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Software package for predicting possible scenarios for changing the geometry of the bottom of shallow water reservoirs using high-performance computing*

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The article is devoted to the study of the model of transport and sedimentation of suspended solids in the coastal zone. The model takes into account the following processes: advection transport due to the movement of the aqueous medium, microturbulent diffusion and gravitational sedimentation of particles of the suspension, as well as a change in the geometry of the bottom caused by the sedimentation of particles of the suspension or the rise of particles of bottom sediments. The article presents the results of a study of the correctness of the initial-boundary-value problem corresponding to the constructed model. Software package has been developed for predicting possible scenarios for changing the geometry of the bottom of reservoirs in shallow water using high-performance computing.

Keywords: distributed computing, high-performance computing, parallel programming, mathematical model, dynamics of sea sediments, bottom relief.

Introduction. The study of dynamic effects in the coastal zones of water bodies is very relevant in connection with the study of sedimentation and sedimentation processes, the generation and evolution of accumulative forms, the determination of the morpholithodynamic regime of the adjacent coastal zone [1]-[4]. The most widely accumulative forms are presented in the conditions of a gentle slope of the underwater and surface parts of the coast, i.e. on shallow shores, where conditions of shallow water and deformed waves dominate at a considerable distance from the coast [5]-[6]. Under the action of waves and currents, the material begins to move, undergoing sorting in accordance with the shape and mass of individual fractions, as well as mechanical and chemical changes. As a result, a flow of suspended solids is formed in the coastal zone, contributing to the formation of sediment.

Non-stationary spatially 3D models of suspension transport have been introduced into the practice of mathematical modeling relatively recently and have been verified by numerically solving a number of model and some real problems [7]-[10]. The authors (Sukhinov A.I., Sukhinov A.A., Chistyakov A.E., Protsenko E.A., Degtyareva E.E., Sidoryakina V.V.) previously proposed spatially 3D model of suspension transport, taking into account the following physical parameters and processes: advection transport due to the movement of the aqueous medium, microturbulent diffusion and gravitational sedimentation of particles of the suspension, as well as a

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change in the geometry of the bottom caused by the sedimentation of particles of the suspension or the rise of particles of bottom sediments. In this work, an analytical study of the correctness of the initial-boundary-value problem corresponding to the constructed model is carried out. Based on the developed parallel algorithms implemented as a complex of programs, numerical experiments were performed for model problems of bottom sediment transport and bottom topography transformation, the results of which are consistent with real physical experiments.

Physical statement of the problem. Comprehensive studies of aquatic ecosystems are an integral part of environmental management. An important place in these studies is the study of the movement of suspensions, which are the starting material for the formation of bottom sediments.

In oceanology, particles of various origin, passively suspended in sea water and having sizes from 0.5 μm to 1 mm, are considered to be suspensions. Particles may consist of organic and inorganic substances. Inorganic minerals consist mainly of clay minerals (silica, alumina, montmorillonite, illite, etc.) and non-clay minerals (quartz, mica, etc.). Organic materials can exist in the form of plants and bacteria.

The main factors of weighing, redistribution and transport of bottom material is the combined effect of waves and currents. A particle involved in the flow moves in the direction of the water flow and, under the influence of variables in magnitude and direction of pulsating velocities, simultaneously continuously makes vertical movements (rises – falls) (Fig. 1).

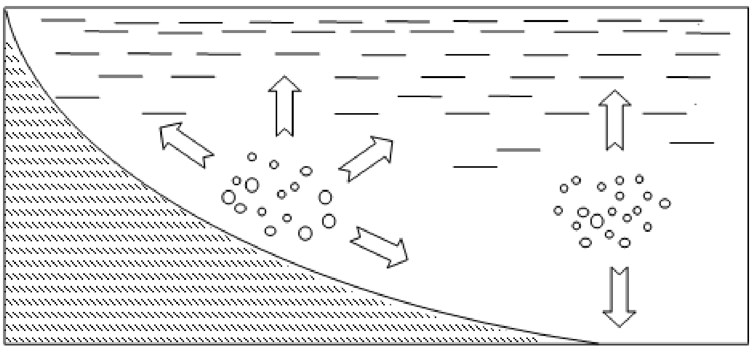

Fig. 1. Scheme of movement of suspended and entrained sediments

In the process of vertical mixing, the particle can sink to the bottom and remain on it until the moment when the lifting force is sufficient to detach it.

Suspension particles differ in size, density, area and, therefore, physico-chemical activity, different residence times in water and sedimentation rate. The mass of suspended solids in the volume of water is characterized by their concentration. The concentration of suspended particles is associated with seasonal factors and the hydrochemical regime of the reservoir, and also depends on anthropogenic factors.

Since it is very difficult to obtain data on the concentration of suspended matter in a pond from field data and in each case the available data may be incomplete, this problem often requires solving using modern methods of mathematical modeling.

Continuous 3Dmodelofdiffusion-convection-suspensionandthecorrespondinginitial-boundaryvalueproblem. Let us consider a continuous mathematical model of suspended matter propagation in an aqueous medium, taking into account diffusion and convection of suspended matter, the effect of gravity on suspension, the presence of a bottom and a free surface. We will use a rectangular Cartesian coordinate system $Oxyz$, where the axis Ox passes on undisturbed water surface and is directed towards the sea, the axis Oz directed vertically down. Let be $h=H+\eta-$ total water depth, [m]; H – depth with undisturbed surface of the reservoir, [m]; η – elevation of the free surface relative to the geoid (sea level), [m].

Let in the closure area $\bar{G}=\{0\le x\le L_x, 0\le y\le L_y, 0\le z\le H(x,y)\}$ there are suspended particles that are at (x,y,z) and at the time t have a concentration $c=c(x,y,z,t)$, [mg / l]; $t-$ temporary variable, [s]. We will also use the notation $L_z\equiv\max\limits_{0\le x\le L_x, 0\le y\le L_y} H(x,y)$.

The behavior of suspended particles will be described by the following system of equations:

$$\begin{cases} \dfrac{\partial c}{\partial t}+\dfrac{\partial(uc)}{\partial x}+\dfrac{\partial(vc)}{\partial y}+\dfrac{\partial((w+w_g)c)}{\partial z}=\mu_h\left(\dfrac{\partial^2 c}{\partial x^2}+\dfrac{\partial^2 c}{\partial y^2}\right)+\dfrac{\partial}{\partial z}\left(\mu_v\dfrac{\partial c}{\partial z}\right)+F, \\ \dfrac{\partial H}{\partial t}=\dfrac{\varepsilon}{\rho}w_g c, \end{cases} \quad (1)$$

where $u,v,w-$ vector components $\vec{U}$ fluid velocity, [m / s]; w_g- hydraulic particle size or sedimentation rate, [m / s]; μ_h, μ_v- coefficients of horizontal and vertical turbulent diffusion of particles, respectively, [m^2 / s]; $F-$ power sources of particles; $\varepsilon-$ bottom porosity.

