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Numerical Modelling of the *FSO Safer* Oil Spill Scenario in the Southern Red Sea

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Abstract

Introduction. This study is devoted to mathematical modeling of accidental oil spill scenarios in the Red Sea in the vicinity of the floating storage and offloading unit *FSO Safer* off the coast of Yemen. Predictive trajectories of oil-slick drift were constructed under the combined effects of advection, diffusion, evaporation, dissolution, and biodegradation. The modelling results made it possible to identify spatiotemporal patterns of slick transformation, determine the areas of greatest environmental risk along the Yemeni coast, and assess potential damage to marine ecosystems and the region's socioeconomic infrastructure.

Materials and Methods. A three-dimensional numerical oil-spill model was developed to predict oil transport and weathering during the first 72 h, which are most critical for mounting an emergency response. The numerical model uses surface currents and temperature from the *HYCOM GOFS 3.1* global analysis and winds from the *ERA5* reanalysis. Oil dynamics were simulated accounting for the multifraction composition of the oil, the mesoscale character of the currents, physicochemical weathering processes (evaporation and dissolution), and biodegradation kinetics coupled with microbial population dynamics.

Results. The results show a slow northeastward displacement of the slick at a velocity of approximately 0.03 m/s (6.5 km over 72 h), with landfall near Ras Isa during the first day. Mass loss over 72 h is 23.4%, of which 96.7% is due to evaporation and 3.3% is due to dissolution, whereas the contribution of biodegradation at realistic microbial biomass levels does not exceed 0.1% over this time horizon. In addition, a regime of persistent northward transport was considered, selected from a three-year series of current and wind data for 2023–2025. From January 29 to February 1, 2023, the slick travels approximately 100 km northward along the Yemeni coast to the Kamaran Islands in three days, while mass loss increases to 32%.

Discussion. Accurate drift forecasting requires up-to-date hydrometeorological data for the scenario date because the trajectory and displacement distance are determined by the current and wind fields. Satellite radar monitoring data are used as initial conditions.

Conclusion. A reproducible predictive oil-spill model based on open data was developed for operational monitoring and environmental risk assessment in the southern Red Sea. The results provide a scientific basis for the urgent implementation of protective measures and for planning regional emergency-response scenarios.

Keywords: oil spill modelling, environmental risk assessment, oil transport and weathering, evaporation, *HYCOM*, *ERA5*, remote sensing

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Оригинальное эмпирическое исследование

Численное моделирование сценария разлива плавучего нефтехранилища *Safer* в южной части Красного моря

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Аннотация

Введение. Работа посвящена математическому моделированию сценариев аварийного разлива нефти в акватории Красного моря в районе расположения плавучего нефтехранилища (*FSO Safer*) у побережья Йемена. Построены прогностические траектории дрейфа нефтяного пятна, формирующиеся под влиянием адвекции, диффузии, испарения, растворения и биоразложения. Результаты моделирования позволили выявить пространственно-временные закономерности трансформации пятна, определить зоны наибольшего экологического риска вдоль йеменского побережья и оценить потенциальный ущерб для морских экосистем и социально-экономической инфраструктуры региона.

Материалы и методы. Разработана трёхмерная численная модель нефтяного разлива для прогноза переноса и выветривания нефти в первые 72 ч, наиболее важные для организации реагирования. Численная модель использует поверхностные течения и температуру глобального анализа *HYCOM GOF3.1*, ветер реанализа *ERA5*. При моделировании динамики нефти учитывался её многофракционный состав, мезомасштабный характер течений, физико-химические процессы выветривания (испарение и растворение), а также кинетика биоразложения с динамикой популяции микроорганизмов.

Результаты исследования. Результаты показывают медленное смещение пятна к северо-востоку со скоростью около 0,03 м/с (6,5 км за 72 ч) с выходом на побережье у Рас-Исы в первые сутки. Потеря массы за 72 ч составляет 23,4 %, из которых 96,7 % приходится на испарение и 3,3 % на растворение, тогда как вклад биоразложения при реалистичной биомассе микроорганизмов на этом горизонте не превышает 0,1 %. Дополнительно рассмотрен режим устойчивого северного переноса, выбранный по трёхлетнему ряду данных о течениях и ветрах периода 2023–2025 гг. С 29 января по 1 февраля 2023 г. пятно за трое суток проходит около 100 км на север вдоль побережья Йемена до островов Камаран, при этом потеря массы возрастает до 32 %.

Обсуждение. Для точного прогноза дрейфа требуются актуальные гидрометеорологические данные на дату сценария, так как траектория и дальность смещения задаются полями течений и ветра. В качестве исходных условий используются данные спутникового радиолокационного мониторинга.

Заключение. На основе открытых данных разработана воспроизводимая прогностическая модель нефтяного разлива для оперативного мониторинга и оценки экологических рисков в южной акватории Красного моря. Полученные результаты служат научным обоснованием для экстренного внедрения защитных мер и планирования региональных сценариев ликвидации последствий ЧС.

Ключевые слова: моделирование нефтяного разлива, оценка экологических рисков, перенос и выветривание нефти, испарение нефти, *HYCOM*, *ERA5*, дистанционное зондирование

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Для цитирования. Сидорякина В.В., Берая В.А., Благин В.А. Численное моделирование сценария разлива плавучего нефтехранилища *Safer* в южной части Красного моря. *Computational Mathematics and Information Technologies*. 2026;10(3):7–21. <https://doi.org/10.23947/2587-8999-2026-10-3-7-21>

Introduction. The vulnerability of marine ecosystems to anthropogenic impacts requires the implementation of advanced methods for forecasting environmental disasters. Among the potential threats, oil spills are among the most

dangerous because they can cause long-term destructive consequences for the environment, the economy, and society [1]. A striking example of such a threat in the Red Sea was the critical condition of the *FSO Safer* floating storage and offloading unit, moored at the Ras Isa terminal (15°07' N, 42°36' E) in the territorial waters of Yemen. Until 2023, the vessel, which held 1.14 million barrels ($1.55 \cdot 10^8$ kg) of crude oil, posed unprecedented risks to the region's unique biodiversity [2]. Although in the summer of 2023 the United Nations successfully transferred this volume to the tanker *Yemen*, which replaced the distressed vessel at the same anchorage, modelling a potential spill of this amount in the Ras Isa area remains scientifically and practically relevant.

The Red Sea is a unique environment for oil-spill modelling due to its narrow topography, seasonally varying wind regime, and complex mesoscale circulation dynamics [3]. These factors significantly complicate prediction of oil transport and transformation and require three-dimensional models that comprehensively account for advection, diffusion, evaporation, dissolution, and biodegradation. Although oil-spill models are widely used worldwide [4, 5], including in the Red Sea (for example, the Lagrangian models of Periañez [6] and Mittal [7]), only a few explicitly account for the specific hydrodynamic regime of this basin or include dynamic kinetics of microbial degradation. The present study is intended to address this gap.

The authors propose a numerical model of an *FSO Safer* spill scenario for a 72-hour response window and assess the associated environmental risks.

