Mathematical model of gas-dynamic and thermal processes in a steam turbine [*]

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The paper covers the calculation of aerodynamic processes in a steam turbine using modern information technology and computational methods for improving the calculation accuracy. The practical significance of the paper is the next: the model of aerodynamic processes in the steam turbine is developed and implemented; limits and prospects of the proposed mathematical model is defined. Aerodynamic processes in the turbine are characterized by uneven steam and heat flows, which significantly affect the reliability and efficiency of the turbine. The calculation was performed taking into account the complex geometry of the turbine and can be applied to any turbine of similar design with minor changes.

Keywords: mathematical model; aerodynamics; steam turbine; computational experiments, steam and heat flows

Introduction. The question about the optimization problem of installation and exploitation of steam tur-bines is the actual. Strict requirements are imposed on the development of modern technologies and industry needs to operate the turbines, associated with the reliability and efficiency of their operation. At present, it is necessary to put into operation more modern turbogenerators to re-place physically and technically obsolete units. Design of steam turbines is carried out on the basis of the theory of gas dynamics, taking into account the results of analysis of heat exchange processes, which allow to assess the reliability and efficiency of installation. Modeling of such complex systems is the problem of optimal control of thermal conditions, which allows choosing the best from different implementations. Optimization of thermal conditions is reduced to solving the problem of thermal conductivity. Mathematical modeling of gas-dynamic and thermal processes in technogenic systems is the relevant, due to which the correctness of accepted engineering ideas can be check and correct errors at the design stage using simple and inexpensive means. After the design stage, it is necessary to determine the real temperature values at significant points of the steam facility and analyze the compliance with required values.

Problem Statement. The main equations of gas dynamics of operating environment are:
– the Navier-Stokes system of equations

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+f_x,$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+f_y,$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+f_z, \tag{1}$$

– the continuity equation

$$\rho'_t+\left(\rho u\right)'_x+\left(\rho v\right)'_y+\left(\rho w\right)'_z=0, \tag{2}$$

– the condition equation

$$P=\varepsilon\frac{\rho}{M}RT, \tag{3}$$

where $\mathbf{V}=\{v_x,v_y,v_z\}$ are velocity vector component values; P is the pressure; ρ is the density; M is the molar mass; R is the universal gas constant; μ is the turbulent exchange coefficient; T is the temperature; ε is a coefficient, describing the deviation of the superheated steam pressure from the ideal gas.

Let us assume that the operating environment is initially at rest. Thus, the initial conditions are following: $u=0$, $w=0$, $P=P_a$, where P_a is the initial pressure.

The system of equations (1), (2) is considered under the following boundary conditions:
– on the impenetrable boundary

$$\rho_w\eta\frac{\partial v_x}{\partial n}=\tau_{x,b}(t),\rho_w\eta\frac{\partial v_y}{\partial n}=\tau_{y,b}(t),\ \rho_w\eta\frac{\partial v_z}{\partial n}=\tau_{z,b}(t),\ \mathbf{V}_n=0,\ P'_n=0;$$

– on the side permeable boundary $\dfrac{\partial v_x}{\partial n}=0,\dfrac{\partial v_y}{\partial n}=0,\dfrac{\partial v_z}{\partial n}=0,\ \dfrac{\partial P}{\partial n}=0$;

– on the source $v_x=U$, $v_y=V,v_z=W,\dfrac{\partial P}{\partial n}=0$;

where *U, V, W* are the velocity vector components on the source; τ_x, τ_z are the components of tangential shear stress.

The Navier-Stocks system of equations (1) in the cylindrical coordinate system will take the form:

$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}+\frac{\partial^2 v_r}{\partial z^2}\right)+f_r,$$

$$\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_\theta v_r}{r}+v_z\frac{\partial v_\theta}{\partial z}=-\frac{1}{r\rho}\frac{\partial P}{\partial\theta}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right)+f_\theta,$$

$$\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+f_z. \tag{4}$$

The position of a point in cylindrical coordinate system is determined by the numbers r,θ,z.

The system of equations (4) in the case of axial symmetry has the form:

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right) + \frac{\partial^2 v_r}{\partial z^2}\right) + f_r,$$

$$\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta v_r}{r} + v_z \frac{\partial v_\theta}{\partial z} = \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right) + \frac{\partial^2 v_\theta}{\partial z^2}\right) + f_\theta,$$

$$\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2}\right) + f_z. \tag{5}$$

The continuity equation (2) in the cylindrical coordinate system has the form:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0,$$

The equation (5) in the case of axial symmetry takes the form:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{\partial(\rho v_z)}{\partial z} = 0. \tag{6}$$

Condition equation. The equation of the specific Gibbs energy is the main for superheated steam. Its consist of two parts, relating to the ideal gas condition γ^0 and describing the real component γ^r:

$$g(p,T)/RT = \gamma(\pi,\tau) = \gamma^0(\pi,\tau) + \gamma^r(\pi,\tau) \tag{7}$$

$$\gamma^0 = \ln\pi + \sum_{i=1}^{9} n_i^0 \tau^{J_i^0} \tag{8}$$

$$\gamma^r = \sum_{i=1}^{43} n_i \pi^{I_i}(\tau - 0,5)^{J_i} \tag{9}$$

where $\pi = p/p*$ and $\tau = T*/T$; $p* = 1MPa$ and $T* = 540\,K$. The coefficients and exponents for the equation (8) are given in the Table 2, and for the equation (9) – in the Table 3.