The terms on the left side (except for the time derivative) of the first equation of system (1) describe the advective transport of particles due to the inertial motion of the aquatic environment and sedimentation under the action of gravity. The terms on the right-hand side describe the diffusion of the suspension. The vertical diffusion coefficient is chosen different from the horizontal diffusion coefficient due to the fact that the effect of the difference of these coefficients is often observed in different environments and can be caused by various factors.

Add to system (1) the initial and boundary conditions (assuming that the deposition of particles on the bottom is irreversible).

As initial conditions at time $t=0$ accept

$$c(x,y,z,0)\equiv c_0(x,y,z); \quad (2)$$

$$H(x,y,0)=H_0(x,y). \quad (3)$$

We set boundary conditions on the edges. $ABCDA_1OC_1D_1$ (set streams of suspensions both towards the coast, and along the coast)

–on the edges $S_1\equiv AA_1OB(x=0, 0\le y\le L_y, 0\le z\le L_z)$, $S_2\equiv AA_1D_1D(y=L_y, 0\le x\le L_x, 0\le z\le L_z)$ and $S_3\equiv BOC_1C(y=0, 0\le x\le L_x, 0\le z\le L_z)$

$$c=c^*, \text{ where } c^*=c^*(x,y,z,t), t\in[0,T]; \quad (4)$$

–on the edges $S_4\equiv DD_1C_1C(x=L_x, 0\le y\le L_y, 0\le z\le L_z)$ and $S_5\equiv A_1OC_1D_1\,(z=0, 0\le x\le L_x, 0\le y\le L_y)$

$$c=0; \quad (5)$$

–on a surface $S_6\equiv ABCD(z=H(x,y,t), 0\le x\le L_x, 0\le y\le L_y)$

$$\frac{\partial c}{\partial n}=-\frac{w_g}{\mu_v}c \text{ or } \frac{\partial c}{\partial z}=-\frac{w_g}{\mu_v}c. \tag{6}$$

The boundary condition (5) takes place with relatively small bottom slopes:

$$\max_{S_6}\sqrt{\left(\frac{\partial H}{\partial x}\right)^2+\left(\frac{\partial H}{\partial y}\right)^2} \ll 1.$$

The following condition of nondegeneracy of the solution domain is set for all (x,y,t) under which the initial boundary problem is posed:

$$H(x,y,t)\geq h_0\equiv const>0,\ 0\leq t\leq T. \tag{7}$$

When studying combined sediment and sediment transport models, it is possible to increase the concentration of suspended particles in the bottom layer due to the rise of sediment particles when the shear stress exceeds a certain critical value [11]-[16]. Then instead of the boundary condition (6) we will consider the boundary condition of the form

$$\frac{\partial c}{\partial z}=\alpha c,\ \alpha=const>0. \tag{8}$$

Linearization of the initial-boundary value problem of transport and sedimentation of suspensions. In order to create a linearized model on a time interval $0\leq t\leq T$ build a uniform grid ω_τ in steps τ, ie many points $\omega_\tau=\{t_n=n\tau, n=0,1,\ldots,N, N\tau=T\}$.

Functions $c^{(n)}(x,y,z,t_{n-1})$ and $H^{(n)}(x,y,t_{n-1})$ we define at each step of the time grid ω_τ. If a $n=1$, then as $c^{(1)}(x,y,z,t_0), H^{(1)}(x,y,t_0)$ it is enough to take the functions of the initial condition, that is $c^{(1)}(x,y,z,0)\equiv c_0(x,y,z)$, $H^{(1)}(x,y,t_0)\equiv H_0(x,y)$ respectively. If $n=2,\ldots,N$, then functions $c^{(n)}(x,y,z,t_{n-1})=c^{(n-1)}(x,y,z,t_{n-1})$ assumed to be known, since problem (1) - (6) for the previous time interval is assumed to be solved $t_{n-2}<t\leq t_{n-1}$.

System (1) in the gap $t_{n-1}<t\leq t_n$ we write in the form:

$$\begin{cases} \dfrac{\partial c^{(n)}}{\partial t}+\dfrac{\partial (uc^{(n)})}{\partial x}+\dfrac{\partial (vc^{(n)})}{\partial y}+\dfrac{\partial \left((w+w_g)c^{(n)}\right)}{\partial z}=\mu_h\left(\dfrac{\partial^2 c^{(n)}}{\partial x^2}+\dfrac{\partial^2 c^{(n)}}{\partial y^2}\right)+\dfrac{\partial}{\partial z}\left(\mu_v\dfrac{\partial c^{(n)}}{\partial z}\right)+F, \\ \dfrac{\partial H^{(n)}}{\partial t}=-\dfrac{\varepsilon}{\rho}w_g c^{(n)} \end{cases} \tag{9}$$

and supplement it with the initial conditions:

$$c^{(1)}(x,y,z,t_0)=c_0(x,y,z), c^{(n)}(x,y,z,t_{n-1})=c^{(n-1)}(x,y,z,t_{n-1}), n=2,\ldots,N. \tag{10}$$

$$H^{(1)}(x,y,t_0)=H_0(x,y), H^{(n)}(x,y,t_{n-1})=H^{(n-1)}(x,y,t_{n-1}), n=2,\ldots,N. \tag{11}$$

The boundary conditions (4) - (6) are assumed to be satisfied for all time intervals $t_{n-1}<t\leq t_n, n=1,2,\ldots,N$.

Defining the function $c^{(n)}(x,y,z,t_{n-1})=c^{(n-1)}(x,y,z,t_{n-1})$ in the time interval $t_{n-1}<t\leq t_n$, can find function $H^{(n)}(x,y,t_{n-1})$. To this end, we integrate both sides of the second equation of system (9) over the variable $t, t_{n-1}<t\leq t_n$. Will get

$$\int_{t_{n-1}}^{t_n} \frac{\partial H^{(n)}}{\partial t} dt = -\frac{\varepsilon}{\rho} w_g \int_{t_{n-1}}^{t_n} c^{(n)} dt. \qquad (12)$$

From equality (12) it is not difficult to get

$$H^{(n)} = H^{(n-1)} - \frac{\varepsilon}{\rho} w_g \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} c^{(n)} dt. \qquad (13)$$

We introduce at each time step $t_{n-1} < t \le t_n,\ n = 1,2,\ldots,N$ region $G_{n-1} = \{0 < x < L_x, 0 < y < L_y, 0 < z < H^{n-1}(x, y, t_{n-1})\}$.

