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Researching the mechanisms of fluid flow in the fracture-porous reservoir based on mathematical modelling *

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Paper covers the researching the process of fluid flow in the reservoir in the fracture-porous reservoir based on mathematical modelling. The model of the dual porosity of Warren and Root was used. The model has two pore systems - a system of fractures and a system of matrices with different values of geometric dimensions and filtration-capacitive properties. The pressure distribution in the "system of fracture-matrix" system is described by the piezoconductivity equations. The paper presents a numerical solution of the problem under consideration. Partial differential equations were approximated using an implicit difference scheme. The matrix sweep method was used for the calculation. Model pressure recovery curves were obtained. Analysis showed that the specific conductivity coefficient depends on the size of the matrix blocks and possible to evaluate the process of manifestation of the effect of dual porosity.

Keywords: fracture-porous reservoir, «system of fracture-matrix», pressure build-up curves, specific conductivity coefficient

Introduction. The role of carbonate reservoirs in the development of the oil industry in Russia is increased. As a rule, productive reservoir with dual porosity are not sufficiently studied in comparison with ordinary sandstone in terrigenous reservoirs [1,2,3,4]. Carbonate reservoirs is characterized by a number of specific features that are associated with the flow of fluid in environment with a dual porosity [5]. The development of methods for mathematical modeling of fluid flow in a given medium is an urgent problem. Due to their physicochemical properties, susceptibility to cracking, and recrystallization, carbonate reservoirs form a complex microstructure of the void space. The main characteristics of such rocks are fracture and cavernousness [6]. The main cause of the appearance of fractures in the rock is deformation phenomena when the stresses resulting from the action of mechanical loads of various nature, as well as tectonic movements and sedimentation processes, change. Fractures are violations of the continuity of the rock. Geometrically, they are characterized by a significant difference in dimensions in the fracture plane (width and length of fractures) and in the perpendicular direction (fracture opening or height). Fractures observed in carbonate rocks can be completely or partially filled with various mineral substances, for example carbonate or sulfates. Along with them, fractures that remain hollow or

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open can be distinguished. Also, fractures can be filled with oil or bitumen. Disclosure of mineral fractures varies in very wide limits: from fractions of a millimeter to 1 cm or more. As a rule, openness of open fractures does not exceed 20-25 microns [1].

The appearance of a system of interconnected fractures in the rock can change the filtration properties of productive deposits [7, 8].

The technology for reservoir development with dual porosity can be implemented only on the basis of a comprehensive study of the mechanisms of filtration in heterogeneous fractured-porous reservoirs. Hydrodynamic methods for studying of fractured reservoirs due to strong heterogeneity differ significantly from traditional methods [9, 10]. Such reservoirs are characterized by an intensive exchange fluid flow between the fractures and porous blocks (matrix), which introduces certain corrections to known methods for determining reservoir parameters.

Fractured layer is characterized by the discreteness of the properties and parameters of the channels due to the presence of two types of voidness. The matrix has smaller pores and is more fine pores (or voids) and has a significant capacity, but low filtration properties. Fractured - low capacity and high filtration properties. Different authors studied the calculation of the flow characteristics under special conditions: Odeh, Kazemi, DeSwaan and Pollard, etc [13, 14, 15, 16]. The authors proposed various development methods based on simplified reservoir models. But, despite the great variety of approaches of different authors they all boil down to either particular cases or exceptions to the Warren-Ruth model. The Warren-Ruta model represents the general case and is the best method for describing the process of fluid filtration in a fractured formation under unsteady filtration conditions [17].

An analytical solution for the Warren and Root model can not be obtained in general form. There is an analytic expression that represents an approximate solution for some particular cases. In this paper, we consider the numerical solution of the filtration process in a fractured-pore-type collector based on the Warren and Root dual porosity model.

Problem statement.

The model considers a porous collector, schematized by the same rectangular parallelepipeds as shown in Figure 1. The collector or matrix has a high porosity and low permeability. The low-permeability matrix is divided by a system of natural fractures that have high permeability and low porosity. It is believed that the movement of fluid to the well is carried out through a system of fractures, and the matrix is a capacitance that continuously feeds the entire system of natural fractures. The redistribution of the fluid between the matrix and the fractures depends on the shape and size of the matrix blocks, the smaller the blocks, the easier the fluid flow between them [16, 17, 18]. The matrix and fracture have individual properties and are characterized by their own permeability, compressibility and porosity in the dual porosity model.