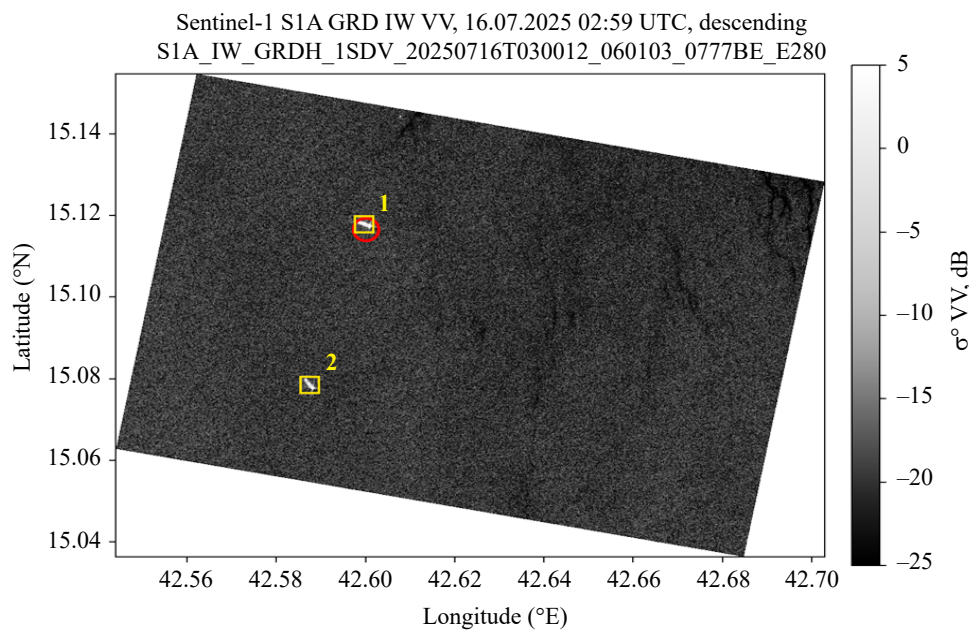


Fig. 1. Sentinel-1A radar image (data obtained from *Copernicus Sentinel-1* [8], *ESA*, *GRD* product; processed by the authors)

Fig. 1 shows the *IW* mode, *VV* polarization, for 16 July 2025, 02:59 UTC, in the area of the *FSO Safer* anchorage off Cape Ras Isa. Brightness represents the normalized radar cross section σ^0 , dB. Label 1 denotes the vessel at the *Safer* anchorage (image-derived length ≈ 380 m), and 2 denotes a second vessel 4.5 km to the southwest (≈ 340 m); the red circle marks the anchorage coordinates from the incident catalogue (15.117° N, 42.600° E). Identification of the vessels as *FSO Safer* (362 m) and the tanker *Yemen* was based on position and size; AIS data were not used.

The key innovations of the study include:

- The locations of the objects under consideration are illustrated by *Sentinel-1* radar images (Fig. 1). In the image acquired on 16 July 2025 near Ras Isa, two large vessels can be distinguished — one at the *Safer* anchorage and another 4.5 km southwest of it — which is consistent with reports that *FSO Safer* and the tanker *Yemen* that received its oil were located in this area. In the image from 5 September 2020, corresponding to the monitoring period before the transfer operation, only one vessel is present at the anchorage. *C-band* radar imaging is independent of illumination and cloud cover and is therefore considered below as the primary source of initial conditions for operational forecasting.
- A conservative finite-volume transport scheme that preserves mass without artificial correction.
- Fraction-specific parameterizations for evaporation (with temperature-dependent vapor pressure), dissolution, and biodegradation; free parameters are calibrated against the scenario mass balance.
- High-fidelity environmental forcing that accounts for diurnal variations in wind, temperature, and currents.

The study not only addresses an urgent practical challenge but also advances oil-spill modelling methodology [8, 9]. The results substantiate the need for proactive measures and provide a scientific basis for regional emergency-response plans, thereby linking oceanographic science with applied environmental management.

Materials and Methods. In this study, a numerical model was developed to simulate an oil-spill scenario from a floating storage and offloading (*FSO*) unit in Yemen in the Red Sea, with parameters calibrated against the scenario mass balance. The methodology integrates advection, diffusion, evaporation, biodegradation, and dissolution. The model is implemented in *Python (NumPy)* with a fixed time step and a conservative flux-form transport formulation that ensures mass conservation and numerical stability; the initial version using *ROMS* fields [10] was used as a control. The simulation setup, governing equations, numerical methods, input data, and validation procedures are described in detail below.

1. Numerical-model setup and parameters. The simulation models a release of $1.55 \cdot 10^8$ kg (1.14 million barrels) of oil at a specified location (14.6°N, 42.7°E) in the Red Sea, with an initial spreading radius of five grid cells. The simulation duration is 72 h with a fixed time step of 15 s, providing a balance between computational efficiency and numerical stability.

Parameters: horizontal diffusion coefficient $D_h = 10$ m²/s, horizontal viscosity $\mu_h = 2.8 \cdot 10^{-7}$ m²/s, microbial decay rate $\lambda = 5.6 \cdot 10^{-6}$ s⁻¹, growth rate $q = 4.2 \cdot 10^{-8}$ s⁻¹ with a 24 h lag phase, evaporation mass-transfer coefficient $K_E = 4.35 \cdot 10^{-4}$ m/s, dissolution rate $k_d = 3.4 \cdot 10^{-8}$ s⁻¹, half-saturation constant $K_s = 1$ kg/m², initial microbial concentration $M_0 = 10^{-3}$ kg/m², and biomass yield $Y = 0.4$. The oil is divided into three fractions with different molecular masses (0.3, 0.5, 0.8 kg/mol), molar fractions (0.2, 0.3, 0.5), solubilities (0.05, 0.02, 0.01 kg/m³), and vapor pressures at 25 °C (5, 3, 1 Pa), recalculated as functions of temperature using the Clausius–Clapeyron equations with heats of vaporization of 40, 50, and 60 kJ/mol. Dynamic weather conditions, including temperature (a baseline of 25 °C with a 5 °C diurnal variation), are specified from *HYCOM* sea-surface temperature with an imposed diurnal cycle of 5 °C amplitude and a maximum 6 h after the start of the calculation; the simulation period is 1–4 May 2020.

2. Input data. The model uses surface currents, temperature, salinity, and sea level from the *HYCOM GOFS* 3.1 global analysis [11] (1/12°, 3 h) and 10 m winds from the *ERA5* reanalysis [12] (0.25°, 1 h) for 1–4 May 2020, bilinearly interpolated onto the regular model grid with spacing of approximately 860 m in longitude and 890 m in latitude (0.008°) over the domain 41.6–43.4° E, 12.8–15.6° N (225×350 cells). The slick drift velocity is the sum of the current velocity and 3% of the wind velocity [13]. Missing temperature values are replaced by 25°C. Cell areas are calculated from the grid metrics (pm, pn). The land mask is constructed from the *GSHHG* coastline; fluxes through faces adjacent to land and through the domain boundaries are set to zero.

3. Governing equations. Oil-spill dynamics are governed by a system of partial differential equations describing the transport and fate of the oil fractions. For each fraction α , the surface oil concentration $c_\alpha(x, y, t)$ kg/m², for $\alpha = 1, 2, 3$, evolves according to [14–32]:

$$\frac{\partial c_\alpha}{\partial t} = -\nabla \cdot (uc_\alpha) + D_h \nabla^2 c_\alpha - E_\alpha - q \frac{Mc_\alpha}{K_s + c_\alpha} - k_d c_\alpha \left(1 - \frac{\phi_\alpha}{S_\alpha}\right), \quad (1)$$

where u is the slick drift velocity, m/s; D_h is the horizontal diffusion coefficient, m²/s; E_α is the evaporation flux, kg/m²·s; q is the microbial growth rate, s⁻¹; M is the microbial concentration, kg/m²; K_s is the half-saturation constant, g/m²; k_d is the dissolution rate, s⁻¹; ϕ_α is the concentration of dissolved oil in the upper layer, kg/m³; and S_α is the solubility, kg/m³.

Let the area covered by the initial unbroken oil slick be denoted by σ (a circle with a radius of five cells centered at the release point); γ is the region describing the surface layers of the water body at the land boundary; and $\vec{n}$ is the outward normal vector to the surface forming the boundary of the computational domain. Equation (1) is supplemented by the following initial and boundary conditions:

$$c_\alpha|_{t=0} = \begin{cases} 0, & (x, y) \notin \sigma, \\ c_{\alpha 0}, & (x, y) \in \sigma, \end{cases} \quad \frac{\partial c_\alpha}{\partial \vec{n}} = 0, \quad (x, y) \in \gamma,$$

where $c_{\alpha 0}$ is the concentration of oil fraction number α in the initial slick, $c_{\alpha 0} = \left(x_\alpha m_\alpha M_0^p / |\sigma| \sum_\beta x_\beta m_\beta \right)$; M_0^p is the spill mass, $M_0^p = 1.55 \cdot 10^8$ kg; x_α, m_α are the molar fractions and molecular masses of the oil fractions; and $|\sigma|$ is the slick area.