Table 1. Relations for calculation the thermodynamic properties by the equation (7)

Characteristic	Ratio
Specific volume	$vP/RT = \pi\gamma_\pi$
Specific internal energy	$u/RT = \tau\gamma_\tau - \pi\gamma_\pi$
Specific entropy	$s/R = \tau\gamma_\tau - \gamma$
Specific enthalpy	$h/RT = \tau\gamma_\tau$
Specific isobaric heat capacity	$c_p/R = -\tau^2\gamma_{\tau\tau}$
Specific isochoric heat capacity	$c_v/R = -\tau^2\gamma_{\tau\tau} + (\gamma_\pi - \tau\gamma_{\pi\tau})^2/\gamma_{\pi\pi}$
Sound velocity	$w^2/RT = \gamma_\pi^2 \left/ \left(\frac{(\gamma_\pi - \tau\gamma_{\pi\tau})^2}{\tau^2\gamma_{\tau\tau}} - \gamma_{\pi\pi}\right)\right.$

In the Table 1:

$$\gamma_\pi = \left(\frac{\partial \gamma}{\partial \pi}\right)_\tau ; \ \gamma_{\pi\pi} = \left(\frac{\partial^2 \gamma}{\partial \pi^2}\right)_\tau ; \ \gamma_\tau = \left(\frac{\partial \gamma}{\partial \tau}\right)_\pi ; \ \gamma_{\tau\tau} = \left(\frac{\partial^2 \gamma}{\partial \tau^2}\right)_\pi ; \ \gamma_{\pi\tau} = \left(\frac{\partial^2 \gamma}{\partial \pi \partial \tau}\right).$$

According to the ratio for the relative volume:

$$PM = \pi \gamma_\pi \rho RT = \left(1 + \sum_{i=1}^{43} n_i I_i \pi^{I_i} \left(\tau - 0,5\right)^{J_i}\right) \rho RT \ \text{ or}$$

$$PM = \left(1 + \sum_{i=1}^{43} n_i I_i \left(p / p^*\right)^{I_i} \left(T^*/T - 0,5\right)^{J_i}\right) \rho RT .$$

Table 2. Coefficients and exponents for the equation (8)

i	J_i^0	n_i^0	i	J_i^0	n_i^0
1	0	$-0,969\cdot10^1$	6	-2	$0,142\cdot10^1$
2	1	$0,100\cdot10^2$	7	-1	$-0,438\cdot10^1$
3	-5	$-0,560\cdot10^{-2}$	8	2	$-0,284$
4	-4	$0,714\cdot10^{-1}$	9	3	$0,212\cdot10^1$
5	-3	$-0,40728$	6	-2	$0,142\cdot10^1$

Table 3. Coefficients and exponents for the equation (9)

i	I_i	J_i	n_i	i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	1	0	$-0,177\cdot10^{-2}$	16	4	1	$-0,788\cdot10^{-9}$	31	10	14	$-0,100\cdot10^{-8}$
2	1	1	$-0,178\cdot10^{-1}$	17	4	2	$0,127\cdot10^{-7}$	32	16	29	$-0,808\cdot10^{-10}$
3	1	2	$-0,459\cdot10^{-1}$	18	4	3	$0,482\cdot10^{-6}$	33	16	50	$0,106$
4	1	3	$-0,575\cdot10^{-1}$	19	5	7	$0,229\cdot10^{-5}$	34	18	57	$-0,336$
5	1	6	$-0,503\cdot10^{-1}$	20	6	3	$-0,167\cdot10^{-10}$	35	20	20	$0,891\cdot10^{-24}$
6	2	1	$-0,330\cdot10^{-4}$	21	6	16	$-0,211\cdot10^{-2}$	36	20	35	$0,306\cdot10^{-12}$
7	2	2	$-0,189\cdot10^{-3}$	22	6	35	$-0,238\cdot10^2$	37	20	48	$-0,420\cdot10^{-5}$
8	2	4	$-0,393\cdot10^{-2}$	23	7	0	$-0,590\cdot10^{-17}$	38	21	21	$-0,590\cdot10^{-25}$
9	2	7	$-0,437\cdot10^{-1}$	24	7	11	$-0,126\cdot10^{-5}$	39	22	53	$0,378\cdot10^{-5}$
10	2	36	$-0,266\cdot10^{-4}$	25	7	25	$-0,389\cdot10^{-1}$	40	23	39	$-0,127\cdot10^{-14}$
11	3	0	$0,204\cdot10^{-7}$	26	8	8	$0,112\cdot10^{-10}$	41	24	26	$0,730\cdot10^{-28}$
12	3	1	$0,438\cdot10^{-6}$	27	8	36	$-0,823\cdot10^1$	42	24	40	$0,554\cdot10^{-16}$
13	3	3	$-0,322\cdot10^{-4}$	28	9	13	$0,198\cdot10^{-7}$	43	24	58	$-0,943\cdot10^{-6}$
14	3	6	$-0,150\cdot10^{-2}$	29	10	4	$0,104\cdot10^{-18}$	31	10	14	$-0,100\cdot10^{-8}$
15	3	35	$-0,406\cdot10^{-1}$	30	10	10	$-0,102\cdot10^{-12}$	32	16	29	$-0,808\cdot10^{-10}$

All thermodynamic properties of superheated water steam can also be obtained from equation (7) using differential relations of thermodynamics.

Turbulence Model. Let's use the Abramovich-Secundov model, which takes into account very important factors such as the presence of rigid walls, flow background, convective and diffusion transfer of turbulent fluctuations:

$$\frac{\partial v_{турб}}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial v_{турб}}{\partial x_i} = \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\left(\left(v_{мол} + k v_{турб}\right)\frac{\partial v_{турб}}{\partial x_i}\right) + v_{турб} f\left(\frac{v_{турб}}{8 v_{мол}}\right) D - \gamma S,$$

$$S = \frac{v_{myp6}\left(v_{mo\pi} + \beta v_{myp6}\right)}{L_{min}^2}, \quad D = \sqrt{\sum_{i=1}^{3}\sum_{j}\frac{\partial v_i}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)}, \quad f(z) = 0,2\frac{z^2 + 1,47z + 0,2}{z^2 - 1,47z + 1}, \quad (10)$$

where $k = 2$, $\gamma = 50$, $\beta = 0,06$; L_{min} is the minimum distance to a solid wall; $v_{mo\pi}$ is the molecular viscosity; v_{myp6} is the turbulent viscosity.

