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Mathematical modeling the density of sea water in the deep pond *

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The paper covers the mathematical model of hydrodynamics of a deep water reservoir-the sea of Japan, taking into account the complex geometry of the shoreline and the bottom, friction on the bottom and wind currents, evaporation, deviation of the pressure field from the hydrostatic approximation, water density as a function of the spatial distribution of temperature and salinity, is proposed and investigated. The models of observations for the functions included in the model problem, including the dependence of the water density distribution on the spatial distribution of its temperature, salinity and ionic composition, are determined and investigated. The application of these models is based on electrical conductivity and temperature, which makes them as accurate as possible in different conditions.

Keywords: mathematical model; deep water reservoir; pressure field; the hydrostatic approximation; spatial distribution of temperature and salinity.

Introduction. Mathematical modeling of biological processes Hydrophysics and kinetics of deep-water reservoirs have been studied by many scientists. Scientists R. A. Ibrayev, Yu. b. Silantyev, V. A. Kalmykov, O. V. Oleinik, R. P. Khodorevskaya, A. A. Zhilkin, I. S. Guliyev, A. Naderi Beni, H. Lahjani, R. Mousavi Harami, K. Apre, S. A. G. Leroy, N. Marriner, M. Berberian, V. Andrieu-Ponel, M. Djamali, A. Mahboubi, P. J. Reimer is devoted to the mathematical fashion to encourage processes of thermohydrodynamics, the analysis of problems of preservation of water resources and the impact of human activities on environmental safety, as well as the study of the status of stocks of valuable commercial fish species. However, despite the large number of works in this area, many effects that are essential to improve the accuracy and reliability of forecasts of changes in the ecological situation of deep water bodies were not taken into account earlier in the post-hydrophysical models [1-6]. As an object of modeling, the sea of Japan is selected, since this reservoir has a complex bathymetry that affects the spatial change of hydrodynamic processes. Databases of data on hydrochemical data of the sea of Japan are in free access and are constantly replenished, which is important for the calibration and verification of the developed mathematical models. The study of the distribution of the density field of sea water is necessary for a more accurate determination of the precipitation of ships in different parts of the sea and allows to improve the means of monitoring the environmental situation in the reservoir, to develop optimal scenarios for the development of biota of deep-water reservoir. Development and numerical implementation of hydrodynamic models is one of

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the stages of creation of projects of global monitoring of hydrodynamics on the planet, as well as an important and first – priority task of environmental safety and economic development of countries.

Statement of the problem. A complete system of equations of sea dynamics with a lateral surface σ, variable depth H in the Cartesian coordinate system with axes z, y, x directed vertically downwards, North and East, respectively (Fig. 1), will take the following form:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-2\Omega(\mathrm{v}\sin\theta-w\cos\theta)+\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial \varphi}{\partial x}=\mu\Delta u+\frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right); \quad (1)$$

$$\frac{\partial \mathrm{v}}{\partial t}+u\frac{\partial \mathrm{v}}{\partial x}+\mathrm{v}\frac{\partial \mathrm{v}}{\partial y}+w\frac{\partial \mathrm{v}}{\partial z}+2\Omega u\sin\theta+\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\partial \varphi}{\partial y}=\mu\Delta \mathrm{v}+\frac{\partial}{\partial z}\left(\nu\frac{\partial \mathrm{v}}{\partial z}\right); \quad (2)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+\mathrm{v}\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}-2\Omega u\cos\theta+\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\partial \varphi}{\partial z}=\mu\Delta w+\frac{\partial}{\partial z}\left(\nu\frac{\partial w}{\partial z}\right); \quad (3)$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho x)}{\partial x}+\frac{\partial(\rho \mathrm{v})}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0; \quad (4)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}+\gamma_T w=\mu_T\Delta T+\frac{\partial}{\partial z}\left(\nu_T\frac{\partial T}{\partial z}\right); \quad (5)$$

