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Review article

UDC 51-77
https://doi.org/10.23947/2587-8999-2023-7-2-40-51

Optimal Control in Neurological Models of Information Warfare

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Abstract
Two neurological models of information warfare are considered. For each of them, the optimal control problem is considered, assuming that the Campaign Planner is associated with the governing body of one of the belligerent parties and distributes the volume of propaganda broadcasting in time.
The cost functional reflects the Planner's desire to maximize the number of their supporters at a given time while minimizing costs during the campaign.
The problem is studied analytically, using the Pontryagin's maximum principle.
Optimal control is obtained for various combinations of parameters.
The "increasing" type of campaign is aimed at ensuring that for most individuals information is received immediately before the finish line, and that the impression of this information does not have time to weaken. In contrast, the strategy of a "decreasing" campaign implies a high role of interpersonal communication: it is based on convincing a significant number of individuals of their position at the very beginning, who will then retell it to their interlocutors.

Keywords: mathematical model, information warfare, optimal control, Pontryagin's maximum principle.

For citation. Petrov AP. Optimal control in neurological models of information warfare. *Computational Mathematics and Information Technologies.* 2023;7(2):40–51. https://doi.org/10.23947/2587-8999-2023-7-2-40-51

Обзорная статья

Оптимальное управление в нейрологических моделях информационного противоборства

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Аннотация
Рассматриваются две нейрологические модели информационного противоборства. Для каждой из них предложено решение задачи оптимального управления. При этом предполагается, что Планировщик кампании ассоциируется с управляющим органом одной из противоборствующих партий и распределяет во времени доступный ему объем пропагандистского вещания. Таким образом, интенсивность пропагандистского вещания одной из сторон противоборства имеет смысл управления.
Целевой функционал отражает стремление Планировщика максимизировать численность своих сторонников в заданный момент времени при минимизации затрат в течение кампании.
Исследование задачи управления проводится аналитически, с помощью принципа максимума Понтрягина.
Получено оптимальное управление для различных комбинаций параметров.

Стратегия пропагандистской кампании, в зависимости от параметров системы, может быть как «нарастающей» (т. е. проходящей с неубывающей интенсивностью пропагандистского вещания), так и «убывающей» (проходящей с невозрастающей интенсивностью). При «нарастающей» кампании информация предоставляется только на финише, с тем, чтобы впечатление от этой информации не успело потерять силу. В основе стратегии «убывающей» кампании — межличностное общение. Сначала нужно убедить в своей позиции как можно больше индивидов, которые затем будут пересказывать ее собеседникам. Параметры системы определяют баланс между этими типами стратегий.

Ключевые слова: математическая модель, информационное противоборство, оптимальное управление, принцип максимума Понтрягина.

Для цитирования. Петров А.П. Оптимальное управление в нейрологических моделях информационного противоборства. *Computational Mathematics and Information Technologies.* 2023;7(2):40–51. https://doi.org/10.23947/2587-8999-2023-7-2-40-51

The following problem is considered in the article. Consider the information war between the two sides. By associating ourselves with one of them, we will try to maximize the number of our supporters at a certain point in time, while minimizing the costs of broadcasting in the media. This moment in time can be considered as an election date. The question is to determine the most profitable strategy: for example, whether to start a campaign with a low intensity of propaganda in the media and strengthen it over time. Or, on the contrary, you need to start a campaign with intensive propaganda, and then reduce it. Or some more complex, non-monotonic strategy is optimal. For simplicity, let›s assume that the propaganda of the other side has a constant intensity.

This paper uses a neurological model of information warfare in society to study this problem [1]. Therefore, from an applied point of view, the analysis is aimed not so much at obtaining quantitative results as at identifying qualitative patterns. For these revealed patterns, the final section presents a transparent sociological interpretation that allows for the practical application of the results. The conclusions obtained in this case do not contradict intuition, but, according to the authors, are not obvious and could hardly be obtained on the basis of general considerations, i.e. without the use of mathematical modeling methods.

Meaningfully, this work relates to such a direction as mathematical modeling of information processes in society [2–4]. In this field, information processes are studied by methods of text analysis [5, 6], network analysis and opinion dynamics [7–11], etc. At the same time, the task of management in the formulation of this work has not been previously considered.

The model of information warfare. The neurological model of information warfare in society [1] (and its application [3]) is based on the traditional neurological scheme [12, 13] and has the form:

$$\frac{d\psi}{dt} = c\left(2\int_{-\psi}^{\infty} n(\varphi)\,d\varphi - N_0\right) + u - b - a\psi. \qquad (1)$$

Here the parameters a, b, c, u are positive, and their meaningful meaning is defined in Rashevsky's theory [12, 13]. They depend both on the stimulus that come from the social environment and on the internal parameters of the neurological system. For the purposes of this work, it is important that a and b are related to the intensity of the mass media supporting the Right and Left parties, respectively.