The turbulence model (10) is considered under the boundary condition:

$$\left(v_{myp6}\right)'_n \left(t, x, y, z\right)\big|_{(x,y,z)\in\Gamma} = 0.$$

Add the initial condition for the model (1):

$$\left(v_{myp6}\right)\left(t, x, y, z\right) = v_0.$$

The following equation was used for calculation the dynamic viscosity values $v_{mo\pi}$:

$$v_{mo\pi} = \overline{v}_0\left(\tau\right)\cdot\overline{v}_1\left(\tau,\delta\right)\cdot v^*, \quad (11)$$

where $v^* = 55{,}071\cdot10^{-6}$ Pa·sec; $\tau = T/T^*$; T is the temperature, [K]; $T^* = 647{,}226$ K; $\delta = \rho/\rho^*$; ρ is the density, [kg/m³]; $\rho^* = 317{,}763$ kg/m³; $\overline{v}_0 = v_0/v^*$; v_0 is the dynamic viscosity of water steam in the zero density limit.

$$\overline{v}_0\left(\tau\right) = \tau^{0,5}\left(\sum_{i=0}^{3}H_i\tau^{-i}\right)^{-1}, \quad (12)$$

where $H_0 = 1$, $H_1 = 0{,}978197$, $H_2 = 0{,}579829$, $H_3 = -0{,}202354$.

$$\overline{v}_1\left(\tau,\delta\right) = \exp\left(\delta\sum_{i=0}^{5}\sum_{j=0}^{6}H_{i,j}\left(1/\tau - 1\right)^i\left(\delta - 1\right)^j\right). \quad (13)$$

Coefficients $H_{i,j}$ for this equation are given in the Table 4.

Table 4. Coefficients and exponents for the equation (13)

i\j	0	1	2	3	4	5	6
0	0,513204	0,215177	-0,281810	0,177806	-0,041766	-	-
1	0,320565	0,731788	-1,070786	0,460504	-	-0,015783	-
2	-	1,241044	-1,263184	0,234037	-	-	-
3	-	1,476783	-	-0,492417	0,160043	-	-0,003629
4	-0.778256	-	-	-	-	-	-
5	0,188544	-	-	-	-	-	-

The error of the dynamic viscosity at the pressures up to 50 MPa and the temperatures up to 475° C is equaled to the 2%, at higher pressures or temperatures – 3%.

Heat conduction problem. Thermal processes in turbine $\overline{G}$ was described by the heat conduction equation:

$$c\,\rho\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) + q_v\left(P\right), \quad (14)$$

which in the case of axial symmetry can be written as:

$$\rho c r T_t' + r v_r T_r' + r v_z T_z' = r\left(\lambda T_z'\right)_z' + \left(\lambda r T_r'\right)_r' + r q_v\left(P\right). \tag{15}$$

In the system (14), (15) T is the temperature; λ is the conductivity of water; ρ is the metal density; c is the heat capacity of metal; r is the polar radius; $q_v\left(P\right)$ is the source function. We will consider the equation (15) with the boundary conditions of third kind:

$$T_n'(x,r,t) = \alpha_n T + \beta_n, \tag{16}$$

where n is the normal vector to the $\overline{G}$.

An initial condition were added to the equation (15):

$$T(x,r,0) = T_0(x,r),\ (x,r) \in \overline{G}. \tag{17}$$

The following equation is used to determine the coefficient of thermal conductivity of water and water steam in international practice:

$$\lambda = \lambda_0(\tau) + \lambda_1(\delta) + \lambda_2(\tau,\delta) \tag{18}$$

where λ is the conductivity; W/(m·K); $\tau = T/T*$; T is the absolute temperature, K; $T* = =647,256$ K; $\delta = \rho/\rho*$; ρ is the density, kg/m³; $\rho* = 317,7$ kg/m³. The thermal conductivity of water steam $\lambda_0(\tau)$ in the ideal gas condition is determined by the equation:

$$\lambda_0(\tau) = \tau^{0,5} \sum_{\kappa=0}^{3} a_\kappa \tau^\kappa,$$

$a_0 = 0,0102811$; $a_1 = 0,0299621$; $a_2 = 0,0156146$; $a_3 = -0,00422464$.

The function $\lambda_1(\delta)$ is defined as: $\lambda_1(\delta) = b_0 + b_1\delta + b_2 \exp\left\{B_1(\delta + B_2)^2\right\}$, where $b_0 = -0,397070$; $b_1 = 0,400302$; $b_2 = 1,06000$; $B_1 = -0,171587$; $B_2 = 2,392190$; and the function $\lambda_2(\tau,\delta)$ has the form: $\lambda_2(\tau,\delta) = \left(\dfrac{d_1}{10} + d_2\right)\delta^{9/5}\exp\left[C_1(1-\delta^{14/5})\right] + d_3 S\delta^Q \exp\left[\left(\dfrac{Q}{1+Q}\right)(1-\delta^{1+Q})\right] + d_4 \exp\left(C_2\tau^{3/2} + \dfrac{C_3}{\delta^5}\right)$,

where Q and S are functions of the value $\Delta\tau = |\tau - 1| + C_4$: $Q = 2 + C_5/\Delta\tau^{0,6}$;

$$S = \begin{cases} 1/\Delta\tau & for\ \tau \geq 1; \\ C_6/\Delta\tau^{0,6} & for\ \tau < 1. \end{cases}$$

The coefficients d_i and C_i have the following values:

$d_1 = 0,0701309$; $d_2 = 0,0118520$; $d_3 = 0,00169937$; $d_4 = -1,0200$; $C_1 = 0,642857$;

$C_2 = -4,11717$; $C_3 = -6,17937$; $C_4 = 0,00308976$; $C_5 = 0,0822994$; $C_6 = 10,0932$.