On the lateral boundaries of the computational domain that pass through water, the following conditions are imposed:

$$c_\alpha = c'_\alpha, \quad \text{if } u_{\vec{n}} < 0,$$

$$\frac{\partial c_\alpha}{\partial \vec{n}} = 0, \quad \text{if } u_{\vec{n}} \geq 0,$$

where $u_{\vec{n}}$ is the projection of the velocity vector onto the outward normal $\vec{n}$ to the boundary, and c'_α are the prescribed concentration values [33].

The change in dissolved-oil concentration ϕ_α in the water column is described by [14]:

$$\frac{\partial \phi_\alpha}{\partial t} = -\nabla \cdot (w \phi_\alpha) + \mu_v \frac{\partial^2 \phi_\alpha}{\partial z^2} + \frac{k_d c_\alpha}{\Delta z} \left(1 - \frac{\phi_\alpha}{S_\alpha} \right) \delta_{k1}, \quad (2)$$

where w is the vertical velocity, m/s; μ_v is the vertical diffusion coefficient, $\mu_v = 10^{-3}$ m²/s; and Δz is the layer thickness. The calculations use: $w = 0$, $\Delta z = 1$.

Let the z axis be directed downward from the free surface, $0 < z < L_z$, $L_z = 20$ m, and let Ω_0 denote the part of the free surface on which the oil forms a slick. Equation (2) is supplemented by the following initial and boundary conditions:

$$\begin{aligned} \phi_\alpha|_{t=0} &= 0, \\ \frac{\partial \phi_\alpha}{\partial z} \Big|_{z=0} &= -\frac{k_d c_\alpha}{\mu_v} \left(1 - \frac{\phi_\alpha}{S_\alpha} \right), \quad (x, y) \in \Omega_0, \quad \frac{\partial \phi_\alpha}{\partial z} \Big|_{z=0} = 0, \quad (x, y) \notin \Omega_0, \\ \frac{\partial \phi_\alpha}{\partial z} \Big|_{z=L_z} &= 0. \end{aligned}$$

The microbial concentration $M(x, y, z)$ evolves according to [14]:

$$\frac{\partial M}{\partial t} = \mu_h \nabla^2 M + Yq \sum_\alpha \frac{c_\alpha M}{K_s + c_\alpha} - \lambda M, \quad (3)$$

where μ_h is the horizontal viscosity, m²/s; λ is the microbial decay rate, s⁻¹; and Y is the biomass yield.

Equation (3) is supplemented by initial and boundary conditions on γ and on the lateral boundaries of the computational domain [14]:

$$M|_{t=0} = M_0, \quad \frac{\partial M}{\partial \vec{n}} = 0,$$

where M_0 is the initial concentration of oil-degrading microorganisms.

The evaporation flux E_α follows the temperature-dependent Mackay–Matsugu formulation [15]:

$$E_\alpha = K_E \frac{P_\alpha(T) X_\alpha m_\alpha}{R(T + 273.15)}, \quad P_\alpha(T) = P_\alpha^{25} \exp \left[-\frac{\Delta H_\alpha}{R} \left(\frac{1}{T + 273.15} - \frac{1}{298.15} \right) \right], \quad (4)$$

where K_E is the mass-transfer coefficient, m/s; P_α^{25} is the vapor pressure at 25 °C, Pa; ΔH_α is the heat of vaporization, J/mol; X_α is the current molar fraction of the fraction in the slick; m_α is the molecular mass, kg/mol; R is the gas constant, $R = 8.314$ J/mol·K; and T is temperature, °C. The molar fractions are recalculated from the slick composition in each cell.

4. Numerical methods. The model uses a finite-difference scheme in which the convective terms are approximated by a first-order upwind scheme with respect to the spatial-grid steps to ensure numerical stability, while boundary fluxes are limited to zero to prevent artificial inflow. Diffusion is approximated using symmetric central-difference schemes, and source/sink terms (evaporation, biodegradation, dissolution) are integrated explicitly with the Euler method. The advective and diffusive terms are approximated using a balance method with flux conservation across cell faces. Consequently, total mass in the discrete model is conserved to round-off accuracy without any correction. At each step, sinks are limited by the amount of oil remaining in the cell after transport, thereby preventing negative concentrations. The mass balance (floating and dissolved oil, evaporation, biodegradation, and dissolution) is output and analyzed at every time step using a relative residual norm, which over 72 h does not exceed 10^{-12} relative to the initial mass. The microbial growth rate increases during the first 24 h, representing the lag phase.

The modelling domain is discretized horizontally on a regular grid of 225×350 cells with spacing of approximately 860 m in longitude and 890 m in latitude (0.008°); 20 cells are used vertically ($H = 20$ m). Oil-concentration distributions are recorded every 12 h, capturing distributions both at the surface and at depth. Dispersion is quantified by calculating the standard deviation of the oil-mass distribution in the longitude and latitude directions at each time step. The numerical simulation yields time series of total oil mass, mass-loss rates (evaporation, biodegradation, dissolution), cumulative mass losses, and dispersion values. Spatial visualizations are generated using a logarithmic scale for oil mass per cell, together with the coastline and a 10 km scale bar. Numerical results are saved as *PNG* files and as a *NetCDF* file containing the time series and final oil distribution.

The methodology combines realistic forcing from open data with a conservative numerical scheme and a reproducible computational pipeline, allowing the results to be verified and refined. It should be noted that the first-order transport scheme introduces numerical diffusion $D_{num} \approx |u| \Delta x / 2$, which for $|u| = 0.1 \div 0.3$ m/s and $\Delta x \approx 0.9$ km amounts to $50 \div 130$ m²/s and is two orders of magnitude larger than the prescribed coefficient D_h ; the simulated slick spreading is

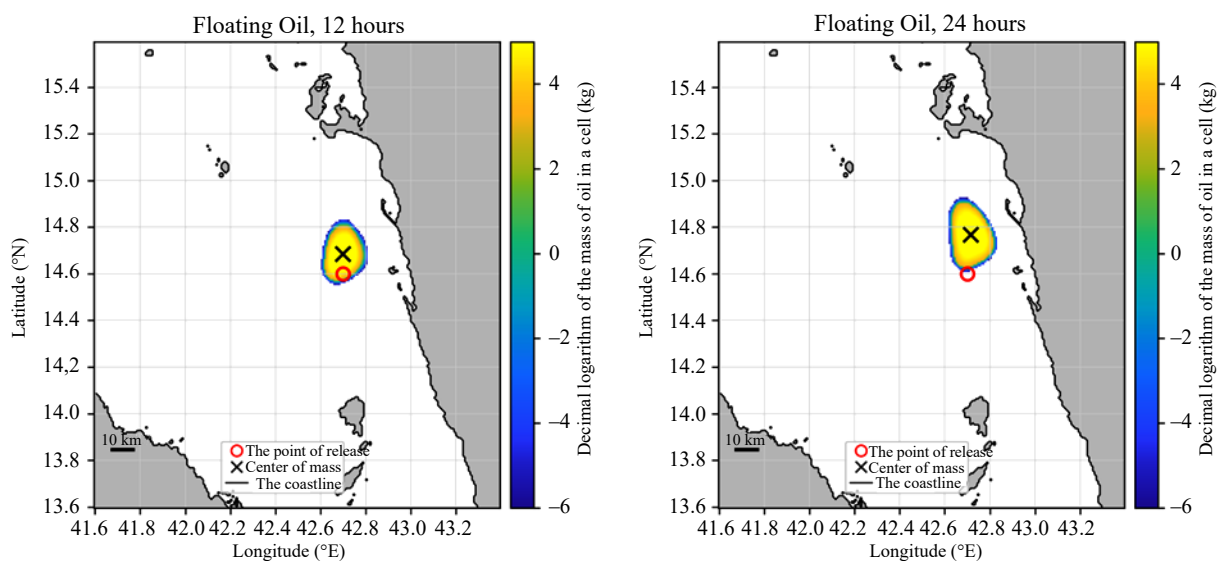
therefore controlled by this numerical diffusion. Transition to a second-order scheme with a limiter is planned for the next version of the model.