In the most general terms, the sociological meaning of the position selection model and equation (1) can be explained as follows. It is assumed that in a society consisting of individuals, there is an information struggle between two sides (parties): Left and Right. Each individual makes a decision to support a particular party based on his attitude and incentives coming from the social environment.

Next, we will consider in turn what these incentives and attitudes are, and then we will describe the decision-making mechanism.

Stimuli in the authors' model are understood as informational influence on an individual by the media and other individuals. Under this influence, an individual can change his party affiliation over time. At the same time, he himself creates incentives for other individuals by agitating them for his current party. As a result, there is a social dynamics described by equation (1). If the intensity of media propaganda is constant, then the dynamic process ends with the formation of a stationary state. Quite often, several stationary states are possible, in this case, which of them is achieved depends on the initial condition. Next, the optimal control problem is formulated, in which the Planner is identified with one of the sides of the warfare (with the Right Party), and the intensity of its propaganda through the media is taken as a control parameter, and the objective function takes into account this intensity (as an indicator of expenses) and the number of supporters of this party at a given time (the end of the warfare).

Now let's look at the model in more detail.

Each of the individuals N_0 is characterized by an attitude, i. e. their predisposition φ to support a particular party, related to their fundamental belonging to a certain ideology, previous experience, social status, etc. This value is assumed to be constant for a period of time, during which the information struggle lasts.

Thus, it is assumed that there is a constant in time, exogenously defined distribution of individuals along the axis of attitudes $n(\varphi)$, while the negative attitude corresponds to the support of the first batch, the positive one corresponds to the support of the second batch, and the absolute value reflects the strength of support.

Here the function $n(\varphi)$ is equal to zero outside the segment at $\varphi_{\min} \le \varphi \le \varphi_{\max}$ and further we will assume that on this segment the function $n(\varphi)$ is positive, except, perhaps, a finite number of isolated points at which it turns to zero.

Obviously:

$$N_0 = \int_{-\infty}^{\infty} n(\varphi)\, d\varphi.$$

The rule by which an individual relates himself to a particular party can be expressed as follows: if the sum of the stimulus and the attitude is negative, then the individual relates himself to the Left party, if positive — to the Right.

The variant of the model considered in this paper assumes that the influence of mass media is evenly distributed throughout society, in particular, this means that selective use of the media is not taken into account (for example, "conservatives read only conservative newspapers, liberals read only liberal newspapers"). The incentive to support a certain party associated with the campaigning carried out by the supporters of this party is assumed to be proportional to its current number.

The numbers of supporters of the Left (L) and Right (R) parties are equal, respectively:

$$L(t) = \int_{\varphi_{\min}}^{-\psi(t)} n(\varphi)\, d\varphi, \tag{2.1}$$

$$R(t) = \int_{-\psi(t)}^{\varphi_{\max}} n(\varphi)\, d\varphi. \tag{2.2}$$

The initial condition $\psi(0)$, necessary for an unambiguous definition $\psi(t)$, is found from the distribution of individuals between parties at the moment t=0 and has the form:

$$L(0) = \int_{-\psi(0)}^{\varphi_{\max}} n(\varphi)\, d\varphi \tag{3}$$

or, equivalently:

$$R(0) = \int_{\varphi_{\min}}^{-\psi(0)} n(\varphi)\, d\varphi.$$

Here $L(0)$ (and also $R(0)$) is an observable quantity and each of these equalities can be considered as an equation for $\psi(0)$, having a unique solution if the function $N(\varphi)$ is positive for almost all φ.

Obviously, $\psi(0) = \varphi_{\max}$, if $X(0) = 0$ and $\psi(0) = \varphi_{\min}$, if $Y(0) = 0$. The realistic assumption is that $0 < X(0) < N_0$, so that $\varphi_{\min} < \psi(0) < \varphi_{\max}$.

Note that, the function $\psi(t)$ can take values in a wider range, up to $-\infty < \psi < \infty$. For example, inequality $\psi(t) > \varphi_{\max}$ corresponds to a situation where absolutely all members of society support the first party, i. e. its advantage in the intensity of propaganda is so great that it outweighs the installation of even the most radical supporter of the second party. This situation seems unrealistic. Therefore, in this analysis, we will choose the areas of parameter changes so that $\varphi_{\min} < \psi(t) < \varphi_{\max}$ for all $0 \le t \le T$.

So, the function $\psi(t)$ is found as a solution to the initial problem (1), (3).