The equation (14) is applicable with the following temperature and pressure values: $p \leq 100$ MPa for $0 \leq T \leq 500°C$; $p \leq 70$ MPa for $500 < T \leq 650°C$; $p \leq 40$ MPa for $650 < T \leq 800°C$.

The error values in the liquid at temperatures 25-200 °C and pressures up to 5 MPa is equaled to the 1.5 % in the calculations, at higher temperatures up to 300 °C – 2 %. The error is equaled to the 1.5% at temperatures up to 550 ° C and the pressure of 0.1 MPa for the water steam, at pressures up to 40 MPa – 3 %. The equation (18), in comparison with the theoretical conclusions, determines the not infinite, but the final value of the thermal conductivity coefficient at the critical point, which does not allow estimating the error value near its critical point.

Splitting schemes into physical processes. According to the pressure correction method [1-4], the initial model of aerodynamics (1)-(3) is devided into three subtasks. The first subtask was represented by the diffusion-convection equation, on the basis of which the velocity field was calculated at the intermediate time step:

$$\frac{\tilde{v}_r - v_r}{\tau} + v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} = \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right) + \frac{\partial^2 v_r}{\partial z^2}\right) + f_r,$$

$$\frac{\hat{v}_\theta - v_\theta}{\tau} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta v_r}{r} + v_z \frac{\partial v_\theta}{\partial z} = \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right) + \frac{\partial^2 v_\theta}{\partial z^2}\right) + f_\theta,$$

$$\frac{\tilde{v}_z - v_z}{\tau} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2}\right) + f_z. \tag{19}$$

The calculation of the pressure distribution (the second subtask):

$$r\frac{\partial \rho}{\partial t} + \frac{\partial(\rho r \tilde{v}_r)}{\partial r} + r\frac{\partial(\rho \tilde{v}_z)}{\partial z} = \tau\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) + \tau r\frac{\partial^2 P}{\partial z^2} \tag{20}$$

or with taking into account the condition equation

$$r\frac{\partial(PM/\varepsilon RT)}{\partial t} + \frac{\partial(\rho r \tilde{v}_r)}{\partial r} + r\frac{\partial(\rho \tilde{v}_z)}{\partial z} = \tau\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) + \tau r\frac{\partial^2 P}{\partial z^2}. \tag{21}$$

A simplified hydrostatic model of the operation environment medium motion was used as an initial approximation for this task. So, the calculation time was significantly reduced.

The distribution of steam velocities on the upper time layer using explicit formulas can be obtained in the third subtask.

$$\frac{\hat{v}_r - \tilde{v}_r}{\tau} = -\frac{1}{\rho}\frac{\partial P}{\partial r}, \quad \frac{\hat{v}_z - \tilde{v}_z}{\tau} = -\frac{1}{\rho}\frac{\partial P}{\partial z}. \tag{22}$$

where τ is a the step at the time coordinate; v is the value of the velocity field at the previous time layer; $\tilde{v}$ is the value of the velocity field at the intermediate time layer; $\hat{v}$ is the value of the velocity field at the current time layer.

Difference scheme for the solution of the diffusion-convection problem at high Peclet numbers. Let's consider the two-dimensional convection-diffusion equation:

$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} = \mu\frac{\partial^2 q}{\partial x^2} + \mu\frac{\partial^2 q}{\partial y^2}, \tag{23}$$

where $t \in [0,T]$, $x \in [0,L_x]$, $y \in [0,L_y]$ $q(0,x,y) = q^0(x,y)$, $q(t,0,y) = q(t,L_x,y) = 0$, $q(t,x,0) = q(t,x,L_y) = 0$.

The estimated domain was covered by an uniform computational grid $\omega = \omega_\tau \times \overline{\omega}_x \times \overline{\omega}_y$, where $\overline{\omega}_x = \{x_i | x_i = ih_x, \ i = 0,1,...,N, \ Nh_x = L_x\}$, $\overline{\omega}_y = \{y_i | y_i = ih_y, \ i = 0,1,...,N, \ Nh_y = L_y\}$, $\omega_\tau = \{t_j | j = 0,1,...\}$, $\tau = t_{n+1} - t_n = const$.

The splitting scheme for the equation (23) at space will have the form:

$$\frac{q_{i,j}^{n+1/2} - q_{i,j}^{n}}{\tau} + \psi_{xL} \frac{q_{i-1,j}^{n-1/2} - q_{i-1,j}^{n-1}}{2\tau} + \psi_{xR} \frac{q_{i+1,j}^{n-1/2} - q_{i+1,j}^{n-1}}{2\tau} +$$

$$+u\frac{q_{i+1,j}^{n} - q_{i-1,j}^{n}}{4h_x} + \psi_{xL}u\frac{q_{i,j}^{n} - q_{i-1,j}^{n}}{h_x} + \psi_{xR}u\frac{q_{i+1,j}^{n} - q_{i,j}^{n}}{h_x} = \frac{3}{2}\mu\frac{q_{i+1,j}^{n} - 2q_{i,j}^{n} + q_{i-1,j}^{n}}{h_x^2} ;$$

$$\frac{q_{i,j}^{n+1} - q_{i,j}^{n+1/2}}{\tau} + \psi_{yL} \frac{q_{i,j-1}^{n} - q_{i,j-1}^{n-1/2}}{2\tau} + \psi_{yR} \frac{q_{i,j+1}^{n} - q_{i,j+1}^{n-1/2}}{2\tau} +$$

$$+v\frac{q_{i,j+1}^{n+1/2} - q_{i,j-1}^{n+1/2}}{4h_y} + \psi_{yL}v\frac{q_{i,j}^{n+1/2} - q_{i,j-1}^{n+1/2}}{h_y} + \psi_{yR}v\frac{q_{i,j+1}^{n+1/2} - q_{i,j}^{n+1/2}}{h_y} = \frac{3}{2}\mu\frac{q_{i,j+1}^{n+1/2} - 2q_{i,j}^{n+1/2} + q_{i,j-1}^{n+1/2}}{h_y^2} , \qquad (24)$$

where $\psi_{xL} = 1$, $\psi_{xR} = 0$ at $u > 0$, and $\psi_{xL} = 0$, $\psi_{xR} = 1$ at $u < 0$; $\psi_{yL} = 1$, $\psi_{yR} = 0$ at $v > 0$, and $\psi_{yL} = 0$, $\psi_{yR} = 1$ at $v < 0$.