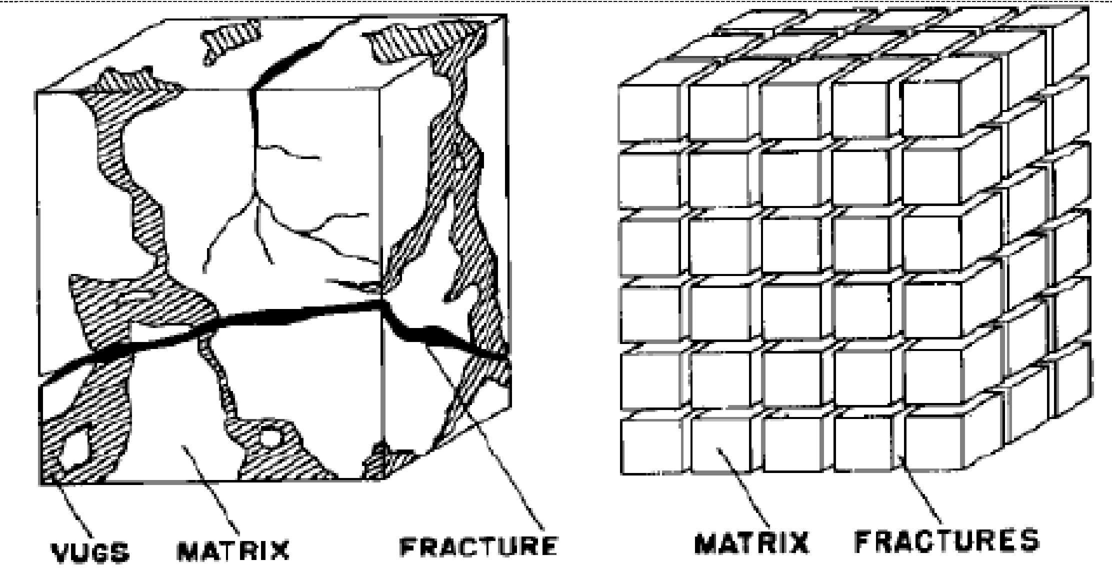

Fig. 1. The dual porosity model

To describe the filtering mechanism in the «system of fractures-matrix» system, the following equations of mathematical physics are used:

$$\varphi_m c_{tm}\frac{\partial P_m}{\partial t} = S\frac{k_m}{\mu}\left(P_f - P_m\right).$$

$$\varphi_f c_{tf}\frac{\partial P_f}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{k_f}{\mu} r\frac{\partial P_f}{\partial r}\right) + S\frac{k_m}{\mu}\left(P_m - P_f\right), \quad (1)$$

The following initial and boundary conditions are considered:

$$P_f\big|_{r=0} = P_0 - \Delta P, \qquad P_f\big|_{r=re} = P_0,$$

$$P_f\big|_{t=0} = P_0, \qquad P_m\big|_{t=0} = P_0, \quad (2)$$

$$2\pi h\frac{k_f}{\mu}\left(r\frac{\partial P_f}{\partial r}\right)_{r=r_w} = Q(t);$$

$$S = \frac{4\cdot n\,(n+2)}{L^2}, \quad (3)$$

$$L = \frac{3\cdot a\cdot b\cdot c}{a\cdot b + b\cdot c + c\cdot a}, \quad (4)$$

where, φ_f – is the porosity of the natural fractures system, φ_m – is the porosity of the matrix, c_{tf} – is the compressibility of the fractures system (1/Pa), c_{tm} – is the compressibility of the matrix (1/Pa), k_f – is the permeability of the fractures system (m^2), k_m – is the permeability of the matrix (m^2), μ – is the oil viscosity (Pa·s), P_f – is the formation pressure in the fractures system (MPa), P_m – is the reservoir pressure in the matrix (MPa), h - is the effective thickness of the formation (m), q - flow rate of liquid (m^3/ day), π ≈ 3.14159, S – is the coefficient of fractured rock (1/ m^2), n – is the number of mutually perpendicular fracture groups, L – is the block size (m), a – is the block side length (m), b – block side width m atrium (m), c – height of the matrix block side (m).