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+\mathrm{v}\frac{\partial S}{\partial y}+w\frac{\partial S}{\partial z}+\gamma_S w=\mu_S\Delta S+\frac{\partial}{\partial z}\left(\nu_S\frac{\partial S}{\partial z}\right), \quad (6)$$

where t is the time; u, v, w are the components of the velocity vector of the water medium; $\Omega = 7{,}2921\cdot 10^{-5}$ c^{-1} is the angular velocity of the earth's rotation; θ is the angle between the angular velocity of the Earth's rotation and the vertical; ρ is the water density; p is the pressure increase over the hydrostatic pressure of the undisturbed liquid; φ is the value of the gravitational potential; μ, ν are the diffusion coefficients in horizontal and vertical directions, respectively [7]; $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplace operator; T, S are the deviations of the temperature and salinity of the water from the mean values $\bar{T}, \bar{S}$; $\gamma_{T,S}(z)$ are given stratification parameters: $\gamma_T = d\bar{T}/dz$, $\gamma_S = d\bar{S}/dz$.

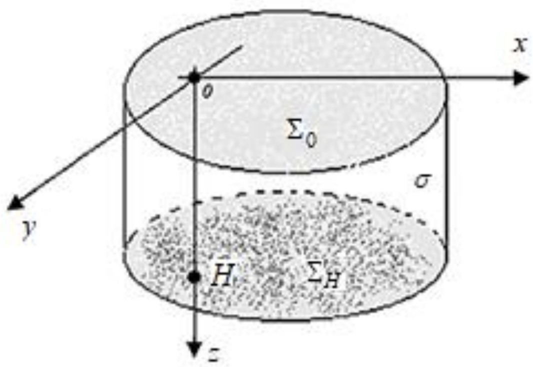

Fig. 1. Scheme of the computational domain

We add boundary conditions to the system (1)-(6):

- on the upper border Σ_0:

$$\rho_{\nu}\mu u'_n(x,y,z,t)=-\tau_x(t); \quad \rho_{\nu}\mu \mathrm{v}'_n(x,y,z,t)=-\tau_y(t); T'_z=f_3; \; p'_n(x,y,t)=0;$$

$$S'_z = f_4\,;\ \rho_v \mu w'_{\boldsymbol{n}}(x,y,z,t) = -\tau_z(t)\,;\ w(x,y,t) = -\psi - p'_t/\rho g\,; \tag{7}$$

– on the side surface σ and bottom Σ_H :

$$\rho_v \mu u'_{\boldsymbol{n}}(x,y,z,t) = -\tau_x(t)\,;\ \rho_v \mu v'_{\boldsymbol{n}}(x,y,z,t) = -\tau_y(t)\,;\ T'_{\boldsymbol{n}} = 0\,;\ p'_{\boldsymbol{n}}(x,y,z,t) = 0\,,\ S'_{\boldsymbol{n}} = 0\,;$$

$$\rho_v \mu w'_{\boldsymbol{n}}(x,y,z,t) = -\tau_z(t)\,;\ V'_{\boldsymbol{n}}(x,y,z,t) = 0\,; \tag{8}$$

where $\boldsymbol{n}$ is the vector of the external normal to the surface $\Sigma(\Sigma = \Sigma_0 \cup \sigma \cup \Sigma_H)$; ρ_v is the he density of the terrigenous suspension, which appears as a result of erosion of the banks and river flow; τ_x, τ_y, τ_z are omponents of the tangential stress (Van-Dorne's law); T'_z, S'_z are changes in temperature and salinity in depth; f_3, f_4 are set functions; ψ is the intensity of evaporation of the liquid; p'_t is the rate of change in pressure; $p'_{\boldsymbol{n}}$ is the normal component of the pressure vector; g is the acceleration of free fall; $T'_{\boldsymbol{n}}, S'_{\boldsymbol{n}}$ are changes in temperature and salinity in the direction of the normal vector; $V'_{\boldsymbol{n}} = \{u'_{\boldsymbol{n}}, v'_{\boldsymbol{n}}, w'_{\boldsymbol{n}}\}$ is the normal component of the water flow velocity vector.