The simulation was performed on the grid by the size of 100×100 computational nodes; the parameters were defined as follows: the dimensions of the computational domain are $L_x = 100$ m, $L_y = 100$ m; the horizontal component is equaled to the 4 m/sec, the vertical component is equaled to the 3 m/sec; the turbulent exchange coefficient is equal to the 0, 0.1, 0.4, 2 m²/sec. The initial distribution was set by the following function at solving the two-dimensional convection-diffusion problem [11-16]:

$$q(x,y) = \begin{cases} \sin(\pi(x-10)/10)\sin(\pi(y-10)/10), \{x,y\} \in D, D: \{x \in [10,20], y \in [10,20]\}, \\ 0, (x,y) \notin D. \end{cases} \qquad (25)$$

Numerical solutions of the convection-diffusion problem at different Peclet numbers using the «cabaret» scheme and the modified scheme (24) are shown in fig. 1.
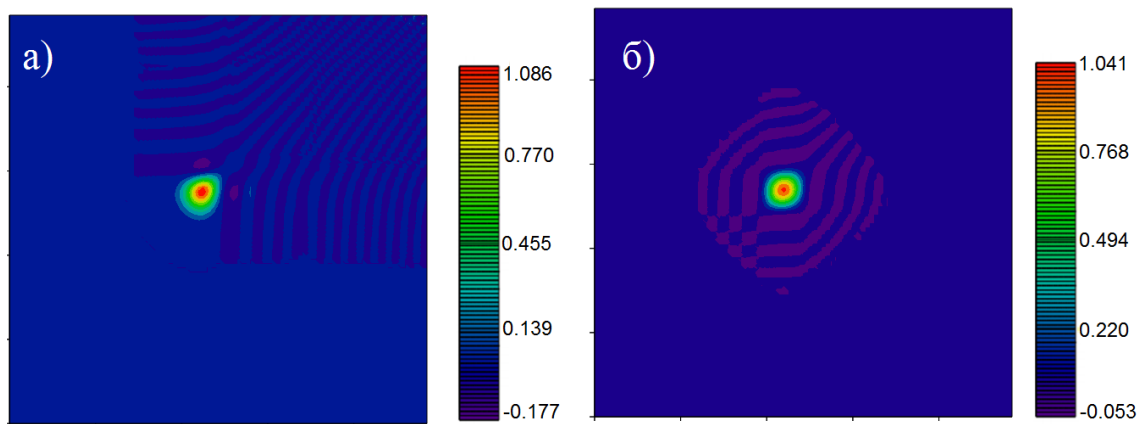

Fig. 1. Numerical solution of the convection problem based on:
a)   the «cabaret» scheme, b) the modified scheme

Discrete Model of Aerodynamics. The estimated domain inscribed in a rectangle. A uniform mesh is introduced for the numerical realization of the discrete mathematical model of the problem in the form:

$$w_h = \{t^n = n\tau, r_i = ih_r, z_j = jh_z; \quad n = \overline{0, N_t}, i = \overline{0, N_r}, j = \overline{0, N_z};$$

$$N_t \tau = l_t, N_r h_r = l_r, N_z h_z = l_z \}, \tag{26}$$

where τ is the time step; h_r, h_z are space steps; N_t is the upper time bound; N_r, N_z are space bounds.

To improve the discrete «smoothness» solution we assume that the cell are not completely filled. [17-22]. The domain Ω_{rz} is the filled part of the domain $D_{rz} \in \left\{ r \in [r_{i-1/2}, r_{i+1/2}], z \in [z_{j-1/2}, z_{j+1/2}] \right\}$.

We introduced the notation for the following domains:

$$D_1 \in \left\{ r \in [r_i, r_{i+1/2}], z \in [z_{j-1/2}, z_{j+1/2}] \right\}; \ D_2 \in \left\{ r \in [r_{i-1/2}, r_i], z \in [z_{j-1/2}, z_{j+1/2}] \right\},$$

$$D_3 \in \left\{ r \in [r_{i-1/2}, r_{i+1/2}], z \in [z_j, z_{j+1/2}] \right\}; D_4 \in \left\{ r \in [r_{i-1/2}, r_{i+1/2}], z \in [z_{j-1/2}, z_j] \right\}.$$

The occupancy coefficients q_0, q_1, q_2, q_3, q_4, for the domains $D_{rz}, D_1, D_2, D_3, D_4$ are introduced as the following: $q_0 = \dfrac{S_{D_{rz}}}{S_{\Omega_{rz}}}$; $q_i = \dfrac{S_{D_i}}{S_{\Omega_i}}$, $i = \overline{1,4}$, where S is an area of the corresponding domain part; Ω_i is a filled part of the domain D_i.