The dual porosity model is characterized by two additional parameters: storativity ratio (ω) and transmissivity ratio (λ). ω - is the fraction of fractures in the total formation system, the higher the coefficient, the greater the fracture-cavernous capacity in the reservoir.

$$\omega = \frac{\varphi_f c_f}{\varphi_f c_f + \varphi_m c_m}, \tag{5}$$

λ - is the ability to filter from the matrix into fractures. This coefficient depends on the size and geometry of the matrix blocks. As the coefficient increases, the ability of the matrix to participate in filtering the system increases. Matrices of low permeability are characterized by lower values of the coefficient.

$$\lambda = S\frac{k_m}{k_f} r_w^2, \tag{6}$$

where, S – is the characteristic coefficient of the fractured rock (1/m^2), r_w – is the radius of the well (m).

Based on various calculations, the following order of magnitude of these parameters was established:

$$10^{-3} < \lambda < 10^{-9} \tag{7}$$

- corresponds to small values of S - blocks of large sizes, small values of k_m - impermeable matrix, and high values of k_f - significant crack opening.

$$10^{-2} < \lambda < 10^{-4} \tag{8}$$

- corresponds to $\varphi_f c_f \gg \varphi_m c_m$, and often $\varphi_f \gg \varphi_m$

The limits of applicability of the parameters $\omega \to 0$, $\lambda \to 0$ and $\omega \to 1$, $\lambda \to \infty$ are due to the basic physical parameters, such as the voidness (porosity), permeability, crack density and block size. In some limiting cases, a system with a double porosity can be reduced to a system with one type of voidness.

Thus, the amount of calculations for the explicit scheme is significantly increased (approximately by 3 orders of magnitude) [19]. Using an implicit difference scheme [20] allows you to select an arbitrary grid, including an uneven grid.

$$\varphi_m c_{tm} \frac{P_{mi}^{j+1} - P_{mi}^{j}}{\tau} = S\frac{k_m}{\mu}\left(P_{mi}^{j+1} - P_{fi}^{j+1}\right) \tag{10}$$

$$\varphi_f c_{tf} \frac{P_{fi}^{j+1} - P_{fi}^{j}}{\tau} = \frac{1}{r}\frac{k_f}{\mu}\frac{1}{h^2}\left(r_{i+\frac{1}{2}} P_{fi+1}^{j+1} - \left(r_{i+\frac{1}{2}} - r_{i-\frac{1}{2}}\right) P_{fi}^{j+1} + r_{i-\frac{1}{2}} P_{fi-1}^{j+1}\right) + \\ + S\frac{k_m}{\mu}\left(P_{mi}^{j+1} - P_{fi}^{j+1}\right) \tag{11}$$

To calculate the implicit scheme, the matrix sweep method was used [19, 20]. Matrix sweep refers to direct methods for solving difference equations. In comparison with other direct methods for solving difference problems, matrix sweeping is more universal, since it allows solving equations with variable coefficients and does not impose strong restrictions on the form of the boundary conditions.

The algorithm of matrix sweep. The system (10-11) can be reduced to a general form:

$$AP_{i-1} - CP_i + BP_{i+1} = -F_i \tag{12}$$

The system of linear algebraic equations with a block tridiagonal matrix needs to be solved. The solution of the system is found recursively by the formulas [21]:

$$\alpha_1 = C_0^{-1}B_0; \quad \beta_1 = C_0^{-1}F_0; \tag{13}$$

$$\alpha_{i+1} = (C_i - A_i\alpha_i)^{-1}B_i, \quad i = 1, 2, \dots N-1$$
$$\beta_{i+1} = (C_i - A_i\alpha_i)^{-1}(A_i\beta_i + F_i), \quad i = 1, 2, \dots N \tag{14}$$

$$P_i = \alpha_{i+1}P_{i+1} + \beta_{i+1}, \quad i = N-1, N-2, \dots 1, 0$$
$$P_N = \beta_{N+1}, \tag{15}$$

where α and β are coefficients. The elements of the tridiagonal matrix are matrices (the dimension of the matrix in question is 2 * 2):

$$A_i = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{r}\frac{k_f}{\mu\varphi_f c_{tf}}\frac{1}{h^2}r_{i-\frac{1}{2}} \end{bmatrix}, \tag{16}$$

$$C_i = \begin{bmatrix} 1 + \frac{Sk_m\tau}{\mu\varphi_m c_{tm}} & -\frac{Sk_m\tau}{\mu\varphi_m c_{tm}} \\ -\frac{Sk_m\tau}{\mu\varphi_f c_{tf}} & 1 + \frac{1}{r}\frac{k_f}{\mu\varphi_f c_{tf}}\frac{1}{h^2}(r_{i+\frac{1}{2}} - r_{i+\frac{1}{2}}) + \frac{Sk_m\tau}{\mu\varphi_f c_{tf}} \end{bmatrix}, \tag{17}$$

$$B_i = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{r}\frac{k_f}{\mu\varphi_f c_{tf}}\frac{1}{h^2}r_{i+\frac{1}{2}} \end{bmatrix}, \tag{18}$$

$$F_i = \begin{bmatrix} P_m \\ P_f \end{bmatrix}. \tag{19}$$

Results of numerical simulation. At the initial time, the production well was put into operation with a flow rate of 100 m^3/day. After working out some period, at the time of 10 days the well is closed on the face. During stopping (q = 0 m^3/day), the pressure in the formation begins to recover. The following initial and boundary parameters are set for the well and the region under consideration:

Table 1. The following initial and boundary parameters

Parameters	Value	Unit of measurement
oil viscosity, μ	2.2E-3	Pa·s
initial fracture pressure, $P_f 0$	250.0E5	MPa
initial pressure in the matrix, $P_m 0$	250.0E5	MPa
permeability of fractures, k_f	1000.0E-15	m^2

permeability of the matrix, k_m	1.0E-15	m^2
compressibility of fractures, c_{tf}	3.0E-9	1/Pa
compressibility of the matrix, c_{tm}	3.0E-10	1/Pa
porosity of the natural fractures system, φ_f	0.01	
porosity of the matrix, φ_m	0.10	
number of mutually perpendicular fracture groups, n	3	
block length, a	100	m
width of the block, b	100	m
block height, c	1	m
reservoir thickness, h	10	m
radius of the well, r_w	0.10	m
well supply circuit, R_e	100.0	m

Analysis of the modeling of hydrodynamic studies by the method of the pressure recovery curve in the production well was carried out [22]. The figures 2 below show the results of numerical simulation.

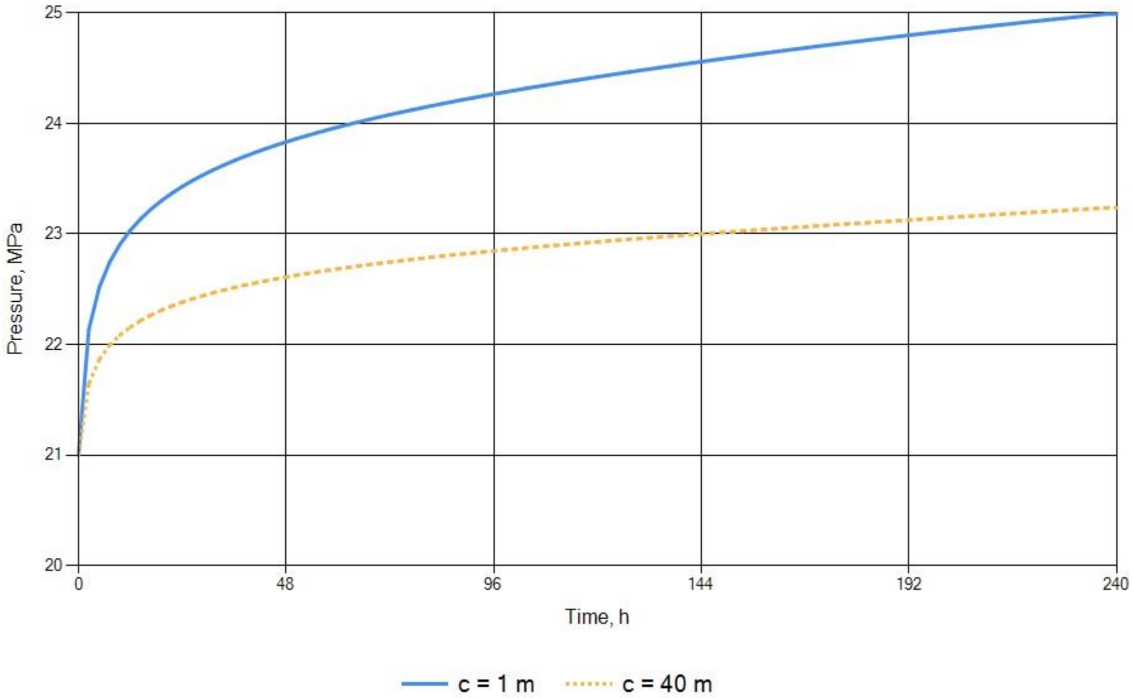

Fig. 2. Results of numerical simulation. Pressure dynamics

The figure 2 shows the calculations for two cases: for a block height of 1 m and for a block height of 40 m. Note that with a block height of 40 m, the pressure is restored more slowly. For the obtained pressure curves, derivatives were constructed.