5. Software implementation and computational resources. The model is implemented in *Python* 3.10.6 as a pipeline of eleven scenarios (input-data download, forcing-field construction, simulation, calibration, figure generation, version comparison, and preparation of the article text) controlled by a single configuration file; all intermediate results are stored in *NetCDF* and *CSV* formats, ensuring reproducibility of every figure and numerical value. The numerical core is written in *NumPy* with vectorized spatial operations; *SciPy* is used to interpolate forcing fields to the computational grid, *xarray* and *netCDF4* to process *NetCDF* data, *Matplotlib* to generate figures, and *Shapely* and *pyproj* to construct the land mask from the *GSHHG* 2.3.7 coastline [16]. Core correctness is checked by nine unit tests (*pytest*), including mass-conservation verification with relative accuracy 10^{-12} and transport of the slick over a prescribed distance in a uniform velocity field; exact versions of all packages are recorded in the *env_lock.txt* file included in the results of each run. Input data — *HYCOM GOFS* 3.1 current, temperature, salinity, and sea-level fields ($1/12^\circ$ resolution, 3 h interval) and *ERA5* near-surface wind obtained through the *Open-Meteo* archive interface [17]—are downloaded automatically for the dates of the simulation window. Calculations were performed on a workstation with an *AMD Ryzen Threadripper 2990WX* processor (32 cores, 64 threads) and 80 GB of *RAM*; no graphics accelerator was used, and each calculation runs as a single process on one core. The computational grid contains 225×350 cells with a size of 0.008° ; 72 h of model time requires 17,280 steps of 15 s each at a Courant number of 0.016. One 72-hour scenario run takes approximately 10 min because computation is restricted to the active slick area. The approximate net CPU time required to obtain the reported results for each scenario was about 7.7 CPU-hours: testing 16 variants of dimensional interpretations and formulations (2.7 h, eight parallel runs), two rounds of calibration of parameters K_E , k_d , and M_0 against the control characteristics of the original implementation (89 runs, 4.8 h), and final calculations for the original and refined parameterizations.

Results.

1. Spatial evolution of the oil slick. The spatial distribution of floating oil shown in Fig. 2 at 12, 24, 36, 48, 60, and 72 h demonstrates dynamic evolution governed by advection, diffusion, evaporation, dissolution, and biodegradation. After 12 h, the oil slick forms an elliptical area centered near the spill location (14.6°N , 42.7°E), with a maximum oil mass per cell of approximately $1.7 \cdot 10^6$ kg and a concentrated core. The center of mass, denoted by the symbol $\langle X \rangle$, shifts northward or northeastward under the effects of the surface current and wind drift (3% of wind speed). After 24 h of drift, the slick expands and its northern edge reaches the coast near Ras Isa.

By 36 and 48 h, the slick continues to disperse, with a maximum oil mass per cell of approximately $1.0 \cdot 10^6$ kg, indicating substantial mass loss due to evaporation and dissolution. The center of mass moves slowly, within a few kilometers, in accordance with changes in the wind and the surface-velocity field derived from *HYCOM*. By 60 and 72 h, the slick intensity decreases, with the maximum mass per cell falling to approximately $7.8 \cdot 10^5$ kg, while the center of mass stabilizes near the coast. Over 72 h, the center of mass travels 13.8 km from the release point (mean velocity 0.03 m/s, azimuth 60°); some oil accumulates along the shoreline because the flux through faces adjacent to land is set to zero.



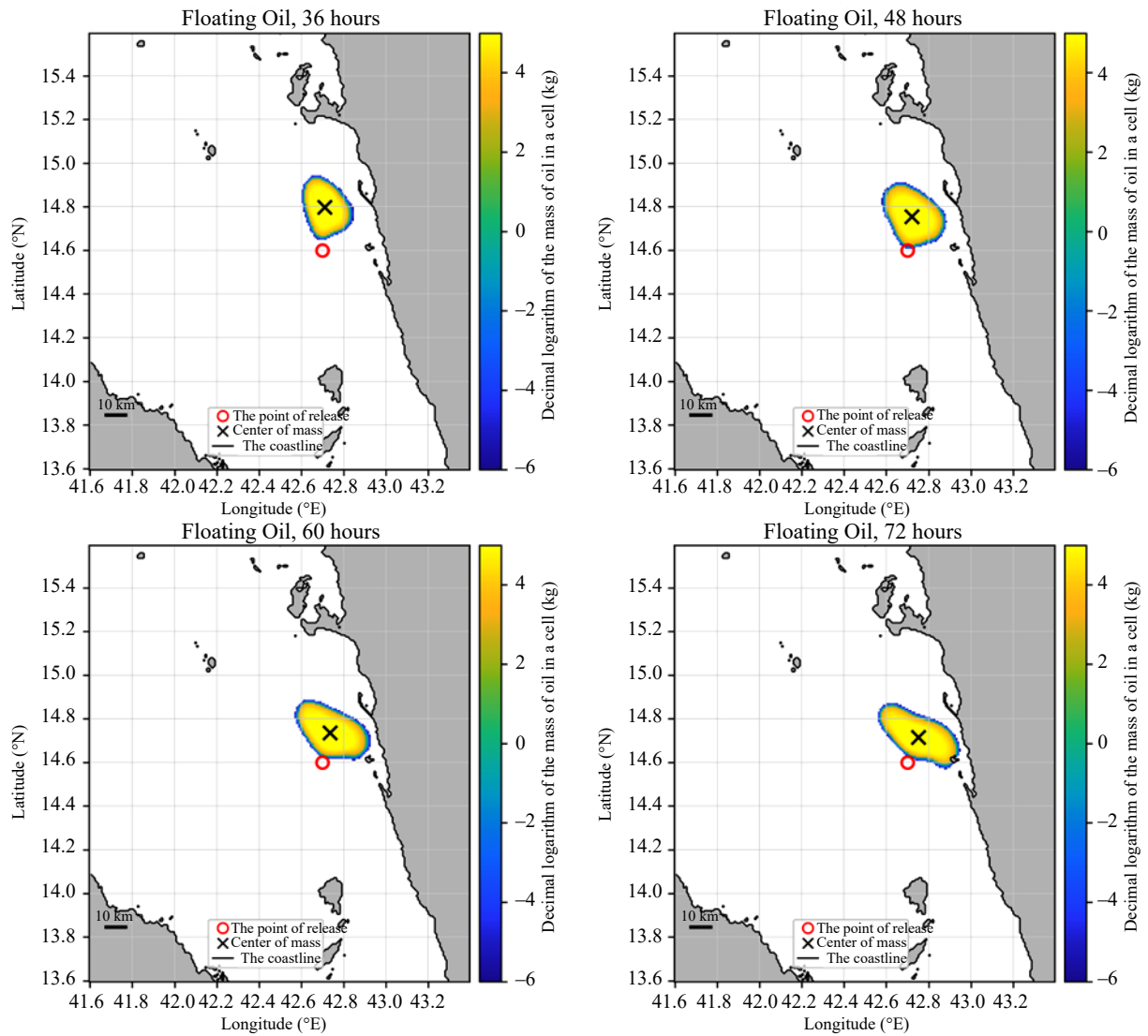


Fig. 2. Snapshots of oil transport at the water surface every 12 hours

2. Time-dependent mass balance and mass-loss processes. The temporal evolution of oil mass is shown in Fig. 3. Total oil mass decreases from the initial 155,040,000 kg to approximately 118,800,000 kg over 72 h, corresponding to a reduction of approximately 23.4%. This decrease is caused by three main loss mechanisms: evaporation, biodegradation, and dissolution.