We have a chain of linear initial-boundary value problems for each time layer, where for the interval $t_{n-1} < t \le t_n,\ n = 1,2,\ldots,N$ view system is considered

$$\begin{cases} \dfrac{\partial c^{(n)}}{\partial t} + \dfrac{\partial (uc^{(n)})}{\partial x} + \dfrac{\partial (vc^{(n)})}{\partial y} + \dfrac{\partial ((w + w_g) c^{(n)})}{\partial z} = \mu_h \left(\dfrac{\partial^2 c^{(n)}}{\partial x^2} + \dfrac{\partial^2 c^{(n)}}{\partial y^2} \right) + \dfrac{\partial}{\partial z} \left(\mu_v \dfrac{\partial c^{(n)}}{\partial z} \right) + F, & (14) \\ (x, y, z) \in G_{n-1},\ G_{n-1} = \{0 < x < L_x, 0 < y < L_y, 0 < z < H^{n-1}(x, y, t_{n-1})\}, & \\ H^{(n)} = H^{(n-1)} - \dfrac{\varepsilon}{\rho} w_g \displaystyle\sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} c^{(n)} dt,\ n = 1,2,\ldots,N. & (15) \end{cases}$$

with initial conditions:

$$c^{(n)}(x, y, z, t_{n-1}) = c^{(n-1)}(x, y, z, t_{n-1}), \qquad (16)$$

$$H^{(n)}(x, y, t_{n-1}) = H^{(n-1)}(x, y, t_{n-1}) \qquad (17)$$

Note that at each time step, the boundary surfaces will change (except the face S_5). Considering the time span $t_{n-1} \le t \le t_n$, we carry out the task of boundary conditions on the edges of the region G_{n-1}:

–on the edges $S_{1,n-1}(x = 0, 0 \le y \le L_y, 0 \le z \le H^{(n-1)}(0, y, t_{n-1}))$, $S_{2,n-1}(y = L_y, 0 \le x \le L_x, 0 \le z \le H^{(n-1)}(x, L_y, t_{n-1}))$ and $S_{3,n-1}(y = 0, 0 \le x \le L_x, 0 \le z \le H^{(n-1)}(x, 0, t_{n-1}))$

$$c^{(n)} = c^*\text{, где } c^* = c^*(x, y, z, t), t \in [t_{n-1}, t_n]; \qquad (18)$$

–on the edges $S_{4,n-1}(x = L_x, 0 \le y \le L_y, 0 \le z \le H^{(n-1)}(L_x, y, t_{n-1}))$ and $S_{5,n-1}(z = 0, 0 \le x \le L_x, 0 \le y \le L_y) \equiv A_1OC_1D_1$

$$c^{(n)} = 0; \qquad (19)$$

–on a surface $S_{6,n-1}(z = H^{(n-1)}(x, y, t_{n-1}), 0 \le x \le L_x, 0 \le y \le L_y)$

$$\frac{\partial c^{(n)}}{\partial n} = -\frac{w_g}{\mu_v} c^{(n)} \text{ or } \frac{\partial c^{(n)}}{\partial z} = -\frac{w_g}{\mu_v} c^{(n)}. \qquad (20)$$

The boundary condition (8) will be replaced by the following

$$\frac{\partial c^{(n)}}{\partial z} = \alpha c^{(n)},\ \alpha = const > 0. \qquad (21)$$

Thus, it is assumed that the bottom relief within a given time step when calculating the distribution of concentrations of suspended matter does not change and is taken from the previous time layer. Initially at this time step $t_{n-1} < t \le t_n,\ n = 1,2,\ldots,N$ the initial boundary value problem is solved for the convection-diffusion equation (14) with a fixed bottom relief function $H^{(n-1)}$, and then an update (recalculation) of the relief function is performed $H^{(n)}$ in accordance with equality (15). The determination of the conditions of existence, uniqueness and continuous dependence of the solution on the input data of the problem is carried out on a fixed time layer in these assumptions and subject to condition (7).

We will not investigate in this paper the existence of solutions of the initial boundary value problems (14) - (20) and (14) - (19), (21). Questions of the existence of solutions of initial-boundary value problems for equations of parabolic type with lower derivatives (diffusion-convection equations) are considered, for example, in monographs [17]-[18].

Investigation of the uniqueness of the solution of the initial-boundary problem of suspension transport. Consider the initial boundary value problem (14) - (20), formulated for an arbitrary time layer $t_{n-1}<t\le t_n$.

Multiply the left and right side of equation (14) by the function $c^{(n)}$ and get:

$$c^{(n)}\frac{\partial c^{(n)}}{\partial t}+c^{(n)}\left(\frac{\partial\left(uc^{(n)}\right)}{\partial x}+\frac{\partial\left(vc^{(n)}\right)}{\partial y}+\frac{\partial\left(\left(w+w_g\right)c^{(n)}\right)}{\partial z}\right)=\mu_h c^{(n)}\left(\frac{\partial^2 c^{(n)}}{\partial x^2}+\frac{\partial^2 c^{(n)}}{\partial y^2}\right)+c^{(n)}\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)+c^{(n)}F\,. \quad (22)$$

The left side of equality (22) can be transformed as follows:

$$\begin{gathered}c^{(n)}\frac{\partial c^{(n)}}{\partial t}+c^{(n)}\left(\frac{\partial\left(uc^{(n)}\right)}{\partial x}+\frac{\partial\left(vc^{(n)}\right)}{\partial y}+\frac{\partial\left(\left(w+w_g\right)c^{(n)}\right)}{\partial z}\right)=\frac{1}{2}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}+c^{(n)}div\left(c^{(n)}\vec{U}\right)=\\=\frac{1}{2}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}+\frac{1}{2}div\left(\left(c^{(n)}\right)^2\vec{U}\right),\end{gathered} \quad (23)$$

where $\vec{U}=\left\|u,v,w+w_g\right\|^T$.

In view of (23), equation (22) will be written as

$$\frac{1}{2}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}+\frac{1}{2}div\left(\left(c^{(n)}\right)^2\vec{U}\right)=\mu_h c^{(n)}\left(\frac{\partial^2 c^{(n)}}{\partial x^2}+\frac{\partial^2 c^{(n)}}{\partial y^2}\right)+c^{(n)}\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)+c^{(n)}F\,. \quad (24)$$

Then we integrate both sides of equation (24) over time on the interval $t_{n-1}\le t\le t_n$ and, after this, by spatial variables in the region G_{n-1}. In the first term, the order of integration is changed by the Fubini theorem [19]. Will get

$$\begin{gathered}\iiint_{G_{n-1}}\frac{1}{2}\left(\int_{t_{n-1}}^{t_n}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}dt\right)dG_{n-1}+\int_{t_{n-1}}^{t_n}\frac{1}{2}\left(\iiint_{G_{n-1}}div\left(\left(c^{(n)}\right)^2\vec{U}\right)dG_{n-1}\right)dt=\\=\int_{t_{n-1}}^{t_n}\left(\iiint_{G_{n-1}}c^{(n)}\mu_h\left(\frac{\partial^2 c^{(n)}}{\partial x^2}+\frac{\partial^2 c^{(n)}}{\partial y^2}\right)dG_{n-1}\right)dt+\int_{t_{n-1}}^{t_n}\left(\iiint_{G_{n-1}}c^{(n)}\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)dG_{n-1}\right)dt+\int_{t_{n-1}}^{t_n}\left(\iiint_{G_{n-1}}c^{(n)}F\,dG_{n-1}\right)dt.\end{gathered} \quad (25)$$