Rashevsky [12, 13] analyzed a stationary equation corresponding to (1) in the case u=b=0 (in terms of this model, this corresponds to a process taking place in the absence of mass media propaganda) and an even function $n(\varphi)$. He showed that for sufficiently small values of the ratio c/a there is a single stationary solution, it is asymptotically stable and corresponds to an equal distribution of individuals between parties, i. e. $R(t)-L(t)\to 0,$ at $t\to\infty.$ This corresponds to the case when the reaction of (each) individual is significantly more determined by his attitude than the opinion of other individuals. And, conversely, with sufficiently large values c/a an individual's reaction is more determined by the opinion of other individuals than by his own attitude. As a result, the balance between the parties is unstable: if one of them has a numerical superiority at t=0, then it allows you to create an advantage in the strength of the incentive (i. e., in the number of individuals campaigning for it), which increases over time. As a result, stable stationary solutions correspond to situations in which one of the parties acquires a substantial majority. It is precisely such cases that are of the greatest interest for analysis and it is to them that the most attention is paid in this work.

Statement of the management problem. We will consider the process of informational warfare in the time interval (0;T), where an exogenously given moment of time T can be conditionally interpreted as "election day". At the same time, the intensity u of the propaganda broadcast of the Right-wing Party is accepted as management and has a limitation:

$$0\le u\le u_m. \tag{4}$$

The target functionality reflects the Planner's desire to maximize the number of his supporters at time T while minimizing costs during the campaign:

$$J=-\frac{k}{2}\int_0^T u^2dt+R(T)\to\max,$$

so

$$J=-\frac{k}{2}\int_0^T u^2dt+\int_{-\psi(T)}^{\varphi_{\max}} n(\varphi)d\varphi\to\max. \tag{5}$$

In this case, we limit ourselves to the case of a uniform symmetric distribution: $\varphi_{\max}=-\varphi_{\min}=\varphi_m$ and $N(\varphi)=\text{const}>0,$ at $-\varphi_m<\varphi<\varphi_m.$

Methods of solving the management problem

A. Basic equations. The Hamiltonian of the optimal control problem (1)–(5) has the form:

$$H(\psi,p,u)=\left[c\left(2\int_{-\psi(t)}^{\varphi_m} n(\varphi)d\varphi-N_0\right)+u-b-a\psi\right]p-\frac{ku^2}{2}.$$

Therefore,

$$\frac{d\psi}{dt}=c\left(2\int_{-\psi}^{\varphi_m} n(\varphi)d\varphi-N_0\right)+u-b-a\psi, \tag{6}$$

$$\frac{dp}{dt}=\left[-2cn(-\psi)+a\right]p, \tag{7.1}$$

$$p(T)=n(-\psi(T)). \tag{7.2}$$

The immediate goal is to maximize the control Hamiltonian. We have:

$$\frac{\partial H}{\partial u}=p-ku.$$

This derivative turns to zero at:

$$u=\frac{p}{k}.$$

Taking into account the constraint (4), we obtain that the optimal control has the form:

$$u^*=\begin{cases}0, & p<0,\\ p/k, & 0\le p\le ku_m,\\ u_m, & p>ku_m.\end{cases} \tag{8}$$

Taking into account the fact that the authors limited themselves to the case of a uniform symmetric distribution: $N(\varphi)=\text{const}>0,$ at $-\varphi_m<\varphi<\varphi_m$, hence:

$$\int_{-\psi}^{\varphi_m} n(\varphi)d\varphi = \begin{cases} 0, & \psi < -\varphi_m, \\ \frac{N_0}{2}\left(1 + \frac{\psi}{\varphi_m}\right), & -\varphi_m \le \psi \le \varphi_m, \\ N_0, & \psi > \varphi_m. \end{cases}$$

Stationary solutions of the differential equation (6) are found from the equation:

$$f(\psi) = a\psi. \tag{9}$$

where

$$f(\psi) = \begin{cases} -cN_0 + u - b, & \psi < -\varphi_m, \\ cN_0 \frac{\psi}{\varphi_m} + u - b, & -\varphi_m \le \psi \le \varphi_m, \\ cN_0 + u - b, & \psi > \varphi_m. \end{cases}$$

It is not difficult to see that if $a > cN_0 / \varphi_m$, then equation (9) has one solution for any constant u. If $a < cN_0 / \varphi_m$, then it has three roots at $a\varphi_m - cN_0 < u - b < -a\varphi_m + cN_0$ and only one root at or $u - b < a\varphi_m - cN_0$ or $-a\varphi_m + cN_0 < u - b$ (this paper does not consider a special case $a = cN_0 / \varphi_m$).