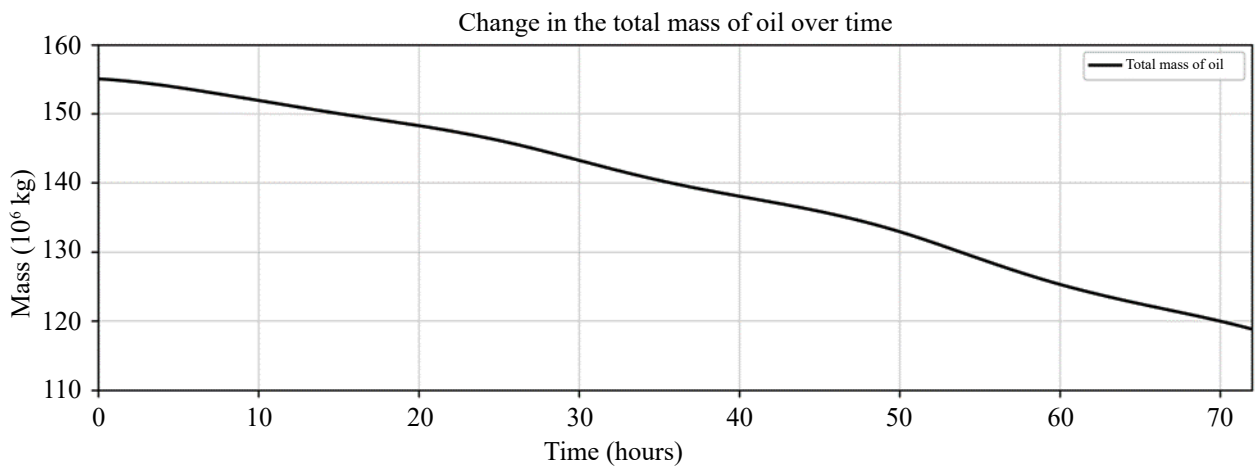


Fig. 3. Total mass of drifting oil from 1 to 3 May 2020

Mass-loss rates (Fig. 4) exhibit oscillatory behavior due to diurnal variations in temperature and wind. Evaporation dominates initially and reaches a peak of approximately 222 kg/s (maximum at 53 h), reflecting the high vapor pressures (5, 3, and 1 Pa, scaled by 1.5) of the oil fractions. With a realistic initial biomass ($M_0 = 10^{-3}$ kg/m²) and a 24 h lag phase, biodegradation is negligible over 72 h (less than $3 \cdot 10^{-3}$ kg/s on average); dissolution averages 4.67 kg/s.

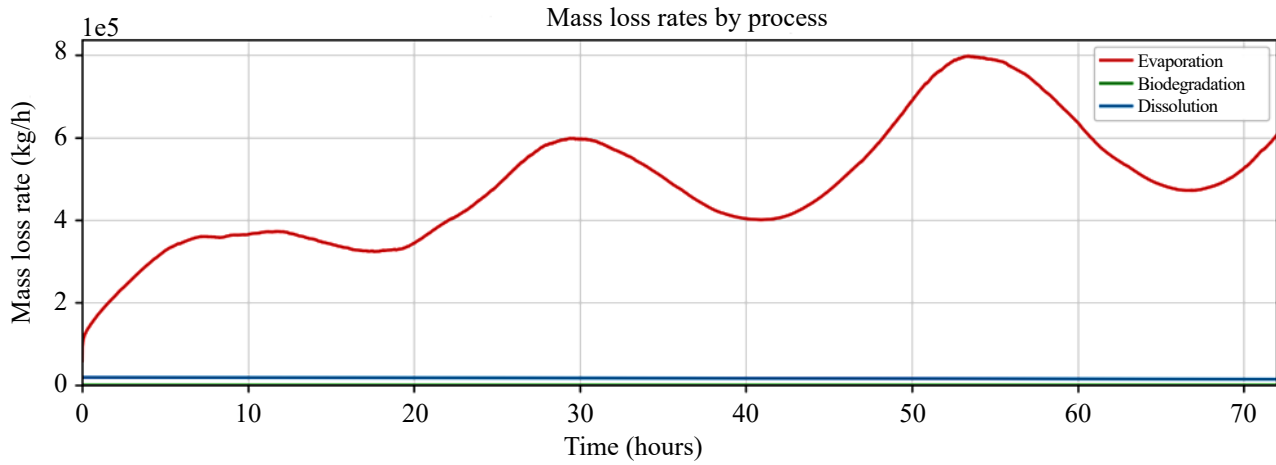


Fig. 4. Mass-loss rate from 1 to 3 May 2020. Evaporation peaks around midday and decreases to a local minimum at night due to diurnal changes in atmospheric temperature

The cumulative mass loss shown as an area plot (Fig. 5) demonstrates that evaporation accounts for the largest fraction (approximately 96.7% of the total loss), followed by dissolution (3.3%), while biodegradation contributes less than 0.1% by 72 h at realistic microbial biomass levels. No artificial mass correction was applied: the conservative flux formulation conserves mass intrinsically, and the mass-balance residual over 72 h does not exceed 10^{-12} of the initial mass.

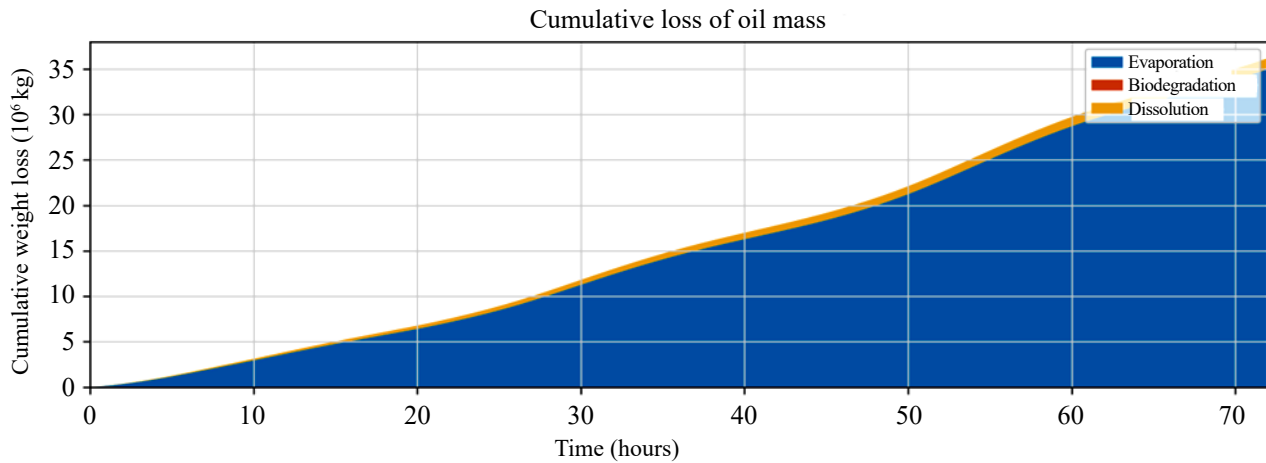


Fig. 5. Evaporation accounts for 96.7% of the losses, dissolution for 3.3%, and biodegradation for less than 0.1%

3. Oil-slick dispersion. Oil-slick dispersion, quantified by the standard deviation in the longitude x and latitude y directions, increases with time (Fig. 6). After 12 h, the longitudinal standard deviation is approximately 2.6 km, increasing to 6.2 km by 72 h (latitudinal value 3.6 → 4.0 km). Longitudinal dispersion (blue line) is slightly greater than latitudinal dispersion (red line), reflecting the anisotropic nature of the current and wind fields. This growth is governed by the numerical diffusion of the first-order upwind scheme ($D_{num} \approx |u| \Delta x / 2$, of order 10^2 m²/s) and by velocity shear; the prescribed coefficient $D_h = 10$ m² m²/s produces a diffusion-induced increase σ of approximately 2.3 km over 72 h, which is smaller than the contribution from numerical diffusion. The continuing increase indicates that the spatial extent of the slick expands even as its mass decreases, owing to the combined effects of environmental forcing and diffusive transport.