At $t = t_0$ the initial conditions for the model (1)-(8) will be as follows:

$$u(x,y,z,t_0) = u_0(x,y,z)\,;\quad v(x,y,z,t_0) = v_0(x,y,z)\,;\ w(x,y,z,t_0) = w_0(x,y,z)\ ;$$

$$T(x,y,z,t_0) = T_0(x,y,z)\,;\ S(x,y,z,t_0) = S_0(x,y,z), \tag{9}$$

where $u_0, v_0,\ w_0,\ T_0,\ S_0$ are given functions.

Model (1)-(9) takes into account the motion of the water flow, microturbulent diffusion, the Coriolis force, the spatial distribution of salinity and TEM-temperature. In the majority of works on Oceanology and thermohydrodynamics the linear variant of water density distribution (LV) depending only on temperature changes is considered:

$$\rho_L = \alpha(1 - \beta T), \tag{10}$$

where $\alpha = 1028$; β is the coefficient expressing the dependence of water density changes on temperature changes T (Table 1).

Table 1. The dependence of the density of water from temperature changes

T, °C	$\beta \cdot 10^4$, °C^{-1}
0	–0,6300
5	0,0350
7,5	0,3675
10	0,7000
12,5	0,9800
15	1,2600
17,5	1,5400
20	1,8200
22,5	2,1675
25	2,5150

This dependence is the maximum permissible simplification, which significantly reduces the accuracy of the models using it. A more accurate model of observations, based on the equation of state (ES-80), used to determine the density of sea water [8], has the form:

$$\rho = \rho(T,S,0)/\left[1 - p/k(T,S,p)\cdot 10^3\right]; \quad (11)$$

$$\rho(T,S,0) = \rho_n + AS - BS^{3/2} + CS^2; \quad (12)$$

$$\rho_n = i_0 + i_1T - i_2T^2 + i_3T^3 - i_4T^4 + i_5T^5; \quad (13)$$

$$A = A_0 - A_1T + A_2T^2 - A_3T^3 + A_4T^4; \quad (14)$$

$$B = B_0 - B_1T + B_2T^2; \quad (15)$$

$$k(T,S,p) = k(T,S,0) + Z_1\cdot 10^{-5}p + X_1\cdot 10^{-10}p^2; \quad (16)$$

$$k(T,S,0) = k_n + aS + bS^{3/2}; \quad (17)$$

$$Z_1 = z_w + \left(z_0 - z_1T - z_2T^2\right)S + z_3S^{3/2}; \quad (18)$$

$$X_1 = x_w - \left(x_0 - x_1T - x_2T^2\right)S; \quad (19)$$

$$z_w = c_0 + c_1T + c_2T^2 - c_3T^3; \quad (20)$$

$$x_w = d_0 - d_1T + d_2T^2; \quad (21)$$

$$k_n = k_{n0} + k_{n1}T - k_{n2}T^2 + k_{n3}T^3 - k_{n4}T^4; \quad (22)$$

$$a = a_0 - a_1T + a_2T^2 - a_3T^3; \quad (23)$$

$$b = b_0 + b_1T - b_2T^2, \quad (24)$$

where $k(T,S,p)$ is the average modulus of elasticity; ρ_n is the density of the standard medium-oceanic clean water; A_{0-4}, B_{0-2}, C, i_{0-5}, $z_{w,0-3}$, $x_{w,0-2}$, k_{n0-n4}, a_{0-3}, b_{0-2}, c_{0-3}, d_{0-2} are specified coefficients.

The observation model (11)-(24) is applicable in the following ranges:

- for salinity $S = 0\ldots 42$ ‰;
- temperature $T = -2\ldots+42$ °C;
- pressure. $p = 0\ldots 1000$ bar.

However, it should be noted that for the application of the model (11)-(24) it is necessary to have average values of temperature and salinity in the nodes of the measuring grid, which does not allow instantaneous calculations.