For the brevity of the record , we introduce the notation:

$$\lambda = a - \frac{cN_0}{\varphi_m}. \tag{10}$$

Tasks (6), (7) take the form:

$$\frac{d\psi}{dt} = \begin{cases} -cN_0 + u - b - a\psi, & \psi < -\varphi_m, \\ -\lambda\psi + u - b, & -\varphi_m \le \psi \le \varphi_m, \\ cN_0 + u - b - a\psi, & \psi > \varphi_m. \end{cases} \tag{11}$$

$$\frac{dp}{dt} = \begin{cases} ap, & \psi < -\varphi_m, \\ \lambda p, & -\varphi_m \le \psi \le \varphi_m, \\ ap, & \psi > \varphi_m. \end{cases} \tag{12}$$

Taking into account the case under consideration $-\varphi_m \le \psi(t) \le \varphi_m$ for all t, we have:

$$\frac{d\psi}{dt} = -\lambda\psi + u - b, \; \psi(0) = \psi^0, \tag{13}$$

$$\frac{dp}{dt} = \lambda p, \;\; p(T) = N_0 / 2\varphi_m. \tag{14}$$

The solution of problem (14) for the conjugate equation has the form:

$$p(t) = \frac{N_0}{2\varphi_m}\exp[\lambda(t - T)]. \tag{15}$$

Thus, the function $p(t)$ either increases strictly (if $\lambda > 0$), or decreases strictly (if $\lambda < 0$), or is constant (if $\lambda = 0$). Let's consider the first two options sequentially.

B. Strictly increasing function $p(t)$

We will assume that:

$$a - cN_0 / \varphi_m > 0. \tag{16}$$

Then there can be three cases (Fig. 1).

Case 1 (Fig. 1 *a*): $ku_m \ge p(T)$, i. e.:

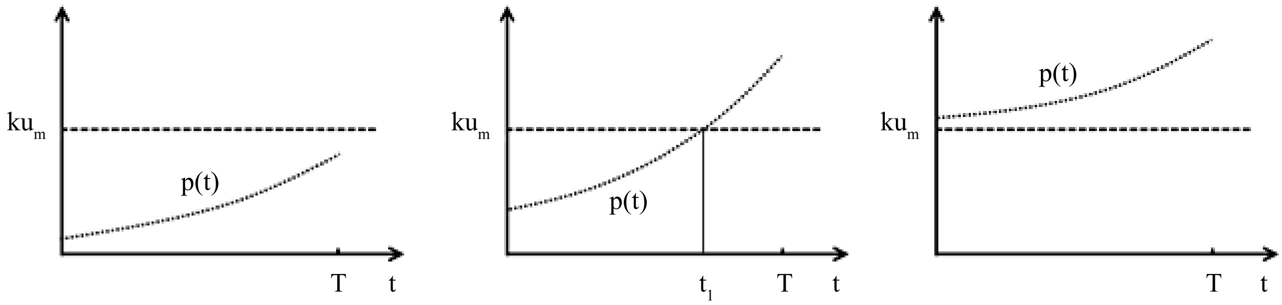
$$N_0 / 2\varphi_m \le ku_m.$$

Case 2 (Fig. 1 *b*): $p(0) < ku_m < p(T)$, i. e.:

$$\frac{N_0}{2\varphi_m}\exp[-\lambda T] < ku_m < \frac{N_0}{2\varphi_m}.$$

Case 3 (Fig.1 *c*): $ku_m \le p(0)$, i. e.:

$$ku_m \le \frac{N_0}{2\varphi_m}\exp[-\lambda T].$$



a) *b)* *c)*

Fig. 1. Three options for the location of the function $y=p(t)$ relative to a straight line $y=ku_m$: cases 1–3

Let's consider each of these cases.

Case 1. $\lambda > 0,\ ku_m \ge p(T)$.

Then $p(t) \le ku_m$ at $t \in [0;T]$. Then it follows from equation (8) that:

$$u^*(t) = \frac{N_0}{2k\varphi_m}\exp[\lambda(t-T)]. \tag{17}$$

Substituting this into (13), we get:

$$\frac{d\psi}{dt} = -\lambda\psi + \frac{N_0}{2k\varphi_m}\exp[\lambda(t-T)] - b, \tag{18}$$

The solution to this Cauchy problem has the form:

$$\psi(t) = \frac{N_0\exp[\lambda(t-T)]}{4\lambda k\varphi_m} - \frac{b}{\lambda} + \left[\psi^0 - \frac{N_0\exp[-\lambda T]}{4\lambda k\varphi_m} + \frac{b}{\lambda}\right]\exp[-\lambda t], \tag{19}$$

at $0 \le t \le T$.