The difference scheme for the system of equations (19) will takes the form:

– for the component v_r:

$$q_0 \frac{v_{r,i,j}^{n+1/2} - v_{r,i,j}^n}{\tau} + q_2 \psi_{rL} \frac{v_{r,i-1,j}^{n-1/2} - v_{r,i-1,j}^{n-1}}{2\tau} + q_1 \psi_{rR} \frac{v_{r,i+1,j}^{n-1/2} - v_{r,i+1,j}^{n-1}}{2\tau} + q_1 v_{r,i+1/2,j}^n \frac{v_{r,i+1,j}^n - v_{r,i,j}^n}{4h_r} +$$

$$+ q_2 v_{r,i-1/2,j}^n \frac{v_{r,i,j}^n - v_{r,i-1,j}^n}{4h_r} + q_2 \psi_{rL} v_{r,i-1/2,j}^n \frac{v_{r,i,j}^n - v_{r,i-1,j}^n}{h_r} + q_1 \psi_{rR} v_{r,i+1/2,j}^n \frac{v_{r,i+1,j}^n - v_{r,i,j}^n}{h_r} =$$

$$= \frac{3}{2} \mu \left(q_1 \frac{r_{i+1} v_{r,i+1,j}^n - r_i v_{r,i,j}^n}{r_{i+1/2} h_r^2} - q_2 \frac{r_i v_{r,i,j}^n - r_{i-1} v_{r,i-1,j}^n}{r_{i-1/2} h_r^2} \right) + q_0 \frac{\left(v_{\theta,i,j}^n \right)^2}{2r} + q_0 \frac{f_{r,i,j}}{2} ;$$

$$q_0 \frac{v_{r,i,j}^{n+1} - v_{r,i,j}^{n+1/2}}{\tau} + q_4 \psi_{zL} \frac{v_{r,i,j-1}^n - v_{r,i,j-1}^{n-1/2}}{2\tau} + q_3 \psi_{zR} \frac{v_{r,i,j+1}^n - v_{r,i,j+1}^{n-1/2}}{2\tau} + q_3 v_{z,i,j+1/2}^n \frac{v_{r,i,j+1}^{n+1/2} - v_{r,i,j}^{n+1/2}}{4h_z} +$$

$$+ q_4 v_{z,i,j-1/2}^n \frac{v_{r,i,j}^{n+1/2} - v_{r,i,j-1}^{n+1/2}}{4h_z} + q_4 \psi_{zL} v_{z,i,j-1/2}^n \frac{v_{r,i,j}^{n+1/2} - v_{r,i,j-1}^{n+1/2}}{h_z} + q_3 \psi_{zR} v_{z,i,j+1/2}^n \frac{v_{r,i,j+1}^{n+1/2} - v_{r,i,j}^{n+1/2}}{h_z} =$$

$$= \frac{3}{2} \mu \left(q_3 \frac{v_{r,i,j+1}^{n+1/2} - v_{r,i,j}^{n+1/2}}{h_z^2} - q_4 \frac{v_{r,i,j}^{n+1/2} - v_{r,i,j-1}^{n+1/2}}{h_z^2} \right) + q_0 \frac{\left(v_{\theta,i,j}^n \right)^2}{2r} + q_0 \frac{f_{r,i,j}}{2} ; \tag{27}$$

– for the component v_θ:

$$q_0 \frac{v_{\theta,i,j}^{n+1/2} - v_{\theta,i,j}^n}{\tau} + q_2 \psi_{rL} \frac{v_{\theta,i-1,j}^{n-1/2} - v_{\theta,i-1,j}^{n-1}}{2\tau} + q_1 \psi_{rR} \frac{v_{\theta,i+1,j}^{n-1/2} - v_{\theta,i+1,j}^{n-1}}{2\tau} + q_1 v_{r,i+1/2,j}^n \frac{v_{\theta,i+1,j}^n - v_{\theta,i,j}^n}{4h_r} +$$

$$+ q_2 v_{\theta,i-1/2,j}^n \frac{v_{\theta,i,j}^n - v_{\theta,i-1,j}^n}{4h_r} + q_2 \psi_{rL} v_{r,i-1/2,j}^n \frac{v_{\theta,i,j}^n - v_{\theta,i-1,j}^n}{h_r} + q_1 \psi_{rR} v_{r,i+1/2,j}^n \frac{v_{\theta,i+1,j}^n - v_{\theta,i,j}^n}{h_r} =$$

$$= \frac{3}{2}\mu\left(q_1 \frac{r_{i+1}v^n_{\theta,i+1,j} - r_i v^n_{\theta,i,j}}{r_{i+1/2}h^2_r} - q_2 \frac{r_i v^n_{\theta,i,j} - r_{i-1}v^n_{\theta,i-1,j}}{r_{i-1/2}h^2_r} \right) - q_0 \frac{v_{\theta,i,j}v_{r,i,j}}{2r} + q_0 \frac{f_{\theta,i,j}}{2} \ ;$$

$$q_0 \frac{v^{n+1}_{\theta,i,j} - v^{n+1/2}_{\theta,i,j}}{\tau} + q_4 \psi_{zL} \frac{v^n_{\theta,i,j-1} - v^{n-1/2}_{\theta,i,j-1}}{2\tau} + q_3 \psi_{zR} \frac{v^n_{\theta,i,j+1} - v^{n-1/2}_{\theta,i,j+1}}{2\tau} + q_3 v^n_{z,i,j+1/2} \frac{v^{n+1/2}_{\theta,i,j+1} - v^{n+1/2}_{\theta,i,j}}{4h_z} +$$

$$+ q_4 v^n_{z,i,j-1/2} \frac{v^{n+1/2}_{\theta,i,j} - v^{n+1/2}_{\theta,i,j-1}}{4h_z} + q_4 \psi_{zL} v^n_{z,i,j-1/2} \frac{v^{n+1/2}_{\theta,i,j} - v^{n+1/2}_{\theta,i,j-1}}{h_z} + q_3 \psi_{zR} v^n_{z,i,j+1/2} \frac{v^{n+1/2}_{\theta,i,j+1} - v^{n+1/2}_{\theta,i,j}}{h_z} =$$

$$= \frac{3}{2}\mu\left(q_3 \frac{v^{n+1/2}_{\theta,i,j+1} - v^{n+1/2}_{\theta,i,j}}{h^2_z} - q_4 \frac{v^{n+1/2}_{\theta,i,j} - v^{n+1/2}_{\theta,i,j-1}}{h^2_z} \right) - q_0 \frac{v^n_{\theta,i,j}v^n_{r,i,j}}{2r} + q_0 \frac{f_{\theta,i,j}}{2} \ ; \tag{28}$$