The results have important implications for oil-spill response in the Red Sea. Rapid initial dispersion and mass loss indicate that containment efforts should be prioritized within the first 24 h. The dominance of evaporation highlights the need for air-quality monitoring, whereas the biodegradation contribution points to the potential of bioremediation strategies. The drift direction is determined entirely by the current and wind fields on the specific dates: under the conditions of 1–4 May 2020, the nearest coast near Ras Isa is affected, so preparedness should rely on operational forecasts rather than on a typical drift direction.

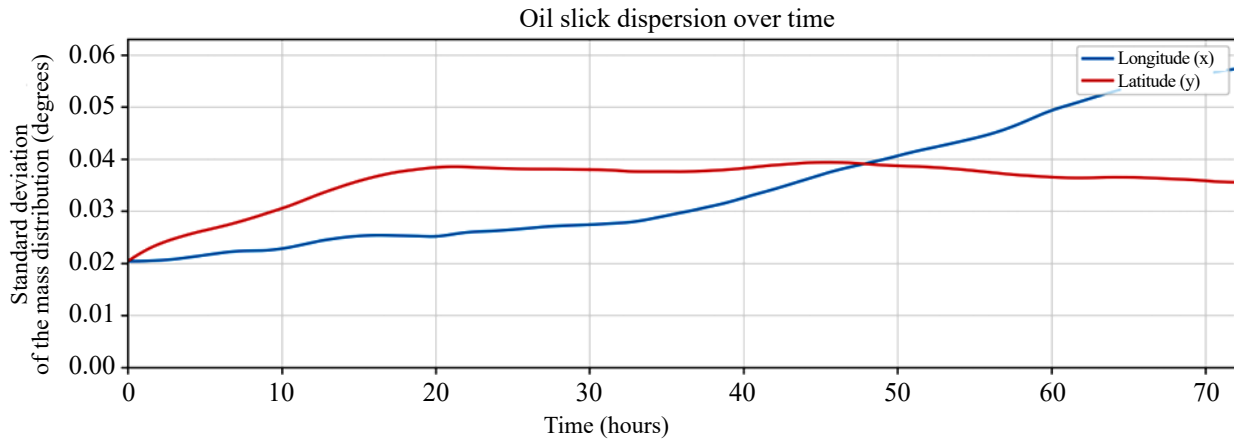


Fig. 6. Oil dispersion during the first three days of May

4. Additional case: persistent northward transport regime. The baseline scenario was calculated for the conditions of 1–4 May 2020. To estimate how far the slick may travel under conditions most favorable for transport, a three-year observational series for 2023–2025 at the release point was analyzed (*HYCOM* surface current averaged over $\pm 0.1^\circ$ square, plus 3% of *ERA5* wind speed, with a 3 h interval). For each 72-hour window shifted by 3 h, the mean drift speed and directional consistency $C = |\bar{V}| / S$, were calculated, where $\bar{V}$ is the mean drift vector over the window ($C = 1$ means that all 24 vectors are directed identically). Of 8,744 windows, 5,667 (64.8%) met the threshold $C \geq 0.9$, and in all three years the northward direction predominated among them (Table 1), consistent with the winter surface inflow into the Red Sea through the Bab el-Mandeb Strait [18, 19]. Among the windows satisfying the threshold, the one with the maximum speed was selected: 29.01.2023 15:00–01.02.2023 15:00 UTC ($S = 0.623$ m/s, of which the current contributes 0.298 m/s, $C = 0.997$, azimuth 350° , expected displacement 160.9 km).

Table 1

72-hour windows with directional consistency $C \geq 0.9$ of drift at the release point by year
(*HYCOM* + *ERA5* series, 3 h interval)

Year	Windows with $C \geq 0.9$	Share of all windows, %	Maximum $S, \text{m/s}$	Predominant azimuth
2023	1953	66.9	0.62	354°
2024	1942	66.3	0.58	3°
2025	1772	61.2	0.61	15°

The calculation for this window was performed with the same model and the same parameters as in the baseline scenario; the computational domain was extended to $14.0\text{--}17.1^\circ \text{N}$. The results are presented in Figs. 7–9 and Table 2. Over 72 h, the center of mass moves 102 km along an azimuth of 346° (mean velocity 0.4 m/s) and, by the end of the simulation, is located at 15.71°N , 42.40°E ; the slick passes between Kamaran Island and the mainland coast (Fig. 9), its northern front reaches 16°N , and part of the oil accumulates along the shoreline (maximum mass per cell approximately $9.1 \cdot 10^6$ kg versus $7.8 \cdot 10^5$ kg in the baseline scenario). Mass loss over 72 h is 32.0% (evaporation 97.7%, dissolution 2.3%; mass at 72 h $105.4 \cdot 10^6$ kg): stretching of the slick along the trajectory increases its area, and the evaporation peak rises to 295 kg/s. The standard deviation of the mass distribution in latitude reaches 13.0 km versus 4.0 km in the baseline scenario, while the longitudinal values remain comparable (8.4 km and 6.2 km).

Table 2

Comparison of the baseline scenario (1–4 May 2020) and the persistent northward transport regime
(29.01.2023 15:00–01.02.2023 15:00 UTC)

Indicator	May 2020	January 2023
Center-of-mass displacement over 72 h, km	6,5	102
Mean drift speed, m/s	0,03	0,47
Drift azimuth, $^\circ$	60	346
Mass loss over 72 h, %	23,4	32,0
Evaporation share of losses, %	96,7	97,7
Dissolution share of losses, %	3,3	2,3
Peak evaporation rate, kg/s	222	295
σ in longitude / latitude at 72 h, km	6,2 / 4,0	8,4 / 13,0