The effect of dual porosity is manifested in the early times in a short period of time (a derivative jump downwards) due to the small size of the matrix blocks.

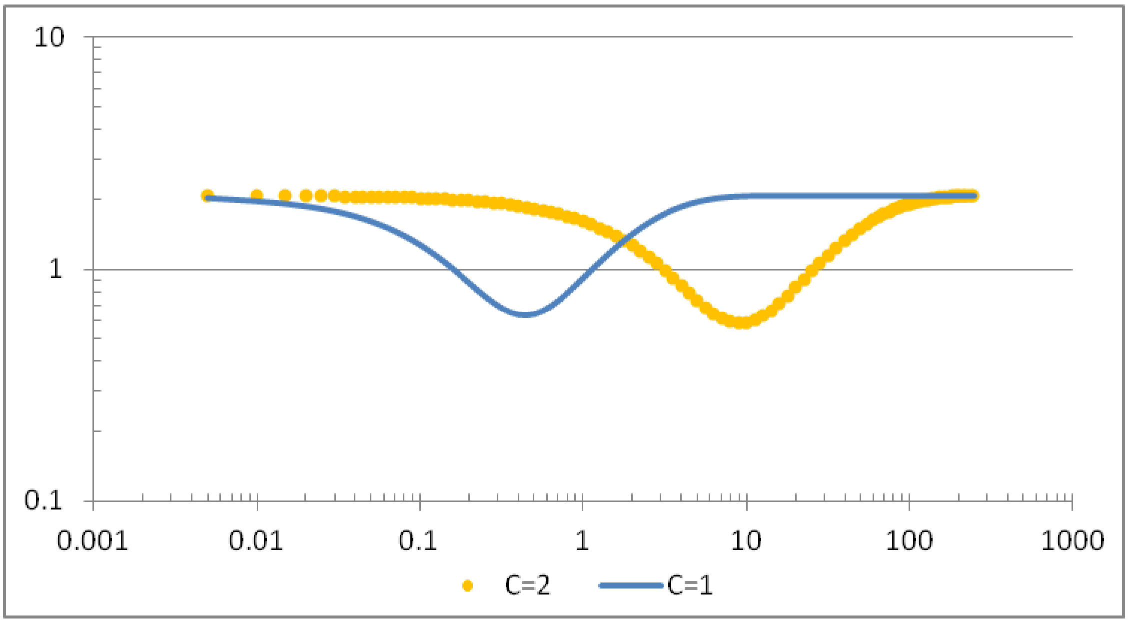

Fig. 3. Log-log plot, block height 1 m and 40 m

For larger block sizes, the effect of dual porosity appears in later times.

Conclusion. The analysis showed that the transmissivity ratio (λ) depends on the size of the matrix blocks, namely: as the size of the matrix block increases, the transmissivity ratio decreases, and the ability of the matrix to participate in filtering the system decreases accordingly.

It was also noted that of the three sides of the block, the height of the matrix block makes a greater contribution. The effect of dual porosity manifests itself in the earlier time region as it increases, and vice versa, the effect of dual porosity is manifested late when height decreases (redistribution of pressure between the crack and the matrix).

When the parameter ω is varied, i.e. change in porosity and compressibility of fractures and matrix, conclusions were drawn too. As this coefficient increases, the volume of fractured-cavernous capacity increases in the reservoir, respectively, the effect of dual porosity occurs later.

As a result of the conducted research on the basis of mathematical modeling, we obtained that the geometry of the conductive cracks distribution, the permeability must be take into account to predict the productivity of wells, the success of various geological and technological measures.

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Исследование механизмов фильтрации в коллекторе трещиновато-порового типа на основе математического моделирования*

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Рассматривается процесс фильтрации жидкости в коллекторах трещиновато-порового типа. Фильтрация описывается с помощью модели двойной пористости, в которой присутствуют сеть естественных трещин и поровый пласт (матрица) с различными фильтрационно-емкостными свойствами. В результате решения задачи получены модельные кривые восстановления давления. Анализ результатов моделирования исследования методом кривой восстановления давления в добывающей скважине показал, что удельный коэффициент проводимости зависит от размеров матричных блоков. Проведен анализ для различных параметров системы, который позволил оценить процесс проявления эффекта двойной пористости.

Ключевые слова: коллекторы трещиновато-порового типа, модель двойной пористости, модельные кривые восстановления давления, удельный коэффициент проводимости

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