In this regard, this paper proposes to modify the system (11)-(24), adding the equation of the form:

$$p = l_0H + l_1H^2, \quad (25)$$

and replacing equation (11) with equation:

$$\rho = \rho(T,S,0)/\left[1 - p/k(T,S,p)\cdot 10^3\right] - \xi H, \quad (26)$$

where ξ s the coefficient obtained experimentally; $l_{0,1}$ are given factors. This modification (ES -18) not only improves the accuracy of predictive modeling, but also allows to use as input data not the temperature and salinity deviations from the mean values, but the data measured in the grid nodes at a specific time. This circumstance allows to optimize the performance of software systems using dependence (11)-(24), and to shorten the program, which is numerically realized in] the developed hydrodynamic model.

The description of input data for modeling. To study the problem of species (1)-(8), taking into account the influence of models of observations (12)-(26) in the computational domain of complex shape-deep-water body of the sea of Japan, the data from the portal of the unified state system of information on the situation in the world ocean «UISGO» [9] are used.

On the coastline of the sea of Japan, there are 18 coastal hydrological stations, forming a constantly growing database «UISGO», the use of which allows for calibration and comparative analysis of models of observations of the species (10), (11) and (26).

The spatial distribution of water temperature on the surface of the sea of Japan throughout the year is characterized by the following large-scale regularity: the maximum values of water temperature in the active layer of the water body are typical for the southern part of the sea. This is mainly due to two reasons, one of which is the inflow of modified sub – tropical waters from the Pacific ocean, and the second is the geographical location of the region and the associated increased solar radiation (compared to the more Northern parts of the sea). The minimum values of water temperature in the active layer of the reservoir are typical for the Northern and North-Western parts of the sea. The warming effect of subtropical waters of Pacific origin is practically not affected. In winter, the lowest values of air temperature are observed over these waters, which leads to intensive cooling of the surface layer of water. In spring and summer, the Northern and North-Western parts of the sea are characterized by weather conditions with high frequency of fogs, which leads to the weakening of solar radiation penetrating into the surface layer of water. All considered factors lead to the fact that in the Northern and North-Western parts of the sea the lowest heat storage of the surface water layer is formed. Figure 2 shows the average monthly temperature values for 2017 in the mountain zone from 0 to 100 meters for the sea of Japan.

