of the theory of bifurcations of maps on R^1. The equilibrium point for iterations (5) has the properties:

$$x^* = K\sqrt[b]{a-1},$$

$$\frac{df(x)}{dx} = \frac{(K^b + x^b)aK^b - ab(Kx)^b}{(K^b + x^b)^2},$$

$$\frac{df(x^*)}{dx} = \frac{a - ba + b}{a} > 0 \text{ when } b < 1.$$

In computational experiments based on the determination of the sign of Lyapunov exponents, the presence of chaotic properties for iterations (5) was established. In a limited range of values of parameter a, which can be applied to invasive populations, period doubling bifurcations occur when b changes. For $b < 1$, the function has no extremes, for $b = 2$, the function has a critical point $x = K$. The 2nd order derivative at the critical point:

$$\frac{d^2 f(x)}{dx^2} = -\frac{a}{4K},$$

So, (5) has a maximum under these conditions. In case (5), a parametric dependence is investigated for the analytical analysis of bifurcations with sufficiently flexible properties. It is possible to avoid a cascade, as well as additional nonlinear effects, internal crises of the chaotic attractor, windows of periodicity and intermittency. All these phenomena here can be carefully excluded from the modeling of biosystems, leaving only the necessary cycles.

Biophysical and commercial interpretation of nonlinear effects in these two models mutually exclude their adequacy. Having considered the change in the behavior of the model (3), we can formulate the hypothesis of "reproductive complexity". Allegedly, an increase in reproductive potential in the biosystem leads to the appearance of population fluctuations, which is expressed in the limit by fluctuations of an aperiodic nature. However, such fluctuations should have an increasing amplitude on average. Accordingly, the average minimum of chaotic fluctuations (the average value of the minimum point for the period) will tend to zero. For biosystems, this means degradation and destruction. In an alternative model, the emergence of a cascade of period cycles 2^n occurs with an increase in the degree of action of limiting environmental factors.

One of the two models will always be fundamentally inadequate. An alternative hypothesis of the essential biophysical interpretation is that the cascade of bifurcations (as well as a number of other complex nonlinear effects and the internal and boundary crisis of the chaotic attractor with the phenomenon of intermittency) for SU-maps has no biophysical interpretation.

Discussion and conclusions. It should be noted that for a number of models there are concomitant nonlinear effects that are undesirable and unnecessary for a number of reasons. Such effects are not confirmed in the analysis of observational data and they need to be excluded from the discussion.

Population cyclicity is an interesting, diverse and far from fully investigated biophysical phenomenon. Cyclicity is observed both in a laboratory aquarium with constant conditions, and in the open ocean (including climate-conditioned). The scientific problem of establishing the physical causes of long-period oscillations in many species is far from being resolved. Research in this area is continued by international teams. It is worth noting some not quite obvious aspects of the problem of describing cyclicity, interesting from the point of view of system analysis.

Population cycles (albeit not in the strictly periodic mathematical sense of a closed trajectory) can be short (weekly) in laboratory conditions. There are examples of long, even secular periods of fluctuations that do not correlate with the length of the life cycle of the species itself. Extreme, in terms of its consequences for forest ecosystems, the phenomenon occurs with the famous fluctuations in the number of the pest of the spruce leaflet *Choristoneura fumiferana* in the forests of North America from the Atlantic to the Pacific Ocean.

Since the phenomenon of population cycles in many species is well described, various mathematical methods and discrete and continuous models have been tried to model the cyclicity inherent in a number of natural populations. The possibility of obtaining cyclic behavior is evaluated for a simple model of the form $x_{n+1} = \psi\ (x_n;\ a)$ positively by many authors. However, the cycles that occur with an increase in parameter a differ not only in the period, but also in the order of traversing their constituent points, that is, the cycles of iterations with the same length (the number of points composing the cycle) period can be qualitatively different — this is one of the differences in the behavior of discrete models.

Figure 1 shows the cyclic trajectory $x_{n+1} = ax_n \exp(-bx_n)$, consisting of four points. In the calculation scenario, there were two other points of the cycle between the extreme upper and lower points of the cycle. The order of traversing the four cyclic points obtained in the computational experiment, when the trajectory passes from the upper branch to the lower one, is universal throughout the cascade. After doubling, the points appear symmetrically in the upper and lower branches until a sharp merger of all the “forked” branches of cyclic points into the Cantor attractor. The order of traversal of branches that form cyclic points is lost only with the formation of the Cantor structure after the unification of branches that were formed during the first doubling of the period.

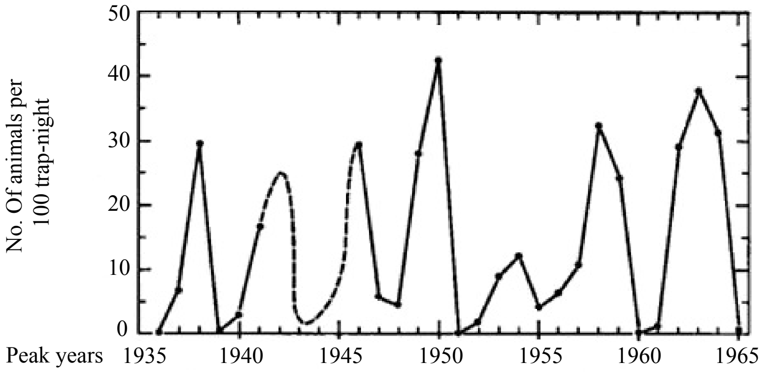

Fig. 1. Cyclical dynamics of the model (3) with a cycle period of “four” (generations (*S*) are indicated along the vertical axis, *t* — along the horizontal axis)

The establishment of the properties of the formation of cycles was perceived on the positive side of confirming the predictive capabilities of such models for populations with non-overlapping generations, and this opinion continues to be expressed, despite the proof of the universal nature of such bifurcations for unimodal functions.