– for the component v_z :

$$q_0 \frac{v^{n+1/2}_{z,i,j} - v^n_{z,i,j}}{\tau} + q_2 \psi_{rL} \frac{v^n_{z,i-1,j} - v^{n-1}_{z,i-1,j}}{2\tau} + q_1 \psi_{rR} \frac{v^{n-1/2}_{z,i+1,j} - v^{n-1}_{z,i+1,j}}{2\tau} + q_1 v^n_{r,i+1/2,j} \frac{v^n_{z,i+1,j} - v^n_{z,i,j}}{4h_r} +$$

$$+ q_2 v^n_{\theta,i-1/2,j} \frac{v^n_{z,i,j} - v^n_{z,i-1,j}}{4h_r} + q_2 \psi_{rL} v^n_{r,i-1/2,j} \frac{v^n_{z,i,j} - v^n_{z,i-1,j}}{h_r} + q_1 \psi_{rR} v^n_{r,i+1/2,j} \frac{v^n_{z,i+1,j} - v^n_{z,i,j}}{h_r} =$$

$$= \frac{3}{2}\mu\left(q_1 \frac{r_{i+1/2}(v^n_{z,i+1,j} - v^n_{z,i,j})}{r_i h^2_r} - q_2 \frac{r_{i-1/2}(v^n_{z,i,j} - v^n_{z,i-1,j})}{r_i h^2_r} \right) + q_0 \frac{f_{z,i,j}}{2} \ ;$$

$$q_0 \frac{v^{n+1}_{z,i,j} - v^{n+1/2}_{z,i,j}}{\tau} + q_4 \psi_{zL} \frac{v^n_{z,i,j-1} - v^{n-1/2}_{z,i,j-1}}{2\tau} + q_3 \psi_{zR} \frac{v^n_{z,i,j+1} - v^{n-1/2}_{z,i,j+1}}{2\tau} + q_3 v^n_{z,i,j+1/2} \frac{v^{n+1/2}_{z,i,j+1} - v^{n+1/2}_{z,i,j}}{4h_z} +$$

$$+ q_4 v^n_{z,i,j-1/2} \frac{v^{n+1/2}_{z,i,j} - v^{n+1/2}_{z,i,j-1}}{4h_z} + q_4 \psi_{zL} v^n_{z,i,j-1/2} \frac{v^{n+1/2}_{z,i,j} - v^{n+1/2}_{z,i,j-1}}{h_z} + q_3 \psi_{zR} v^n_{z,i,j+1/2} \frac{v^{n+1/2}_{z,i,j+1} - v^{n+1/2}_{z,i,j}}{h_z} =$$

$$= \frac{3}{2}\mu\left(q_3 \frac{v^{n+1/2}_{z,i,j+1} - v^{n+1/2}_{z,i,j}}{h^2_z} - q_4 \frac{v^{n+1/2}_{z,i,j} - v^{n+1/2}_{z,i,j-1}}{h^2_z} \right) + q_0 \frac{f_{z,i,j}}{2} \ . \tag{29}$$

The difference scheme for the calculation of pressure will takes the form:

$$q_0 r_i \frac{P^{n+1}_{i,j} / (\varepsilon T)^{n+1}_{i,j} - P^n_{i,j} / (\varepsilon T)^n_{i,j}}{\tau} + q_1 \frac{r_{i+1}\tilde{v}^n_{r,i+1,j}\rho^n_{i+1,j} - r_i \tilde{v}^n_{r,i,j}\rho^n_{i,j}}{2h_r} +$$

$$+ q_2 \frac{r_i \tilde{v}^n_{r,i,j}\rho^n_{i,j} - r_{i-1}\tilde{v}^n_{r,i-1,j}\rho^n_{i-1,j}}{2h_r} + q_3 r_i \frac{\tilde{v}^n_{z,i,j+1}\rho^n_{i,j+1} - \tilde{v}^n_{z,i,j}\rho^n_{i,j}}{2h_z} +$$

$$+ q_4 r_i \frac{\tilde{v}^n_{z,i,j}\rho^n_{i,j} - \tilde{v}^n_{z,i,j-1}\rho^n_{i,j-1}}{2h_z} = q_1\tau \frac{r_{i+1/2}(P^n_{i+1,j} - P^n_{i,j})}{h^2_r} - q_2\tau \frac{r_{i-1/2}(P^n_{i,j} - P^n_{i-1,j})}{h^2_r} +$$

$$+ q_3\tau r_i \frac{P^n_{i,j+1} - P^n_{i,j}}{h^2_z} - q_4\tau r_i \frac{P^n_{i,j} - P^n_{i,j-1}}{h^2_z} \ . \tag{30}$$

The difference schemes for the system of equations (22) will takes the form:

$$q_0 \frac{\hat{v}_{r,i,j} - \tilde{v}_{r,i,j}}{\tau} = -q_1 \frac{P^{n+1}_{i+1,j} - P^{n+1}_{i,j}}{2\rho h_r} - q_2 \frac{P^{n+1}_{i,j} - P^{n+1}_{i-1,j}}{2\rho h_r} ,$$

$$q_0 \frac{\hat{v}_{z,i,j} - \tilde{v}_{z,i,j}}{\tau} = -q_3 \frac{P_{i,j+1}^{n+1} - P_{i,j}^{n+1}}{2\rho h_z} - q_4 \frac{P_{i,j}^{n+1} - P_{i,j-1}^{n+1}}{2\rho h_z} \; . \tag{31}$$

Software implementation. The initial parameters of the model are: the rotation frequency of the turbine; the range of exit velocities at the turbine blades; the number of blades; the number of nozzle channels; the width of the nozzle channel to the impeller; the steam pressure on impellers.