The first term on the left side of (25) is obviously equal to

$$\iiint_{G_{n-1}}\frac{1}{2}\left(\int_{t_{n-1}}^{t_n}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}dt\right)dG_{n-1}=\iiint_{G_{n-1}}\frac{1}{2}\left(\left(c^{(n)}\right)^2\left(x,y,z,t_n\right)-\left(c^{(n)}\right)^2\left(x,y,z,t_{n-1}\right)\right)dG_{n-1}\,. \quad (26)$$

Next, we turn to the transformation of the second term of the left-hand side of equality (25). Taking into account the Ostrogradsky-Gauss formula and the boundary conditions (18) - (20), it can be written as [20]

$$\int_{t_{n-1}}^{t_n}\left(\frac{1}{2}\iiint_{G_{n-1}} div\left(\left(c^{(n)}\right)^2\vec{U}\right)dG_{n-1}\right)dt=\frac{1}{2}\int_{t_{n-1}}^{t_n}\left(\iint_{S_{1,n-1}}\left(c^*\right)^2\left(\vec{U^*},\vec{n}\right)dydz\right)dt+\frac{1}{2}\int_{t_{n-1}}^{t_n}\left(\iint_{S_{6,n-1}} c^2 w_g dxdy\right)dt+$$
$$+\int_{t_{n-1}}^{t_n}\left(\iint_{S_{3,n-1}}\left(c^*\right)^2\left(\vec{U^*},\vec{n}\right)dxdz\right)dt+\frac{1}{2}\int_{t_{n-1}}^{t_n}\left(\iint_{S_{2,n-1}}\left(c^*\right)^2\left(\vec{U^*},\vec{n}\right)dxdz\right)dt=-\frac{1}{2}\int_{t_{n-1}}^{t_n}\left(\iint_{S_{1,n-1}}\left(c^*\right)^2 udydz\right)dt- \quad (27)$$
$$-\frac{1}{2}\int_{t_{n-1}}^{t_n}\left(\iint_{S_{3,n-1}}\left(c^*\right)^2 vdxdz\right)dt+\frac{1}{2}\int_{t_{n-1}}^{t_n}\left(\iint_{S_{2,n-1}}\left(c^*\right)^2 vdxdz\right)dt+\frac{1}{2}\int_{t_{n-1}}^{t_n}\left(\iint_{S_{6,n-1}}\left(c^{(n)}\right)^2 w_g dxdy\right)dt$$

where $\vec{U}^*$ is the known velocity of the aquatic environment on the faces, where the boundary conditions of the first kind are specified; in fact, this is all the side faces, except $S_{4,n-1}$ and top cover $S_{5,n-1}$, on which the suspension concentration is zero, and therefore the flows through them are zero.

Let us turn to the transformation of the right side of (25). There is equality

$$\iiint_{G_{n-1}}\left[c^{(n)}\left(\mu_h\frac{\partial}{\partial x}\left(\frac{\partial c^{(n)}}{\partial x}\right)+\mu_h\frac{\partial}{\partial y}\left(\frac{\partial c^{(n)}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)\right)\right]dG_{n-1}=$$
$$=\iiint_{G_{n-1}}\left[\mu_h\frac{\partial}{\partial x}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial x}\right)+\mu_h\frac{\partial}{\partial y}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial y}\right)+\frac{\partial}{\partial z}\left(c^{(n)}\mu_v\frac{\partial c^{(n)}}{\partial z}\right)\right]dG_{n-1}- \quad (28)$$
$$-\iiint_{G_{n-1}}\left[\mu_h\left(\frac{\partial c^{(n)}}{\partial x}\right)^2+\mu_h\left(\frac{\partial c^{(n)}}{\partial y}\right)^2+\mu_v\left(\frac{\partial c^{(n)}}{\partial z}\right)^2\right]dG_{n-1}.$$

Let be $\vec{Q}=\left\{Q_x,Q_y,Q_z\right\}=\left\{\mu_h c^{(n)}\frac{\partial c^{(n)}}{\partial x},\mu_h c^{(n)}\frac{\partial c^{(n)}}{\partial y},c^{(n)}\mu_v\frac{\partial c^{(n)}}{\partial z}\right\}$. Then, by virtue of the Ostrogradsky-Gauss theorem, we have:

$$\iiint_{G_{n-1}}\left[\mu_h\frac{\partial}{\partial x}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial x}\right)+\mu_h\frac{\partial}{\partial y}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial y}\right)+\frac{\partial}{\partial z}\left(c^{(n)}\mu_v\frac{\partial c^{(n)}}{\partial z}\right)\right]dG_{n-1}=\iiint_{G_{n-1}} div\vec{Q}dG=$$
$$=\iint_{S_{2,n-1}}Q_y dxdz+\iint_{S_{4,n-1}}Q_x dydz+\iint_{S_{3,n-1}}Q_y dxdz+\iint_{S_{1,n-1}}Q_x dydz+\iint_{S_{6,n-1}}Q_z dxdy+\iint_{S_{5,n-1}}Q_z dxdy= \quad (29)$$
$$=\iint_{S_{2,n-1}}Q_y dxdz+\iint_{S_{3,n-1}}Q_y dxdz+\iint_{S_{1,n-1}}Q_x dydz+\iint_{S_{6,n-1}}Q_z dxdy.$$

Transforming each term from the right-hand side of (29) subject to the conditions on the boundary (18) - (20), we obtain

$$\iiint_{G_{n-1}}\left[\mu_h\frac{\partial}{\partial x}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial x}\right)+\mu_h\frac{\partial}{\partial y}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial y}\right)+\frac{\partial}{\partial z}\left(c^{(n)}\mu_v\frac{\partial c^{(n)}}{\partial z}\right)\right]dG_{n-1}= \quad (30)$$
$$=\iint_{S_{2,n-1}}c^*\mu_h\frac{\partial c^*}{\partial y}dxdz+\iint_{S_{3,n-11}}c^*\mu_h\frac{\partial c^*}{\partial y}dxdz+\iint_{S_{1,n-1}}c^*\mu_h\frac{\partial c^*}{\partial x}dydz-\iint_{S_{6,n-1}}w_g\left(c^{(n)}\right)^2dxdy.$$