Cyclic density changes are characteristic of small mammals of the Arctic islands and have often been observed in them, but the cycles do not persist and are easily destroyed by any external disturbance [12]. In addition to the length of the period p, iteration cycles $x_{n+1} = \psi\ (x_n)$ differ from each other by the relative location of their constituent points during traversal. Using a typical example of the dynamics of an Arctic rodent, we see that population cycles are monotonous permutations with increasing. The main peak in rodents falls at the end of the four-year period of rodent oscillations, and such cycles with a maximum at the end can be obtained in the order of the Sharkovsky theorem, but by other mathematical methods.

Using the example of longer cycles of steppe rodents in modern Kazakhstan, it is obvious that there is a stage of minimum abundance, a stage of rapid growth and peak, which is replaced by a prolonged depression with a minimal condition. For many insect pests, fading “sawtooth” series of population peaks are observed [13].

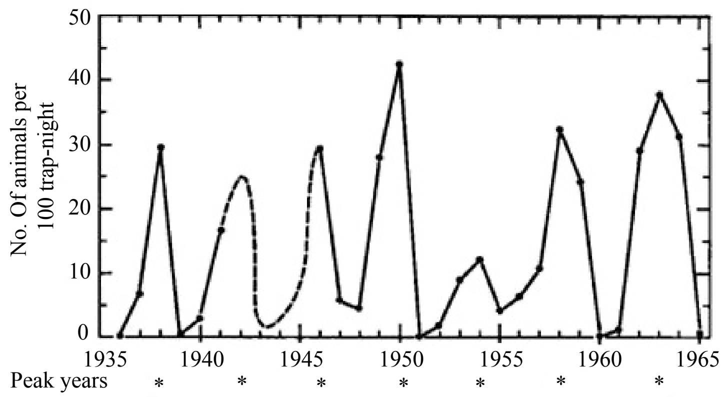

Fig. 2. Four-year cyclicity in the dynamics of the Arctic mammal population according to monitoring data [12]

A series of population outbreaks is also a form of cyclical changes, but such dynamics are not reflected by discrete iterative models. The series of oscillations were described on the basis of continuous models with a delay in operation [14].

Biophysical models in nature management that do not take into account the effect of an aggregated group (or the factor of the presence of a critically low community population) are practically beyond the scope of the possibility of interpreting the results of modeling in ecology. There are a number of other examples where the principles of ecology do not agree with the properties of the mathematical apparatus [15]. It is known that the more species there are in an ecosystem, the more stable it is [16], which means the ability to keep its state unchanged for a long time [17]. But with an increase in the dimension of the phase space of mathematical models, the possibility of trajectory behavior only becomes more complicated.

The tasks of regulating biophysical processes are only becoming more complicated due to unforeseen disturbances, therefore, the development of computational methods for analyzing the nonlinearity of situations with a description of the logic of the impact is relevant. Evolutionarily developed long-term modes of functioning of trophic chains, which include regular cycles of populations, are destroyed without maintaining species diversity [19]. Excessive exploitation of valuable populations violates the regulatory mechanisms that maintain the balance of the species ratio, which leads to the occupation of an ecological niche by harmful invaders and the spread of the invasive combworm Mnemiopsis leidy [20].

The relative position of the extremes of y functions, which are used to link the main values of the reproductive process relative to stationary points, is an important characteristic for dynamics, since it affects the nature of the boundaries of the attractor attraction areas and the occurrence of alternative cycles. For the found scenarios of transition to aperiodic dynamics and back to regular dynamics, a generalized strictly mathematical description has not yet been developed to explain the properties of the transition to chaos.

Optimization errors entail the phenomenon of structural collapse, which must be determined in a timely manner by characteristic features. Optimization for the economy of regions carries a risk, according to the theory of maximum supported withdrawal, implemented for an indefinite period of time in the practice of its application and for populations. The collapse of stocks means a long stop of fishing and depression of the economy. Regulated fishing leads to unexpected degradation of biological resources quite often. Mathematically, this is reflected by the case when an unstable equilibrium in the model is represented by a repeller point. In the ecological reality of invasive processes, this is a blurred area, not a point [21], as the result is the action of stochastic factors that cannot be taken into account directly.

The complexity of managing biosystems with the uncertainty of stochastic effects is that crisis situations are diverse in key features and are often caused by unforeseen factors of hydrology. Similar examples of models are shown in the studies of A.V. Nikitina [22] to analyze the effects of foreign invasive biota actively affecting the bottom biosystems of the Azov and Caspian Seas that have developed in long isolation [23] due to the construction of canals and active shipping.

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