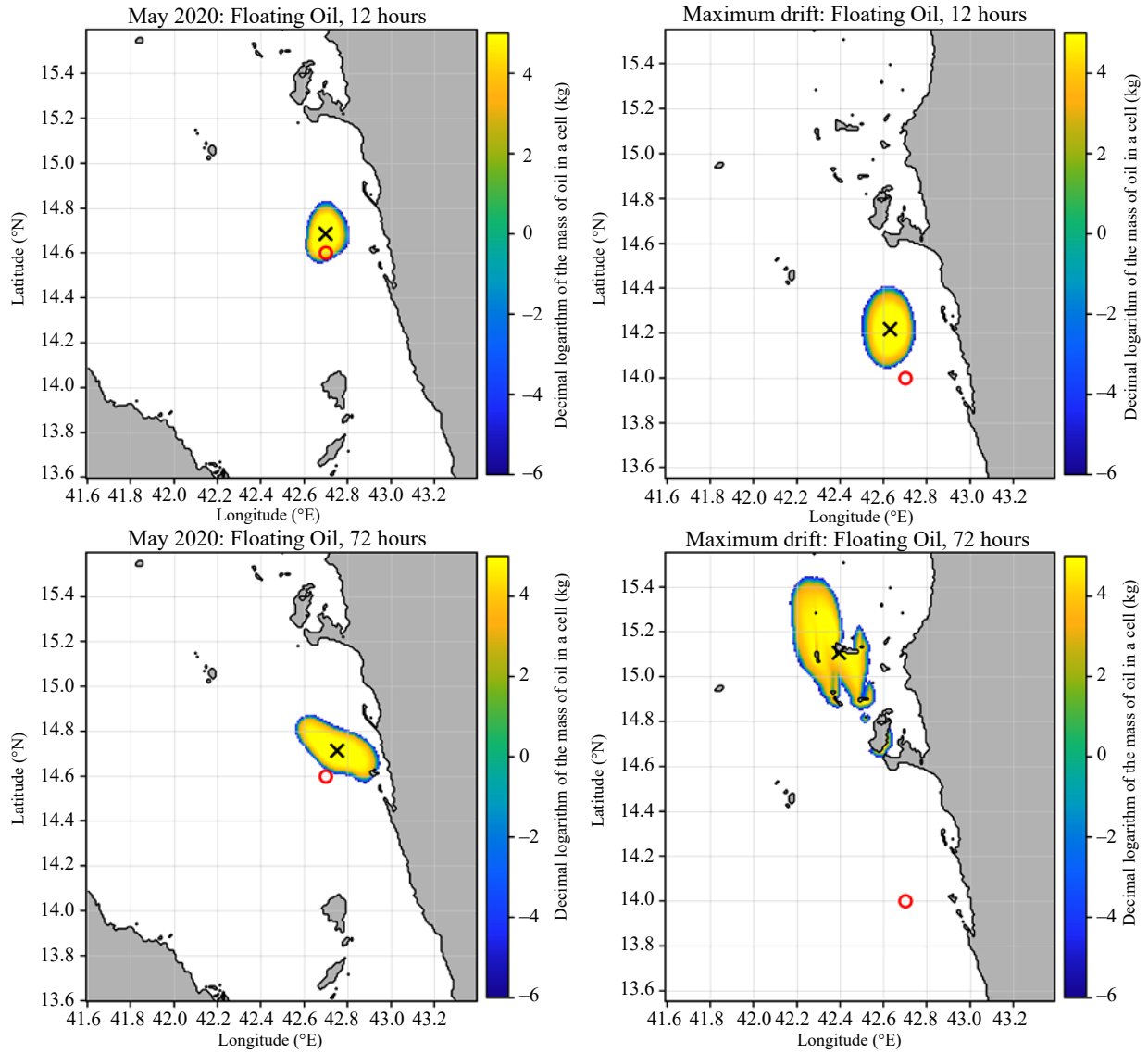


Fig. 7. Distribution of floating oil after 12 and 72 hours for the baseline scenario (left, 1–4 May 2020) and the persistent northward transport regime (right, 29.01.2023 15:00–01.02.2023 15:00 UTC). Red circle — release point; cross — center of mass; scale — decimal logarithm of oil mass per cell, kg

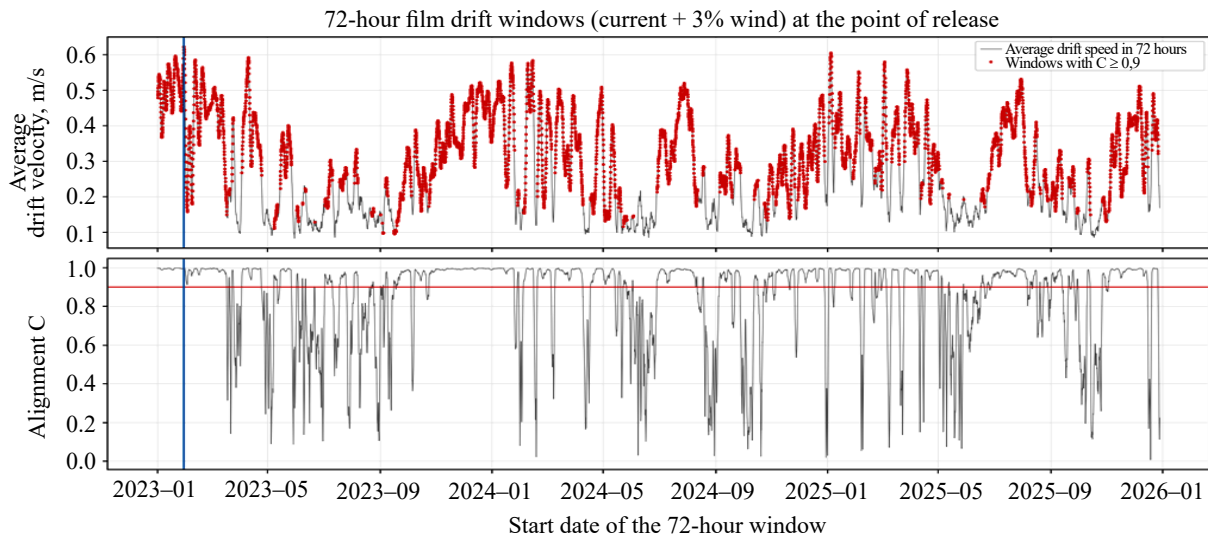


Fig. 8. Mean slick drift speed over 72 hours (top) and directional consistency C (bottom) for all 2023–2025 windows at the release point; dots mark windows with $C \geq 0.9$, and the vertical line denotes the selected window beginning at 29.01.2023 15:00 UTC

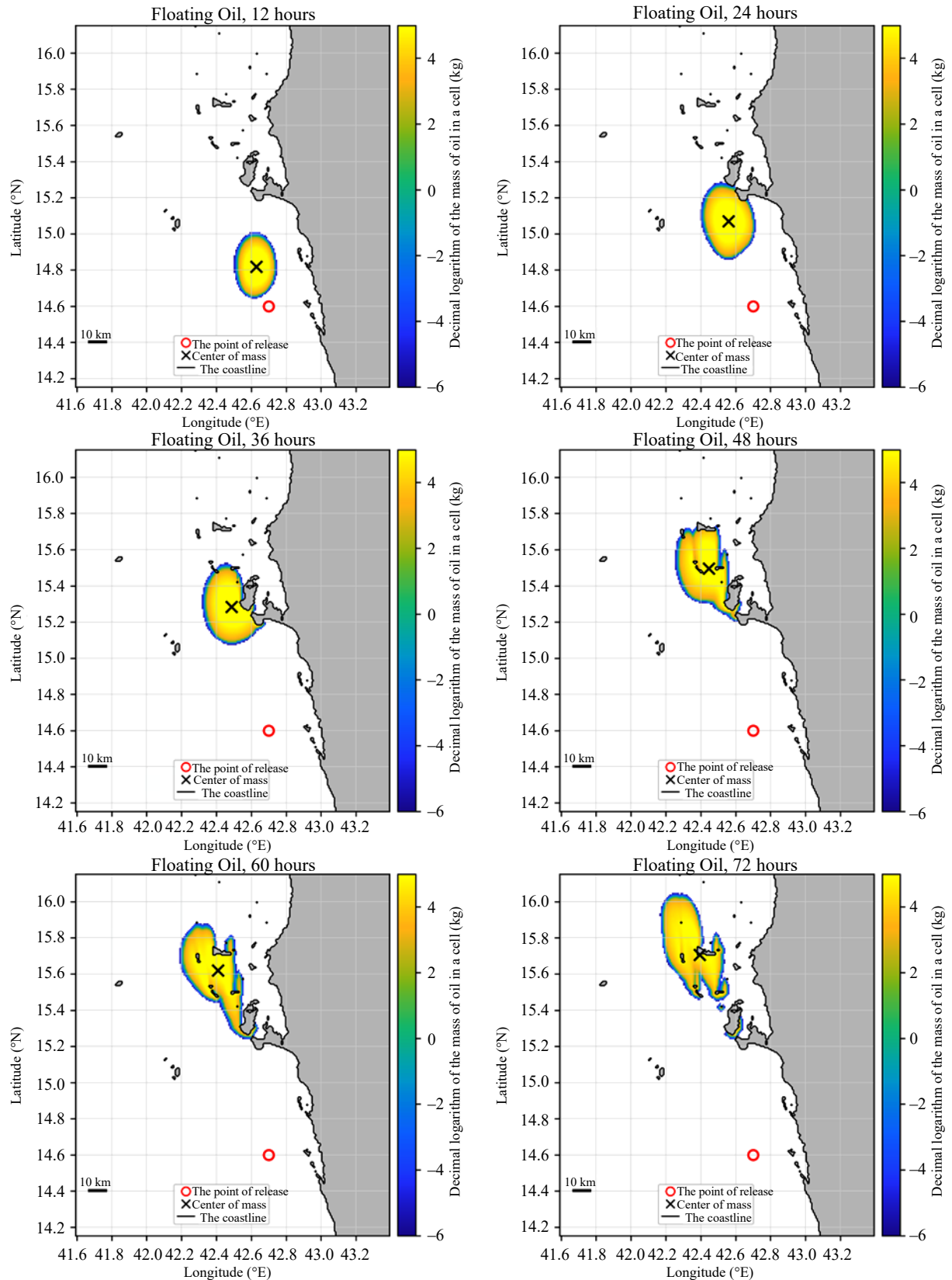


Fig. 9. Distribution of floating oil after 12, 24, 36, 48, 60, and 72 hours for the persistent northward transport regime (29.01.2023 15:00–01.02.2023 15:00 UTC)

In Fig. 9, the slick moves northward along the Yemeni coast, passes between Kamaran Island and the mainland, and reaches 16° N by 72 h. The red circle marks the release point, the cross marks the center of mass, the scale shows the decimal logarithm of oil mass per cell in kg, and the scale bar represents 10 km.