Case 2. $\lambda > 0,\ p(0) < ku_m < p(T)$.

Then there is a point $t_1 \in (0;T)$, such that $p(t_1) = ku_m$ (Fig. 1 *b*). In other words, t_1 is determined by equating the function $p(t)$, given by expression (15) to the value ku_m. We have an equation for this value $N_0/(2\varphi_m)\exp[\lambda(t_1-T)] = ku_m$, i. e.:

$$t_1 = T - \frac{1}{\lambda}\ln\frac{N_0}{2\varphi_m ku_m}. \tag{20}$$

Calculated in this way t_1 is positive due to the condition $p(0) < ku_m < p(T)$, defining Case 2 (non-positivity would mean that $p(0) > ku_m$, i. e., Case 3 takes place). For optimal control, we obtain:

$$u^* = \begin{cases} p(t)/k, & 0 \le t \le t_1, \\ u_m, & t_1 < t \le 1. \end{cases}$$

Substituting $u=u^*$ into equation (13), we obtain for the function $\psi(t)$ the Cauchy problem with the right part given separately on the segments $0 \le t \le t_1$ and $t_1 \le t \le 1$:

$$\frac{d\psi}{dt} = -\lambda\psi + \frac{N_0}{2k\varphi_m}\exp[\lambda(t-T)] - b,\ 0 \le t \le t_1,\ \psi(0) = \psi^0, \tag{21}$$

$$\frac{d\psi}{dt} = -\lambda\psi + u_m - b,\ t_1 \le t \le 1,\ \psi(t_1+0) = \psi(t_1-0). \tag{22}$$

Sequentially solving problems (21), (22), we obtain:

$$\psi(t)=\frac{N_0\exp[\lambda(t-T)]}{4\lambda k\varphi_m}-\frac{b}{\lambda}+\left[\psi^0-\frac{N_0\exp[-\lambda T]}{4\lambda k\varphi_m}+\frac{b}{\lambda}\right]\exp[-\lambda t],\ \ 0\le t\le t_1; \tag{23}$$

$$\psi(t)=\frac{u_m-b}{\lambda}+\left[\psi(t_1)-\frac{u_m-b}{\lambda}\right]\exp[-\lambda(t-t_1)],\ \ t_1\le t\le 1.$$

Case 3. $\lambda>0,\ \ ku_m\le p(0)$.

In this case $p(t)\ge ku_m$ for $t\in[0;T]$, in particular $p(T)=N_0/2\varphi_m>ku_m$. It follows from (8) that the optimal control then has the form:

$$u*(t)=u_m,\ \ t\in[0;T]. \tag{25}$$

Substituting u=u^* into equation (13), we obtain for the function $\psi(t)$ задачу the Cauchy problem:

$$\frac{d\psi}{dt}=-\lambda\psi+u_m-b,\ \ \psi(0)=\psi^0,$$

the solution of which has the form:

$$\psi(t)=\frac{u_m-b}{\alpha}+\left[\psi^0-\frac{u_m-b}{\alpha}\right]\exp[-\alpha t]. \tag{26}$$

C. Strictly decreasing function $p(t)$

We will assume that:

$$\lambda=a-cN_0/\varphi_m<0. \tag{27}$$

For convenience, we will rewrite the formula (15):

$$p(t)=\frac{N_0}{2\varphi_m}\exp[\lambda(t-T)].$$

There may be three cases (Fig. 2).

Case 4 (Fig. 2 *a*): $ku_m\ge p(0)$, i. e.:

$$ku_m\ge\frac{N_0}{2\varphi_m}\exp[-\lambda T].$$

Case 5 (Fig. 2 *b*): $p(T)<ku_m<p(0)$, i. e.:

$$\frac{N_0}{2\varphi_m}<ku_m<\frac{N_0}{2\varphi_m}\exp[-\lambda T].$$

Case 6 (Fig. 2 c): $ku_m\le p(T)$, i. e.:

$$ku_m\le\frac{N_0}{2\varphi_m}.$$

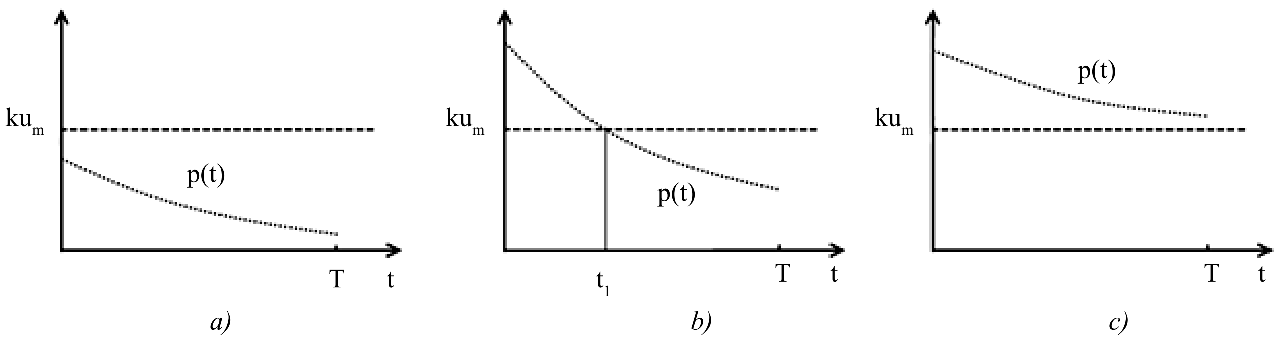


Fig. 2. Three options for the location of the function y=$p(t)$ relative to a straight line y=ku_m: cases 4–6

Let's consider these cases sequentially.

Case 4. $\lambda<0,\ \ p(0)\le ku_m$.

Thus, $\frac{N_0}{2\varphi_m}\exp[-\lambda T]\le ku_m$. In this case $p(t)\le ku_m$ for $t\in[0;T]$. Then it follows from (8) that:

$$u*(t)=\frac{N_0}{2k\varphi_m}\exp[\lambda(t-T)]. \tag{28}$$

Substituting $u=u^*$ into equation (13), we obtain the Cauchy problem for the function $\psi(t)$:

$$\frac{d\psi}{dt} = -\lambda\psi + \frac{N_0}{2k\varphi_m}\exp[\lambda(t-T)] - b, \quad \psi(0) = \psi^0. \tag{29}$$

Its solution at has the form:

$$\psi(t) = \frac{N_0\exp[\lambda(t-T)]}{4\lambda k\varphi_m} - \frac{b}{\lambda} + \left[\psi^0 - \frac{N_0\exp[-\lambda T]}{2\lambda k\varphi_m} + \frac{b}{\lambda}\right]\exp[-\lambda t]. \tag{30}$$

Case 5. $\lambda < 0,\ p(T) < ku_m < p(0)$.

Thus, $N_0/(2\varphi_m) < ku_m < N_0/(2\varphi_m)\exp[-\lambda T]$. Then there is a point $t_1 \in (0;T)$, such that $p(t_1) = ku_m$, t_1 is determined by equating the function $p(t)$, given by formula (15) to the value ku_m. We will get:

$$t_1 = T - \frac{1}{\lambda}\ln\frac{N_0}{2\varphi_m ku_m}. \tag{31}$$

Then the optimal control has the form:

$$u^* = \begin{cases} u_m, & 0 \le t \le t_1, \\ p(t)/k, & t_1 < t \le 1. \end{cases} \tag{32}$$

Substitute this in (13). We will get:

$$\frac{d\psi}{dt} = -\lambda\psi + u_m - b, \quad 0 \le t \le t_1, \quad \psi(0) = \psi^0. \tag{33}$$

$$\frac{d\psi}{dt} = -\lambda\psi + \frac{N_0}{2k\varphi_m}\exp[\lambda(t-T)] - b, \quad t_1 \le t \le 1, \quad \psi(t_1+0) = \psi(t_1-0). \tag{34}$$

Having solved these problems at the appropriate time intervals, we get:

$$\psi(t) = \frac{u_m - b}{\lambda} + \left[\psi^0 - \frac{u_m - b}{\lambda}\right]\exp[-\lambda t], \tag{35}$$

$$\psi = \left\{\frac{\frac{N_0}{2k\varphi_m}\exp[\alpha(t-T)]}{2\alpha} - \frac{b}{a}\right\} + C\exp[-\alpha(t-T)], \tag{36}$$

where

$$C = \frac{u_m - b}{\alpha}\exp[\alpha(t_1 - T)] + \left[\psi^0 - \frac{u_m - b}{\alpha}\right]\exp[-\alpha T] - \\ - \exp[\alpha(t_1 - T)]\left\{\frac{N_0\exp[\alpha(t_1 - T)]}{4\alpha k\varphi_m} - \frac{b}{\alpha}\right\}. \tag{37}$$

Case 6. $\lambda < 0,\ ku_m \le p(T)$.