Taking into account (26), (28), (29) and (30), equality (25) takes the form

$$\frac{1}{2}\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2(x,y,z,t_n)dG_{n-1}-\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{1,n-1}}\left(\frac{1}{2}\left(c^*\right)^2u+c^*\mu_h\frac{\partial c^*}{\partial x}\right)dydz\right)dt-$$
$$-\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{3,n-1}}\left(\frac{1}{2}\left(c^*\right)^2v+c^*\mu_h\frac{\partial c^*}{\partial y}\right)dxdz\right)dt+\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{2,n-1}}\left(\frac{1}{2}\left(c^*\right)^2v-c^*\mu_h\frac{\partial c^*}{\partial y}\right)dxdz\right)dt+$$
$$+\frac{3}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}w_g\left(c^{(n)}\right)^2dxdy\right)dt+\int\limits_{t_{n-1}}^{t_n}\left[\iiint\limits_{G_{n-1}}\left(\left(\frac{\partial c^{(n)}}{\partial x}\right)^2+\mu_h\left(\frac{\partial c^{(n)}}{\partial y}\right)^2+\mu_v\left(\frac{\partial c^{(n)}}{\partial z}\right)^2\right)dG_{n-1}\right]dt= \tag{31}$$
$$=\frac{1}{2}\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2(x,y,z,t_{n-1})dG_{n-1}+\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}c^{(n)}FdG_{n-1}\right)dt.$$

Identity (31) will be fundamental in the study of uniqueness and obtaining an a priori estimate of the norm for the solution of the initial-boundary value problem (14) - (20). In the case of replacing the boundary condition (20) with the boundary condition (21), the quadratic functional (31) changes as follows:

$$\frac{1}{2}\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2(x,y,z,t_n)dG_{n-1}-\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{1,n-1}}\left(\frac{1}{2}\left(c^*\right)^2u+c^*\mu_h\frac{\partial c^*}{\partial x}\right)dydz\right)dt-$$
$$-\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{3,n-1}}\left(\frac{1}{2}\left(c^*\right)^2v+c^*\mu_h\frac{\partial c^*}{\partial y}\right)dxdz\right)dt+\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{2,n-1}}\left(\frac{1}{2}\left(c^*\right)^2v-c^*\mu_h\frac{\partial c^*}{\partial y}\right)dxdz\right)dt+$$
$$+\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}\left(\frac{1}{2}w_g-\alpha\mu_v\right)\left(c^{(n)}\right)^2dxdy\right)dt+\int\limits_{t_{n-1}}^{t_n}\left[\iiint\limits_{G_{n-1}}\left(\left(\frac{\partial c^{(n)}}{\partial x}\right)^2+\mu_h\left(\frac{\partial c^{(n)}}{\partial y}\right)^2+\mu_v\left(\frac{\partial c^{(n)}}{\partial z}\right)^2\right)dG_{n-1}\right]dt= \tag{32}$$
$$=\frac{1}{2}\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2(x,y,z,t_{n-1})dG_{n-1}+\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}c^{(n)}FdG_{n-1}\right)dt.$$

Suppose that equation (14) with the same conditions (16) - (20) satisfies two different solutions of the problem $c_1=c_1(x,y,z,t), c_2=c_2(x,y,z,t)$. For their difference $\tilde{c}=c_1-c_2$ The following initial boundary problem is valid:

$$\frac{\partial\tilde{c}}{\partial t}+\frac{\partial(u\tilde{c})}{\partial x}+\frac{\partial(v\tilde{c})}{\partial y}+\frac{\partial\left((w+w_g)\tilde{c}\right)}{\partial z}=\mu_h\left(\frac{\partial^2\tilde{c}}{\partial x^2}+\frac{\partial^2\tilde{c}}{\partial y^2}\right)+\frac{\partial}{\partial z}\left(\mu_v\frac{\partial\tilde{c}}{\partial z}\right), \tag{33}$$

$$\tilde{c}(x,y,z,0)=0,(x,y,z)\in\overline{G}_{n-1}, \tag{34}$$

- on the edges $S_{1,n-1},S_{2,n-1},S_{3,n-1},S_{4,n-1},S_{5,n-1}$

$$\tilde{c}=c^*-c^*=0; \tag{35}$$

- on a surface $S_{6,n-1}$

$$\frac{\partial\tilde{c}}{\partial z}=-\frac{w_g}{\mu_v}(c_1-c_2)=-\frac{w_g}{\mu_v}\tilde{c}. \tag{36}$$

For function $\tilde{c}$ equality (33) will take the form in view of equalities (34) - (36)

$$\frac{1}{2}\iiint\limits_{G_{n-1}}\tilde{c}^2(x,y,z,t_n)dG_{n-1}+\frac{3}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}w_g\tilde{c}^2dxdy\right)dt+$$
$$+\int\limits_{t_{n-1}}^{t_n}\left[\iiint\limits_{G_{n-1}}\left[\mu_h\left(\frac{\partial\tilde{c}}{\partial x}\right)^2+\mu_h\left(\frac{\partial\tilde{c}}{\partial y}\right)^2+\mu_v\left(\frac{\partial\tilde{c}}{\partial z}\right)^2\right]dG_{n-1}\right]dt=0. \tag{37}$$

Insofar as $w_g>0$ and other known quantities under the sign of integrals are positive $\mu_h>0,\ \mu_v>0,$ then equality (36) is satisfied only under the condition

$$\tilde{c}(x,y,z,t)\equiv 0,\ (x,y,z)\in G_{n-1},\ t_{n-1}<t\le t_n, \qquad (38)$$

which completes the proof of the uniqueness of the solution of the initial-boundary value problem (14) - (20).

In the case of replacing the boundary condition (20) by the relation (21), instead of the expression (37), we obtain the equality

$$\frac{1}{2}\iiint_{G_{n-1}}\tilde{c}^2(x,y,z,t_n)dG_{n-1}+\int_{t_{n-1}}^{t_n}\left(\iint_{S_{6,n-1}}\left(\frac{1}{2}w_g-\alpha\mu_v\right)\tilde{c}^2dxdy\right)dt+ \\ +\int_{t_{n-1}}^{t_n}\left[\iiint_{G_{n-1}}\left[\mu_h\left(\frac{\partial\tilde{c}}{\partial x}\right)^2+\mu_h\left(\frac{\partial\tilde{c}}{\partial y}\right)^2+\mu_v\left(\frac{\partial\tilde{c}}{\partial z}\right)^2\right]dG_{n-1}\right]dt=0. \qquad (39)$$

Demandfulfillmentofinequality

$$\frac{1}{2}w_g-\alpha\mu_v\ge 0,\ (x,y,z)\in S_{6,n-1},\ t_{n-1}<t\le t_n$$

or

$$\alpha\le\frac{w_g}{2\mu_v},\ (x,y,z)\in S_{6,n-1},\ t_{n-1}<t\le t_n, \qquad (40)$$

then all the terms in equation (39) are non-negative and equality to zero is possible if and only if $\tilde{c}(x,y,z,t)\equiv 0,\ (x,y,z)\in G_{n-1},\ t_{n-1}<t\le t_n,$ which means the uniqueness of the solution in this case too.

Reasoning is similarly repeated for all layers of the time grid ω_τ. The change in the boundary conditions associated with the continuous change of the bottom relief depending on the time variable requires additional research and is beyond the scope of this article.