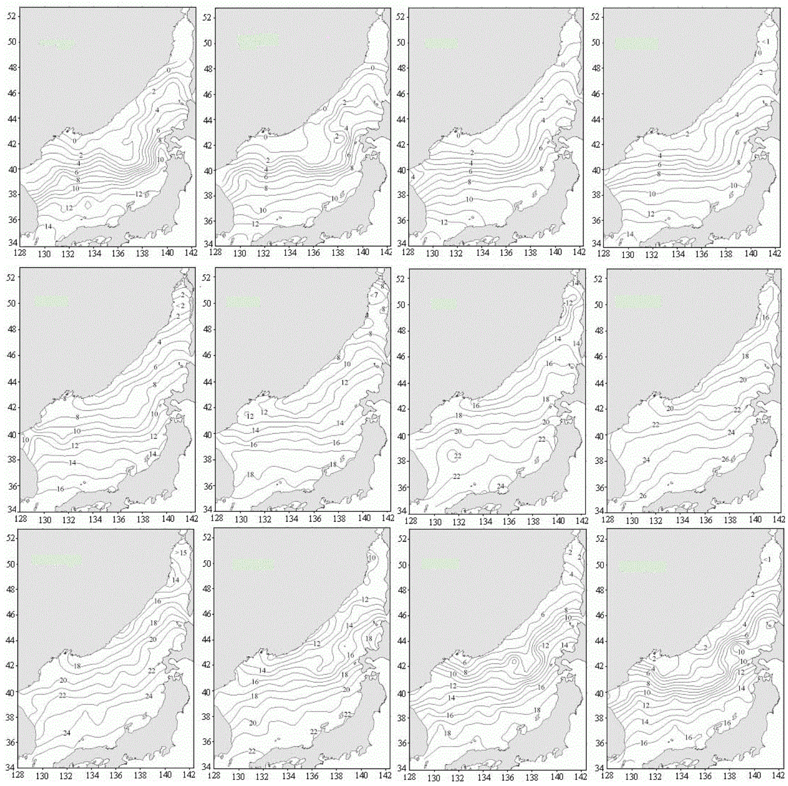

Fig. 2. The distribution of water temperature on the surface of the sea of Japan monthly from January to December

There are two types of spatial distribution of salinity in the layer of 0-100 meters in the waters of the sea of Japan during the year. The first type of salinity distribution manifests itself from January to June. During this period of the year, maximum values are observed in the southern and Eastern parts of the sea. The main feature of the second type of salinity distribution, which is allocated from July to December, is the presence of maximum values in the Central part of the sea. As you get closer to the coast the salinity is lowered. Starting from the 200 m horizon, seasonal changes in salinity become unreliable. Therefore, for these horizons, only the average annual salinity distribution in the sea of Japan is considered. In the layer of 600-800 m, it is very difficult to distinguish the spatial laws of the distribution of salinity of the waters of the sea of Japan. Limits spatial changes in salinity become comparable to the point-ness of registrations salinity. Thus, until the mid-1970s, salinity was determined by the method of chlorine titration, the accuracy of which did not exceed 0.02‰. From the mid-70s to the end of the 80s to determine the salinity, mainly used electric meters with an accuracy of up to 0.005‰. Only in the last decade of the 20th century, water salinity is really determined to an accuracy of 0.001-0.003‰. Average long-term data show that in the deep and bottom layers of the sea of Japan variations of salinity are in the range of 34,050-34,075‰. In the fields of salinity is quite difficult to identify regional patterns. This is mainly due to the lack of accuracy of the predominant number of salinity definitions currently available at great depths of the sea. However, as follows from viscotec governmental registrations of salinity made in recent years, the salinity of the deep sea-the deep waters of the sea of Japan lies in a fairly narrow range (34,065 – 34,069‰). Figure 3 shows the average monthly salinity values for 2017 on the horizon from 0 to 100 meters for the sea of Japan.

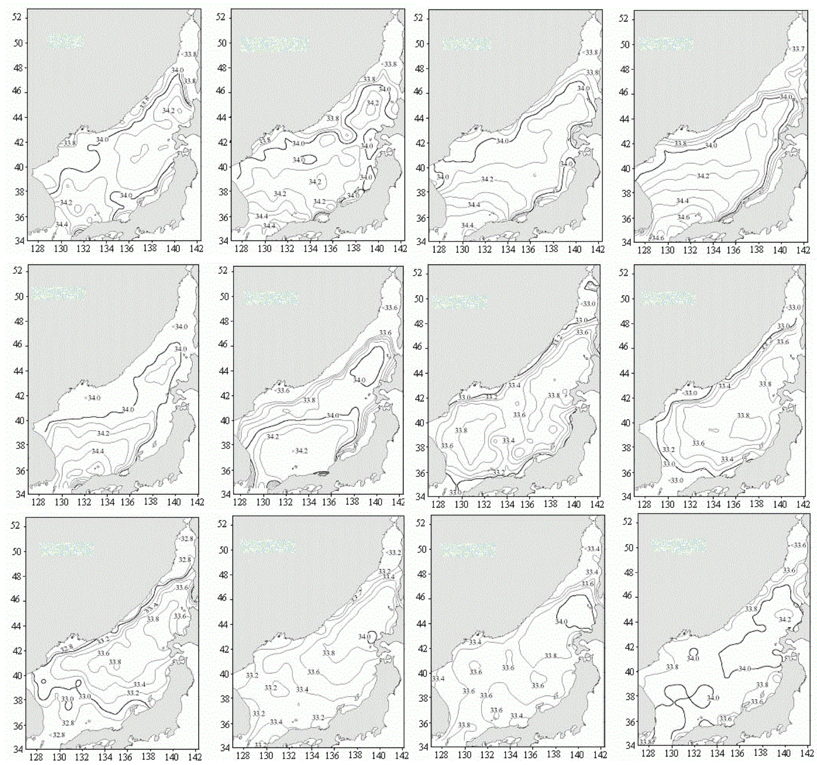

Fig. 3. Distribution of water salinity on the surface of the sea of Japan monthly from January to December