In this case $p(t)/k \ge u_m$ for $t \in [0;T]$. It follows from (8) that the optimal control has the form:

$$u^*(t) = u_m \text{ for all } t \in [0;T]. \tag{38}$$

Substituting this into (13), we get the Cauchy problem:

$$\frac{d\psi}{dt} = -\lambda\psi + u_m - b, \quad \psi(0) = \psi^0, \tag{39}$$

the solution of which is the function:

$$\psi(t) = \frac{u_m - b}{\alpha} + \left[\psi^0 - \frac{u_m - b}{\alpha}\right]\exp[-\alpha t]. \tag{40}$$

The main conclusions of this section are:

– at sufficiently high values of the relaxation parameter a and/or sufficiently small values of the intensity of information transmission in interpersonal communication (parameter c), the optimal strategy is non-decreasing (Fig. 1);

– in the opposite case, the optimal strategy is non-increasing (Fig. 2).

Extended model: a society consisting of two groups. This section is devoted to the analysis of a model that considers society in more detail compared to the model from the previous section. The main goal is to determine whether the conclusion of the previous section on the influence of parameters a, c on the nature of the optimal campaign is preserved for this more complex model.

The model has the form:

$$\frac{d\psi_1}{dt} = f_1(\psi_1, \psi_2, u) - a\psi_1, \tag{41}$$

$$\frac{d\psi_2}{dt} = f_2(\psi_1, \psi_2, u) - a\psi_2, \tag{42}$$

$$J(n,u) = -\frac{k}{2}\int_0^T u^2 dt + \int_{-\psi_1(T)}^{\varphi_m} n_1(\varphi)d\varphi + \int_{-\psi_2(T)}^{\varphi_m} n_2(\varphi)d\varphi \to \max, \tag{43}$$

where

$$f_1(\psi_1, \psi_2, u) = c\left[\gamma\left(2\int_{-\psi_1}^{\varphi_m} n_1(\varphi)d\varphi - N_1\right) + (1-\gamma)\left(2\int_{-\psi_2}^{\varphi_m} n_2(\varphi)d\varphi - N_2\right)\right] + (u-b),$$

$$f_2(\psi_1, \psi_2, u) = c\left[(1-\gamma)\left(2\int_{-\psi_1}^{\varphi_m} n_1(\varphi)d\varphi - N_1\right) + \gamma\left(2\int_{-\psi_2}^{\varphi_m} n_2(\varphi)d\varphi - N_2\right)\right] + (u-b)$$

and there is still a limitation:

$$0 \le u \le u_m. \tag{44}$$

Here it is assumed that the system consists of two groups of individuals. Each of them is characterized by its own distribution $n_1(\varphi)$, $n_2(\varphi)$. At the same time, each individual communicates more with members of his group than a stranger, which is described by the parameter γ (at the same time $\gamma=1$ corresponds to the fact that there is no intergroup communication, but $\gamma=0.5$ corresponds to homogeneous communication when groups are actually absent. Next, we will assume that $0.5 < \gamma < 1$). Let's denote the number of the first group by N_1, the number of the second group by N_2. Let's assume that the distribution of individuals by installation within each of the groups is similar to the distribution from the model discussed in the previous section. In other words, the distributions are uniform over some exogenously given interval $(-\varphi_m, \varphi_m)$, so that:

$$\int_{-\psi}^{\varphi_m} n_i(\varphi)d\varphi = \begin{cases} 0, & \psi < -\varphi_m, \\ \dfrac{N_i}{2}\left(1 + \dfrac{\psi}{\varphi_m}\right), & -\varphi_m \le \psi \le \varphi_m, \\ N_i, & \psi > \varphi_m, \end{cases}$$

(here and in formulas (45) i=1,2). We have for the numbers of supporters of the Right and Left parties in the first and groups:

$$R_i(t) = \int_{-\psi_i(t)}^{\varphi_m} n_i(\varphi)d\varphi, \tag{45.1}$$

$$L_i(t) = \int_{-\varphi_m}^{-\psi_i(t)} n_i(\varphi)d\varphi. \tag{45.2}$$

In addition, it is assumed here that the groups are equally exposed to the media propaganda of each of the parties.

Taking into account the selected type of functions $f_i(\psi_1, \psi_2, u), n_i(\varphi)$ formulas (41)–(43) take the form:

$$\frac{d\psi_1}{dt} = c\left[\gamma N_1 \frac{\psi_1}{\varphi_m} + (1-\gamma)N_2\frac{\psi_2}{\varphi_m}\right] + (u-b) - a\psi_1, \quad -\varphi_m \le \psi_1 \le \varphi_m, \tag{46}$$

$$\frac{d\psi_2}{dt} = c\left[(1-\gamma)N_1 \frac{\psi_1}{\varphi_m} + \gamma N_2\frac{\psi_2}{\varphi_m}\right] + (u-b) - a\psi_2, \quad -\varphi_m \le \psi_2 \le \varphi_m, \tag{47}$$

$$J(n,u) = -\frac{k}{2}\int_0^T u^2 dt + \frac{N_1}{2}\frac{\varphi_m + \psi_1(T)}{\varphi_m} + \frac{N_2}{2}\frac{\varphi_m + \psi_2(T)}{\varphi_m} \to \max. \tag{48}$$