Theorem. Let given a system of equations

$$\begin{cases}\dfrac{\partial c^{(n)}}{\partial t}+\dfrac{\partial\left(uc^{(n)}\right)}{\partial x}+\dfrac{\partial\left(vc^{(n)}\right)}{\partial y}+\dfrac{\partial\left(\left(w+w_g\right)c^{(n)}\right)}{\partial z}=\mu_h\left(\dfrac{\partial^2c^{(n)}}{\partial x^2}+\dfrac{\partial^2c^{(n)}}{\partial y^2}\right)+\dfrac{\partial}{\partial z}\left(\mu_v\dfrac{\partial c^{(n)}}{\partial z}\right)+F,\\ (x,y,z)\in G_{n-1}\ ,G_{n-1}=\left\{0<x<L_x,0<y<L_y,0<z<H^{n-1}\left(x,y,t_{n-1}\right)\right\},\\ H^{(n)}=H^{(n-1)}-\dfrac{\varepsilon}{\rho}w_g\displaystyle\sum_{n=1}^{N}\int_{t_{n-1}}^{t_n}c^{(n)}dt,\ n=1,2,\ldots,N\end{cases}$$

in a simply connected domain $\Omega_{n-1}=G_{n-1}\times\left(t_{n-1}<t<t_n\right),\ G_{n-1}=\left(0<x<L_x,0<y<L_y,0<z<H^{(n-1)}\left(x,y,t_{n-1}\right)\right),$ with a fairly smooth boundary defined by the smoothness of the function $z=H^{(n-1)}(x,y),\ 0\le x\le L_x,\ 0\le y\le L_y$ with initial and boundary conditions (16) - (20). Let solution functions $c^{(n)}\left(x,y,z,t_{n-1}\right)$, water velocity vector $\left\|u,v,w+w_g\right\|^T$, initial condition $c^{(n-1)}\left(x,y,z,t_{n-1}\right)$, right side $F(x,y,z,t),$ boundary condition $c^*(x,y,z,t)$, coefficient of vertical turbulent exchange $\mu_v=\mu_v(z),(x,y,z)\in G_{n-1}$ satisfy the following smoothness conditions: $c^{(n)}\left(x,y,z,t_{n-1}\right)\in C^2\left(\Omega_{n-1}\right)\cap C\left(\bar{\Omega}_{n-1}\right),\ grad\,c^{(n)}\in C\left(\bar{\Omega}_{n-1}\right),$

$\left\| u,v,w+w_g \right\|^T \in C^1(\Omega_{n-1}) \cap C(\bar{\Omega}_{n-1})$, $c^{(n-1)}(x,y,z,t_{n-1}) \in C(\bar{G}_{n-1})$, $F(x,y,z,t) \in C(\Omega_{n-1})$, $\mu_v(x,y,z) \in C^1(G_{n-1}) \cap C(\bar{G}_{n-1})$, $c^*(x,y,z,t) \in C(S_{n-1}) \times [t_{n-1} \le t \le t_n]$, $S_{n-1} = \bar{G}_{n-1} \backslash G_{n-1}$, $\frac{\partial c^*}{\partial n} \in C\left(\left(0 \le x \le L_x,\ 0 \le y \le L_y,\ z = H^{(n-1)}(x,y)\right) \times [t_{n-1} \le t \le t_n]\right)$, as well as the conditions of consistency of the boundary and initial conditions, $c^*(x,y,z,0) = c_0(x,y,z)$, $(x,y,z) \in S_{n-1} \backslash \left(0 < x < L_x, 0 < y < L_y,\ z = H^{(n-1)}(x,y)\right)$, $\frac{\partial c_0}{\partial z} = -\frac{\mu_v}{w_g} c^*$, $\left(0 < x < L_x, 0 < y < L_y,\ z = H^{(n-1)}(x,y)\right)$, then the solution to this problem exists and is unique.

Comment. If the boundary condition (20) is replaced by the boundary condition (21), inequality (40) should be added as a sufficient condition for the fulfillment of the previous theorem.

Investigation of the continuous dependence of the solutions of the initial-boundary value problem of suspension transport on the initial, boundary conditions and the function of the right-hand side. The next stage is connected with the study of the continuous dependence of the solution on the functions of the right-hand side, the boundary and initial conditions for the system (14) - (15).

We will assume that

$$c^* \ge c_0^* \equiv const > 0,$$
$$0 \le x \le L_x, 0 \le y \le L_y, 0 < z < H^{(n)}(x,y,t_{n-1}), t_{n-1} \le t \le t_n. \quad (41)$$

For convenience, we introduce the following notation: the union of all parts of the lateral surface the area borders G_{n-1} denote as $S_{c,n-1}$, and the lower base of the area $G_{n-1} - S_{b,n-1}$. By virtue of the smoothness conditions listed under the conditions of the Theorem, extremes of functions on bounded closed sets are reached.:

$$M_{1,n-1} \equiv \max_{\Omega_{n-1}} \left\{ \left| c^{(n)} \right| \right\},\ M_{2,n-1} \equiv \max_{S_{n-1}} \left\{ \left| \frac{\partial c^{(n)}}{\partial x} \right|, \left| \frac{\partial c^{(n)}}{\partial y} \right| \right\},$$
$$M_{3,n-1} \equiv \max_{S_{c,n-1}} \{\mu_h\},\ M_{4,n-1} \equiv \max_{S_{c,n-1} \times [t_{n-1} \le t \le t_n]} \{|u|, |v|\},\ M_{5,n-1} \equiv \min_{G_{n-1}} \{\mu_h, \mu_v\}. \quad (42)$$

We will focus on equation (31) if the boundary condition (20) is used, and equality (32) in the case of the boundary condition (21). Drawing on the Friedrichs inequality, we have a chain of inequalities:

$$\iiint_{G_{n-1}} \left(\mu_h \left(\frac{\partial c^{(n)}}{\partial x} \right)^2 + \mu_h \left(\frac{\partial c^{(n)}}{\partial y} \right)^2 + \mu_v \left(\frac{\partial c^{(n)}}{\partial z} \right)^2 \right) dG_{n-1} \ge$$
$$\le \min_{G_{n-1}} \{\mu_h, \mu_v\} \iiint_{G_{n-1}} \left(\left(\frac{\partial c^{(n)}}{\partial x} \right)^2 + \left(\frac{\partial c^{(n)}}{\partial y} \right)^2 + \left(\frac{\partial c^{(n)}}{\partial z} \right)^2 \right) dG_{n-1} \ge \quad (43)$$
$$\ge M_{5,n-1} \left(\pi^2 \left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{\left(H^{(n-1)}\right)^2} \right) \right) \iiint_{G_{n-1}} \left(c^{(n)}\right)^2 dG_{n-1}.$$