Analysis of observation models. Three points in the sea of Japan with coordinates (latitude/longitude/identifier) were arbitrarily taken to analyze the results of modeling the density of sea water):

1. 39.5/134.5/13194;
2. 42.5/136.5/16726;
3. 50.5/141.5/20201.

The simulation result for point 1 is shown in Fig. 4, for point 2-in Fig. 5 and for point 3-in Fig. 6, the figures show the values of the difference between the field data and the results of the numerical experiment, obtained on the basis of models LV, CE-80 and LE-18.

It should be noted that on the horizon of 0 meters (the surface of the reservoir) models LV, CE-80 and LE-18 models show fairly accurate and close results. However, on the horizon of 10 meters, the difference between the simulation results becomes much larger and increases with depth.

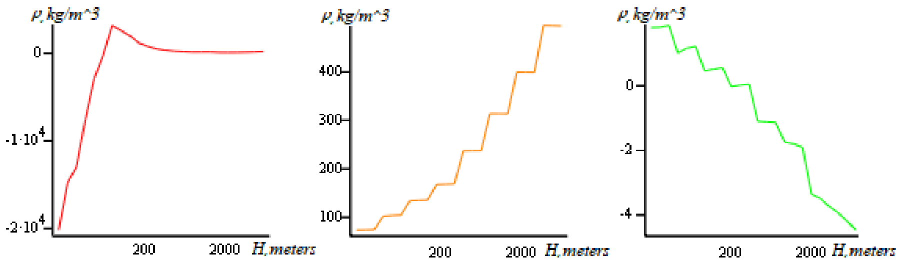

Fig. 4. Comparison of field and model data for calculation of water density fields at point 1 (39.5/134.5/13194) for LV, CE-80 and LE-18 dependencies

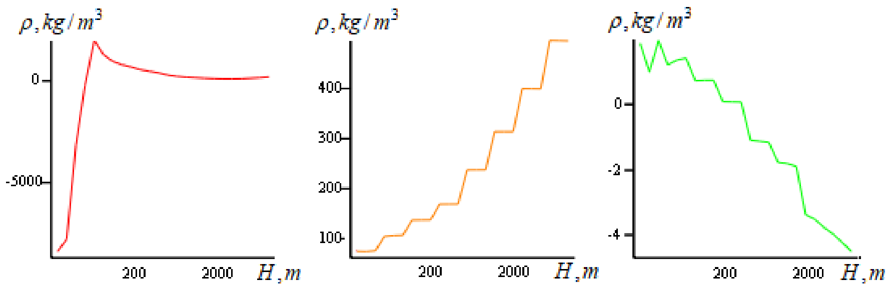

Fig. 5. Comparison of field and model data for calculation of water density fields at point 2 (42.5/136.5/16726) for LV, CE-80 and LE-18 dependencies

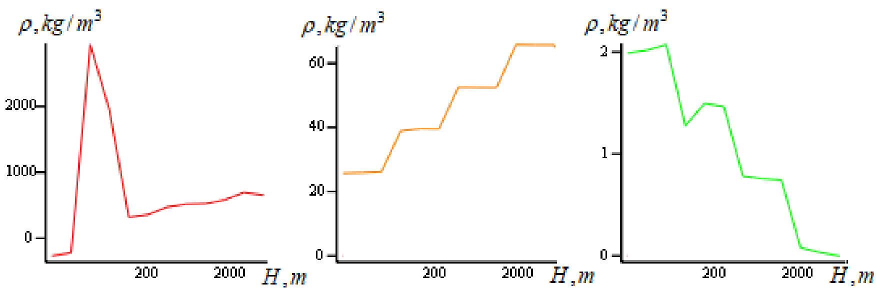

Fig. 6. Comparison of field and model data for calculation of water density fields at point 3 (50.5/141.5/20201) for LV, CE-80 and LE-18 dependencies