The Hamiltonian has the form:

$$H(\psi_1, \psi_2, p_1, p_2, u) = \left\{ c\left[\gamma N_1 \frac{\psi_1}{\varphi_m} + (1-\gamma) N_2 \frac{\psi_2}{\varphi_m} \right] + (u-b) - a\psi_1 \right\} p_1 +$$
$$+ \left\{ c\left[(1-\gamma) N_1 \frac{\psi_1}{\varphi_m} + \gamma N_2 \frac{\psi_2}{\varphi_m} \right] + (u-b) - a\psi_2 \right\} p_2 - \frac{ku^2}{2}.$$

The immediate goal is to maximize the control Hamiltonian. Here:

$$\frac{\partial H}{\partial u} = p_1 + p_2 - ku.$$

This function has a maximum when:

$$u = \frac{p_1 + p_2}{k}.$$

Taking into account the constraint (44), we obtain optimal control, which has the form:

$$u^* = \begin{cases} 0, & p_1 + p_2 < 0, \\ (p_1 + p_2)/k, & 0 \le p_1 + p_2 \le ku_m, \\ u_m, & p_1 + p_2 > ku_m. \end{cases} \tag{49}$$

The conjugate system has the form:

$$\frac{dp_1}{dt} = -\left(\frac{\gamma c N_1}{\varphi_m} - a\right) p_1 - \frac{(1-\gamma) c N_1}{\varphi_m} p_2, \quad p_1(T) = \frac{N_1}{2\varphi_m},$$

$$\frac{dp_2}{dt} = -\frac{(1-\gamma) c N_2}{\varphi_m} p_1 - \left(\frac{\gamma c N_2}{\varphi_m} - a\right) p_2, \quad p_2(T) = \frac{N_2}{2\varphi_m}.$$

It is not difficult to see that due to the positivity of the values $p_1(T)$, $p_2(T)$ the functions $p_1(t)$, $p_2(t)$ are positive for $0 \le t \le T$. It is also obvious that each of the derivatives dp_1/dt, dp_2/dt is positive for sufficiently large values of parameter a and/or small values of parameter c and negative in the opposite case. Consequently, the optimal control given by formula (49) is non-decreasing at sufficiently high values of the relaxation parameter a and non-increasing at sufficiently small values. Thus, the model of this section retains the basic property of the simpler model discussed in the previous section.

Let's give a meaningful interpretation of the results obtained. It follows from the formulas obtained for both models considered that the parameters affect the optimal strategy as follows:

– relaxation parameter a: large values contribute to an increasing campaign, small values to a decreasing one;

– the duration of the confrontation T and the intensity b of the broadcast of the opposing party does not affect the choice of strategy (provided that this intensity is constant);

– the intensity of information transmission through interpersonal communication (parameter c), the size of the society, as well as the consolidation of the group parameter φ_{max}: large values contribute to a decreasing campaign, small values contribute to an increasing one.

Information is distributed through the media and through interpersonal communication (rumors): from individual to individual. At the same time, the influence of information on a particular individual after he receives it gradually decreases, so he can be "turned over" by more recent enemy information.

Accordingly, the "growing" campaign is focused on ensuring that for most individuals, information is received immediately before the finish line, and the impression of it does not have time to weaken. The flip side of such a strategy is that individuals will not have time to widely disseminate the information received "by word of mouth", because each act of interpersonal communication requires a certain amount of time. In contrast, the strategy of a "decreasing" campaign implies an effective role of interpersonal interaction and is based on convincing a significant number of individuals of their position at the very beginning, who will then retell it to others. This strategy also has a downside: over time, the interest of individuals in this position will fade, so that by the end of the campaign, the enemy can "turn over" them due to more intensive broadcasting. Thus, the conclusions obtained in the work are not obvious, but they do not contradict intuitio.

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Received 24.04.2023.

Revised 30.05.2023.

Accepted 31.05.2023.

Conflict of interest statement

The author does not have any conflict of interest.

The author has read and approved the final manuscript.

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Поступила в редакцию 24.04.2023.
Поступила после рецензирования 30.05.2023.
Принята к публикации 31.05.2023.

Конфликт интересов
Автор заявляет об отсутствии конфликта интересов.

Автор прочитал и одобрил окончательный вариант рукописи.