Let us turn to equation (26), from which, by virtue of (42) and (43), we obtain the inequality:

$$\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2 dG_{n-1}+2M_{5,n-1}\left(\pi^2\left(\frac{1}{L_x^{\,2}}+\frac{1}{L_y^{\,2}}+\frac{1}{\left(H^{(n-1)}\right)^2}\right)\right)\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2 dG_{n-1}\right)dt+$$
$$+3\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{b,n-1}} w_g\left(c^{(n)}\right)^2 dxdy\right)dt\le\iiint\limits_{G_{n-1}}c_0^{\,2}dG_{n-1}+M_{4,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left(c^*\right)^2 dS_{n-1}\right)dt+ \quad (44)$$
$$+2M_{2,n-1}M_{3,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left|c^*\right|dS_{n-1}\right)dt+2M_{1,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}\left|F\right|dG_{n-1}\right)dt.$$

From inequality (44) there are two inequalities

$$\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2 dG_{n-1}\le\iiint\limits_{G_{n-1}}c_0^{\,2}dG_{n-1}+M_{4,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left(c^*\right)^2 dS_{n-1}\right)dt+$$
$$+2M_{2,n-1}M_{3,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left|c^*\right|dS_{n-1}\right)dt+2M_1\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}\left|F\right|dG_{n-1}\right)dt. \quad (45)$$

and

$$\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2 dG_{n-1}\le M_{6,n-1}\left(\iiint\limits_{G_{n-1}}c_0^{\,2}dG_{n-1}+\right.$$
$$\left.+M_{4,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left(c^*\right)^2 dS_{n-1}\right)dt+2M_{2,n-1}M_{3,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left|c^*\right|dS_{n-1}\right)dt+2M_{1,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}\left|F\right|dG_{n-1}\right)dt\right). \quad (46)$$

where $M_{6,n-1}=\frac{1}{2M_{5,n-1}}\left(\pi^2\left(\frac{1}{L_x^{\,2}}+\frac{1}{L_y^{\,2}}+\frac{1}{\left(H^{(n-1)}\right)^2}\right)\right)^{-1}$.

From the obtained inequalities, it follows the continuous dependence (stability) of the solution of problem (14) - (20) on the functions: the initial condition, the boundary conditions and the right-hand side L_2 for any point in time $0<T<+\infty$, as well as in time integral L_2.

Obviously, with the fulfillment of inequality (45) and the conditions of the Theorem, the initial boundary value problem (14) - (19), (20) will also have a solution that depends continuously on the functions: the initial condition, the boundary conditions and the right side in the corresponding norms.

Description of the parallel algorithm. A software package implemented in C++ has been developed for constructing turbulent flows of an incompressible velocity field of the water medium on high-resolution grids for predicting sediment transport and possible scenarios for changing the geometry of the bottom area of shallow water reservoirs. Parallel algorithms implemented in the software package for solving model problems arising in the process of sampling systems of grid equations were developed using MPI technology.

To solve this problem, we used an adaptive modified alternately triangular method of minimal corrections. In parallel implementation, the methods of decomposition of grid areas for computationally time-consuming diffusion-convection problems are used, taking into account the architecture and parameters of a multiprocessor computer system. The decomposition of the calculated two-dimensional region is performed using two spatial variables . The peak performance of a multiprocessor computer system is 18.8 teraflops. As computing nodes are used 128 similar 16-core Blade servers HP ProLiant BL685c, each of which is equipped with four 4-core processors AMD Opteron 8356 2.3 GHz and RAM in the amount of 32GB.

The software package is used to calculate the geometry of the bottom of shallow water reservoirs and includes the following blocks:

– control unit, it contains a cycle on the time coordinate and calls the functions: calculation of the speed field without taking into account pressure, calculation of the elevation function, speed field refinement, calculation of the depth field and data input / output functions;

– a block for constructing grid equations for the velocity field without taking into account the pressure in accordance with the finite-difference scheme. The coefficients and the right part of the corresponding grid equation presented in canonical form are considered and written to the array;

– block for constructing grid equations for calculating the elevation function;

– block for calculating the velocity field taking into account the elevation of the level (result of this block is the calculation of the values of the velocity vector field on the next time layer);

– block for calculating grid equations by adaptive modified alternately-triangular method of rapid descent;

– block of output values of the velocity field in the file.

Numerical experiments modeling the sediment transport and the dynamics of changes of the bottom topography. After the development of the software package, a series of numerical experiments was performed to simulate the dynamics of changes in the bottom relief of a complex configuration in the coastal zone of the reservoir. The model problems assumed the presence of obstacles on the bottom surface (boulders, underwater breakwaters, breakwaters, dumps, jetties, spurs, etc.) and various irregularities underlying its surface. As an example, the paper presents the results of modeling the dynamics of changes in the bottom for the case when there are obstacles on its surface in the form of pointed structures-intermittent bun. Due to the retention of sediment, boons not only stop the movement of the material carried by the waves along the shore, but also contribute to its deposition. These structures are one of the best means to protect the coast and prevent the invasion of the sea to the mainland.

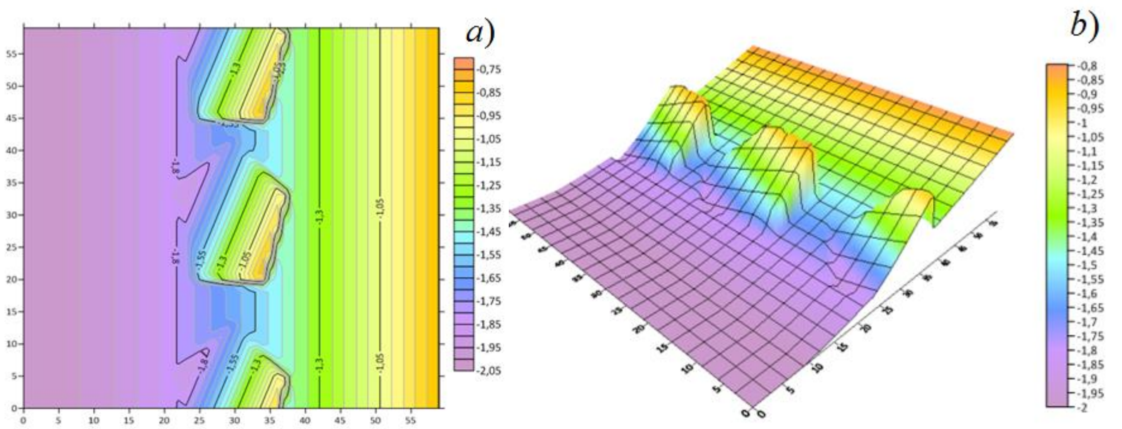

Fig.2. The geometry of the calculated area at the initial moment of modeling (a-the position of the isolines of the depth function, and-the bottom relief)

The simulation area under consideration has dimensions of 55 m by 55 m horizontally and 2 m vertically (in depth), the peak point rises above sea level up to 1 m.Assume that the liquid is at rest at the initial moment of time. The size of the calculated grid is equal to 110by 110, the step on spatial variables is 0.05 m, the step on time is 0.01 s, the wind speed is 5 m/s and is directed from left to right.