The criterion for checking the adequacy of the considered models of observations VI-da (10), (11) and (26) is the estimation of modeling error with simultaneous account of the field data on the available $n=10000$ measurements, which is calculated by the formula^

$$\delta=\sqrt{\sum_{k=1}^{n}\left(\rho_{k\,nat}-\rho_k\right)^2}\Bigg/\sqrt{\sum_{k=1}^{n}\rho_{k\,\text{nat}}^2}\cdot 100\%\,, \qquad (27)$$

where $\rho_{k\,\text{nat}}$ are the full-scale measurements of water density values; ρ_k шы density value calculated using models (10), (11), (26).

For the comparative analysis of the observation models, the full-scale values of the seawater density in the nodes of the computational grid for the high seas on the horizons from 0 to 100 meters, taken from the database «UISGO» (Fig. 7).

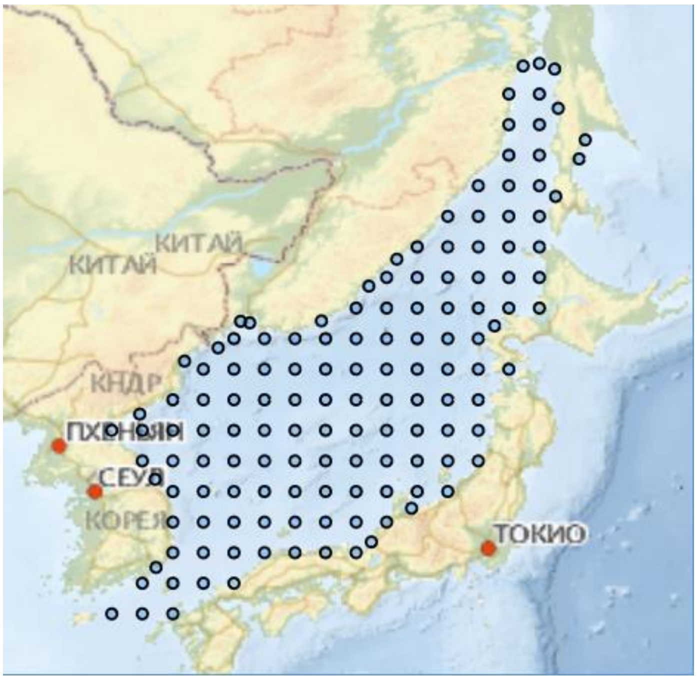

Fig. 7. The nodes of the computational grid for the sea of Japan

Denote $\delta_{(i)}$ is the error value calculated by the formula (27) using the i-th dependence, $i\in\{10,11,26\}$, as a model of observations. Accordingly, for models (10), (11) and (26) we obtain the mean error values:

$$\delta_{(10)}=8{,}854\,;$$

$$\delta_{(11)}=0{,}259\,;$$

$$\delta_{(26)}=0{,}044\,.$$

Graphs of error calculated by the formula (27) for models of the form (11) and (26) are presented in figures 8 and 9, respectively.