The speed of steam movement is indicated by the color. The measure units is meters per second. Due to the developed software, we can calculate movement of the operating environment within areas with cylindrical symmetry and determine the velocity and pressure field patterns within axial steam turbines. The permeability of the medium was used in the mathematical model for describing the geometry of the nozzle lattice. The movement of steam in several stages at the same time can be calculated on the basis of the developed software. The distribution fields of the main physical computing quantities inside for the first operating chamber are shown in Fig. 2 (1 is the longitudinal and radial component of the velocity vector, m/sec; 2 is the rotation speed, revs/min; 3 is the pressure, atm.; 4 is the turbulent viscosity, Pa·sec; 5 is the temperature, °C; 6 is the density, kg/m³).
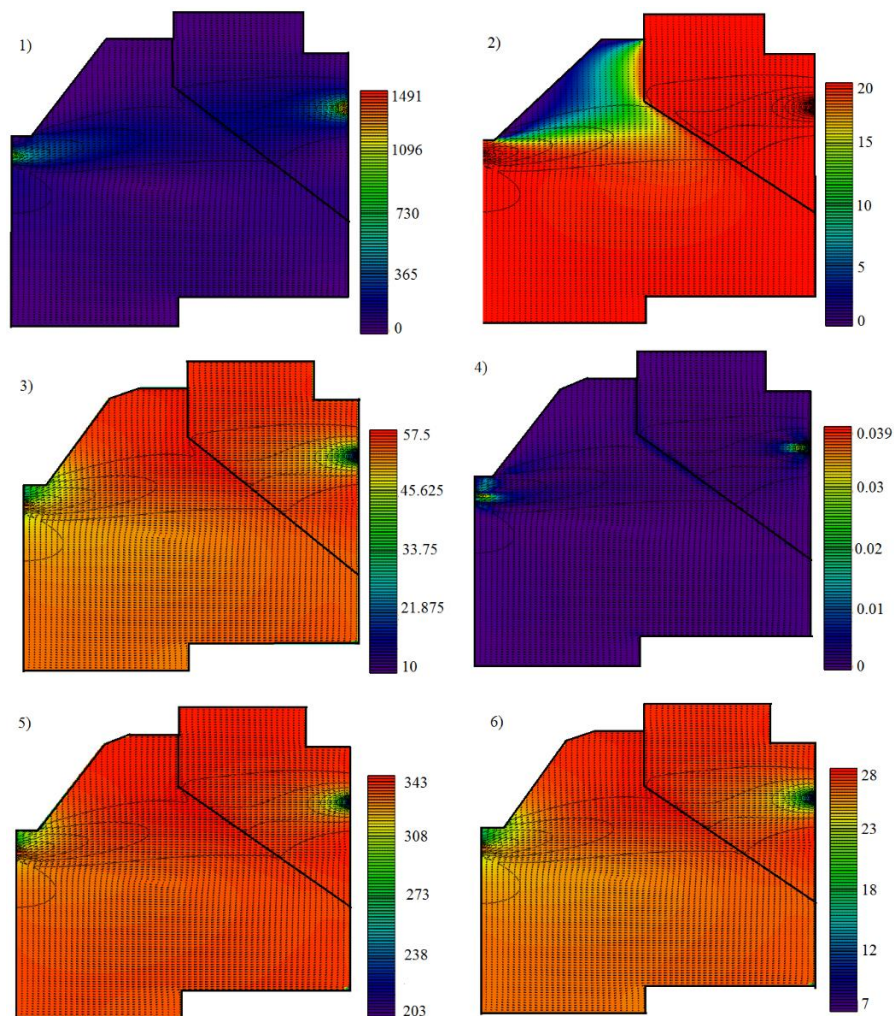

Fig. 2. Distribution fields of the main physical computing quantities
inside for the first operating chamber

Conclusion. The mathematical model of gas-dynamic processes in a steam turbine was developed. The main equations of the model are the Navier-Stokes equations, the continuity and condition equations for the real gas. The initial system of equations was written in a cylindrical coordinate system taking into account the axial symmetry of the steam flow field in the turbine. The splitting schemes into physical processes (the pressure correction method) were used for approximation problem in the temporary variable. According to this method, the problem is solved in three stages. The diffusion-convection equation with large Peclet numbers is necessary for calculating the velocity field components without taking into account the pressure. New difference schemes, based on the «cabaret» scheme modernization, were applied for solving this problem. These difference schemes are in 2-2.5 times more accurately solve the diffusion-convection problem in comparison with traditional «cabaret» schemes, which are effective in solving such problems. Software implementation of the problem was performed on the basis of the devel-oped algorithms, and the results of numerical calculations of aerodynamic processes in the steam turbine were obtained.

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Математическая модель газодинамических и тепловых процессов в паровой турбине *

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В работе представлен расчет аэродинамических процессов в паровой турбине с использованием современных информационных технологий и вычислительных методов повышения точности расчетов. Практическая значимость работы заключается в следующем: разработана и реализована модель аэродинамических процессов в паровой турбине, определены пределы и перспективы предложенной математической модели. Аэродинамические процессы в турбине характеризуются неравномерностью паровых и тепловых потоков, что существенно влияет на надежность и эффективность работы турбины. Расчет выполнен с учетом сложной геометрии турбины и может быть применен к любой турбине аналогичной конструкции с незначительными изменениями.

Ключевые слова: математическая модель, аэродинамика, паровая турбина, вычислительные эксперименты, паровые и тепловые потоки

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