Fig.2 shows the initial position of the isolines of the function of depths and bottom relief, the feature of which is the presence of three bottom bun. Fluctuations in the isoline of the depth function are observed in the Central part of the calculation. These structures have a length of up to 15 m and are located at a distance not exceeding 10 m from each other. The structures are completely submerged in the reservoir, and their maximum height is 1.25 m.

Modeling the process of sediment transport showed that over time there is a «smoothing» of the roughness on the surface, the formation of sediments, a decrease in the depth of the slope of the bottom of the coastal zone and, as a result, a gradual shallowing of the considered area of the reservoir. So, after 5 minutes after the beginning of modeling the isolines of the depth function in the center of the calculated area became undulating, and in the area of the location of the boon-sinuous.

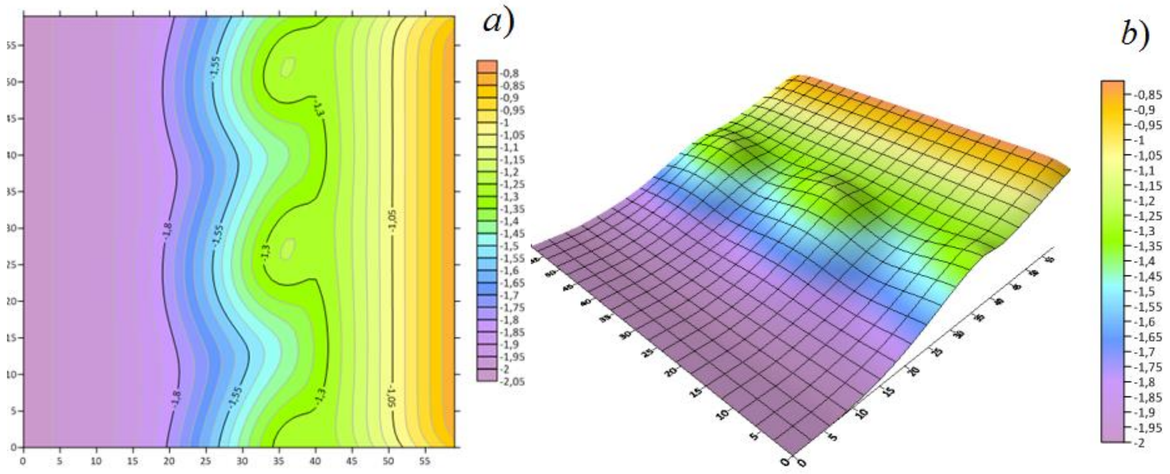

Fig.3. The geometry of the calculated area 5 minutes after the start of the simulation (a – the position of the isolines of the depth function, b – the bottom relief)

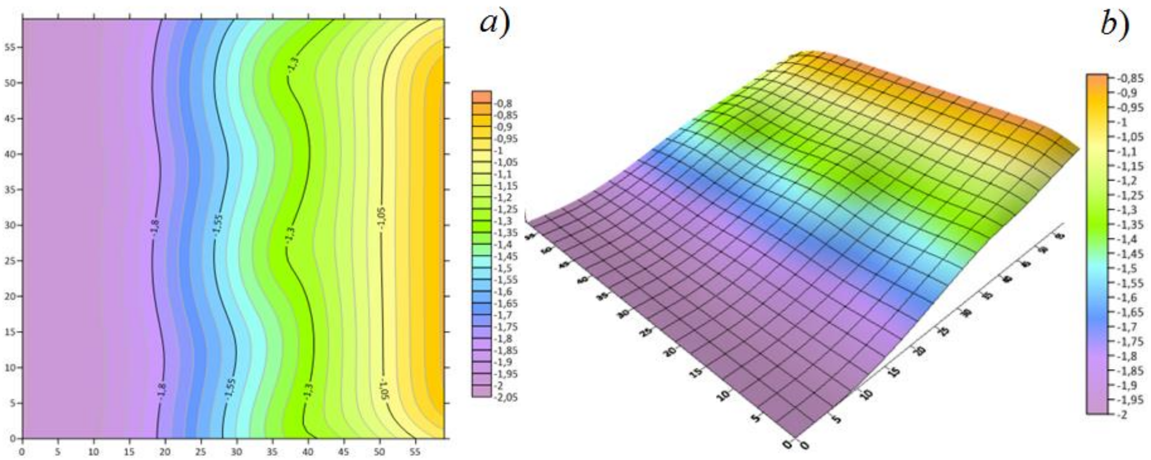

Fig. 4. The geometry of the calculated area 15 minutes after the start of the simulation (a – the position of the isolines of the depth function, b – the bottom relief)

Sediment deposition occurs in the interbunar compartments, and the pointed peaks of the boon have deformed and taken the form of gentle hills. As a result of these processes, there is a decrease in the depth of the coastal zone and the increase in the beach area (Fig. 3).

More clearly, the results are presented in Fig. 4, when the simulation time was 15 min. the isolines of the depth function acquire a soft wavelike shape over the entire calculation area, including the areas of peak values. There was an active process of long-distance movement of sediments and lowering the depth level. So, during the specified estimated time in the interbunny compartments, the depth decrease was about 0.5 m. the height of the slides is reduced, and the slides themselves become more «smoothed» appearance.

Conclusion. The paper presents a software package for predicting possible scenarios for changing the geometry of the bottom of shallow water reservoirs using high performance computing. Numerical experiments were performed for model problems of bottom sediment transport and bottom relief transformation, the results of which are consistent with real physical experiments. The proposed mathematical model and the developed software package allow us to predict the dynamics of the behavior of the bottom surface, the appearance of sea braids and ridges, their growth and transformation.

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Программный комплекс прогнозирования возможных сценариев изменения геометрии дна мелководных водоемов с использованием высокопроизводительных вычислений*

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Настоящая работа посвящена изучению модели переноса и осаждения взвешенных веществ в прибрежной зоне. Комплексные исследования водных экосистем являются неотъемлемой частью рационального природопользования. В этих исследованиях важное место занимает изучение перемещения взвесей, являющихся исходным материалом для образования донных осадков.Основными факторами взвешивания, перераспределения и транспорта донного материала является комбинированное воздействие волн и течений. Частица, вовлеченная внутрь потока, движется в направлении водного потока и, находясь под воздействием переменных по величине и направлению пульсационных скоростей, одновременно непрерывно совершает вертикальные движения (поднимается – опускается).Модель учитывает следующие процессы: адвективный перенос, обусловленный движением водной среды, микротурбулентную диффузию и гравитационное осаждение частиц взвеси, а также изменение геометрии дна, вызванное осаждением частиц взвеси или подъемом частиц донных отложений.В статье представлены результаты исследования корректности начально-краевой задачи, соответствующей построенной модели. Разработан программный пакет для прогнозирования возможных сценариев изменения геометрии дна водоемов на мелководье с использованием высокопроизводительных вычислений.

Ключевыеслова:распределенные вычисления, высокопроизводительные вычисления, параллельное программирование, математическая модель, динамика морских наносов, рельеф дна.

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