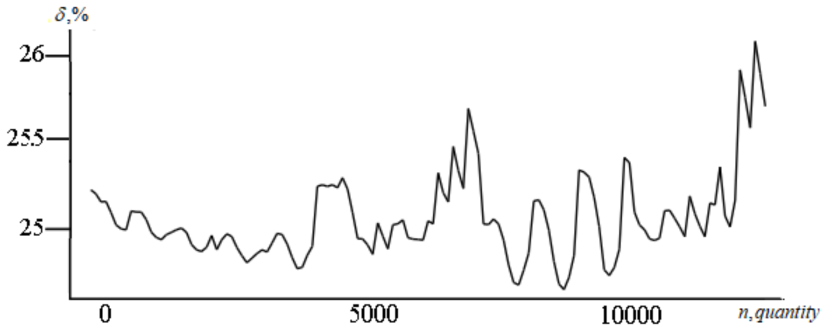

Fig. 8. Graph of error changes for the model CE-80

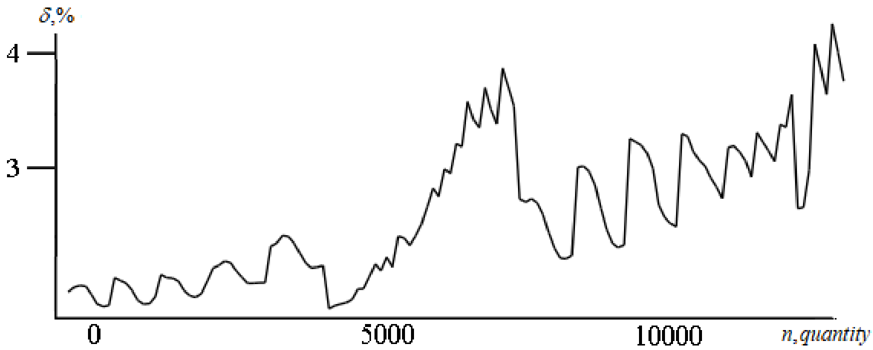

Fig. 9. Graph of error change for the model LE-18

It is obvious that the universal model of the species (10) is significantly inferior to the exact news of the model of the species (11), which in turn is a rough approximation to the developed model of observations of the species (26).

Conclusion. The mathematical model of hydrodynamics of a deep water reservoir-the sea of Japan, taking into account the complex geometry of the shoreline and the bottom, friction on the bottom and wind currents, evaporation, deviation of the pressure field from the hydrostatic approximation, water density as a function of the spatial distribution of temperature and salinity, is proposed and investigated. The models of observations for the functions included in the model problem, including the dependence of the water density distribution on the spatial distribution of its temperature, salinity and ionic composition, are determined and investigated. The application of these models is based on electrical conductivity and temperature, which makes them as accurate as possible in different conditions.

Taking into account the dependence of the density of sea water on the spatial distribution of its temperature, salinity and ionic composition increases the accuracy of mathematical modeling of the processes of hydrodynamics of the reservoir. Modeling of water flow movement in deep-water reservoirs taking into account variable density of sea water allows to improve accuracy of calculations of TRANS-loads for vessels, to calculate moments of formation of natural catastrophes, such as floods, coast abrasion, eutrophication, Zamora, and in real time to carry out calculations for allocation of the zones polluted following catastrophes of technogenic character, for example, emergency oil spills.

In the future, it is possible to develop and implement an algorithm for the PLIS of a modified alternating triangular method for solving the grid equations arising from the discretization of the hydrodynamic model problem [10-17]. In order to improve the efficiency of the computational method for the implementation of the hydrodynamic model, algorithms and library elements focused on multiprocessor computing systems will be developed in the future.

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Математическое моделирование плотности морской воды глубоководного водоема*

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В работе предложена и исследована математическая модель гидродинамики глубоководного водоема – Японское море, учитывающая сложную геометрию береговой линии и дна, трение о дно и ветровые течения, испарение, отклонение значения поля давления от гидростатического приближения, плотность воды как функцию пространственного распределения температуры и солености. Определены и исследованы модели наблюдений для функций, входящих в модельную задачу, включая зависимость распределения плотности воды от пространственного распределения ее температуры, солености и ионного состава. Применение данных моделей опирается на электропроводность и температуру, что делает их максимально точными в различных условиях.

Ключевые слова: математическая модель; поле давления; гидростатическая аппроксимация.

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