The input data used to select the window were surface currents from the *HYCOM* global analysis (*GOFS* 3.1 until September 2024 and *ESPC-D-V02* thereafter) and near-surface wind from the *ERA5* reanalysis at the release point for 2023–2025 with a 3 h interval [12, 11]. Over this three-year series, the maximum directionally consistent drift over 72 h is approximately 160.9 km northward; the model simulation for the selected window yields a 102 km displacement of the center of mass, with the northern front of the slick reaching 16° N (Fig. 9). Under this regime, the impact zone includes the coast and islands of Yemen: the Ras Isa terminal, Kamaran Island, the port of As Salif, and the approaches to Hodeidah, together with their mangroves, reefs, and fishing grounds. Under the May 2020 conditions, the same slick remains within 15 km of the release point; therefore, operational forecasts should rely on current wind and current fields rather than on an average or typical direction.

Discussion.

1. Key findings and model performance. Simulation of the *FSO Safer* spill scenario shows slow northeastward drift of the slick (0.03 m/s, azimuth 60°), landfall within the first day, and substantial mass loss (23.4% over 72 h), caused primarily by evaporation and, to a lesser extent, dissolution. The model reproduces the principal characteristics of the spill using open data. These results are consistent with recent studies of oil spills in semi-enclosed basins, while fraction-resolved weathering and biodegradation kinetics make it possible to separate the contributions of individual processes to mass loss under southern Red Sea conditions.

The dominance of evaporation is consistent with observations from the Deepwater Horizon spill [20] and with empirical oil-evaporation models [21], while the calculated rates are higher due to Red Sea temperatures (25–35 °C) and the presence of low-molecular-weight fractions. With realistic biomass, biodegradation contributes no more than 0.1% of the losses during the first three days, which is consistent with a lag phase of one day or longer [22, 23]; microbial remediation is relevant on timescales of weeks rather than hours.

2. Comparison with existing literature. Unlike Lagrangian models previously applied to the Red Sea, the proposed Eulerian model provides an explicit mass balance at every time step. The observed anisotropic dispersion (longitudinal > latitudinal) reflects current-field shear and numerical diffusion rather than physical turbulent diffusion. The proximity of the coast (less than 20 km from the release point) makes oil stranding likely within the first day whenever drift has an eastward component.

3. Implications for risk mitigation. Slick expansion (6.2 km longitudinal dispersion over 72 h) and the loss of nearly one quarter of the mass through evaporation over three days mean that containment measures should be deployed within the first 24 hours. Priority areas include:

- Mangrove forests and coral reefs along the Yemeni coast near Ras Isa and Kamaran Island, which are highly sensitive to oil toxicity.
- Coral reefs near the spill site, where delayed response may lead to irreversible biodiversity loss.
- Desalination plants, disruption of which would aggravate the regional water shortage.

The results obtained by the authors support the following recommendations:

- Pre-positioning of containment booms and dispersants along predicted trajectories.
- Real-time monitoring of winds and currents for dynamic updating of spill forecasts.
- Explicit treatment of oil stranding and use of a second-order transport scheme in the next version of the model.

4. Limitations and future research. Although the model captures the key physicochemical processes, it does not account for emulsification, spreading, and photo-oxidation, which may additionally modify oil viscosity and toxicity [24], or for shoreline stranding as a separate reservoir;

5. Broader significance. The study provides a reproducible open-data oil-spill model for the southern Red Sea under conflict conditions, directly supporting United Nations Sustainable Development Goal 14 (Life Below Water). By quantifying seasonal variability in trajectories (for example, winter versus summer winds), the authors provide an oil-pollution distribution scenario for regional emergency planning that allows operational forecasting at the required resolution. The model can be applied to other high-risk regions, including the Persian Gulf, for predictive environmental-safety applications.

6. Application prospects and regional characteristics. The developed model is intended to forecast the consequences of technological disasters in the southern Red Sea, including spills caused by deliberate damage to oil-transport and storage facilities: floating storage units, tankers, terminals, and pipelines. Damage to vessels in the Bab el-Mandeb Strait area since November 2023 [25] and damage in the area of the Ras Isa oil terminal on 17–18 April 2025, which according to a UN statement involved substantial damage to port infrastructure and a possible oil release into the Red Sea [26], demonstrate that such a scenario is possible. In operational mode, the problem's initial conditions (position, area, and estimated mass of the slick) and boundary conditions (coastline position and already contaminated sections) are to be determined from Earth remote-sensing data: *Sentinel-1* radar imagery provides the slick contour independently of cloud

cover and time of day [27], while Sentinel-2 and commercial optical imagery refine it under clear atmospheric conditions. The model thus becomes a component of the chain “satellite detection — drift and weathering forecast — warning of response services”, and a 72-hour scenario calculation, which takes approximately 10 min on a single CPU core, can be updated with each new satellite overpass.

The calculations highlight two regional features. The first is the high evaporation intensity at surface temperatures of 25–35°C [28]: during the first day, 5.2% of the spilled oil mass evaporates ($8.0 \cdot 10^6$ kg), during the second day 7.7% ($12.0 \cdot 10^6$ kg), and during the third day 9.7% ($15.0 \cdot 10^6$ kg) as the slick area grows; over 72 h, $35.0 \cdot 10^6$ kg of hydrocarbons enter the atmosphere, corresponding to 22.6% of the initial volume, with a maximum rate of approximately 222 kg/s. This means that a substantial fraction of the light components enters the atmosphere already during the first day, and air-quality monitoring over the water area and coastal settlements should begin simultaneously with marine response measures. The second feature is the sand and dust storms characteristic of Red Sea coasts [29]: they reduce atmospheric transparency and complicate optical remote sensing, whereas C-band radar imaging is practically insensitive to dust loading [27]. Joint work on remote-sensing problems under these conditions is planned to determine experimentally the dust-loading level (aerosol optical depth and the corresponding atmospheric transparency) at which useful results can still be obtained from optical channels, and the dust-content percentage at which slick detection becomes possible only by radar methods. The answer to this question will determine the set of satellite data suitable for specifying the initial conditions of the model in operational mode.

Conclusion. The computer simulation provides an analysis of an oil-spill scenario in Yemen, capturing the complex interaction of physical and biological processes. The spatial evolution shows a slowly northeastward-moving slick that reaches the coast near Ras Isa within the first day and decreases in intensity over 72 h; under the persistent winter-transport regime (29 January–1 February 2023), the same slick travels approximately 100 km northward in three days, reaching the Kamaran Islands, with a mass loss of 32%. The temporal analysis confirms a 23.4% reduction in mass, with evaporation as the dominant loss mechanism (96.7%), while the conservative scheme preserves mass without artificial correction. Dispersion metrics indicate an increasing spatial footprint consistent with the environmental forcing. The results are applicable to environmental-management tasks and emphasize the urgency of early response and the need for atmospheric monitoring over the water area.

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Determining the Principal Directions of a Point Set in the Plane and in Space

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Abstract

Introduction. The problem of determining the principal directions of a point set in the plane or in space is solved numerically. The task is to determine the orientation of an extremal line passing through the centroid of the set and minimizing or maximizing the functional defined as the sum of the squared distances from all points of the set to the line. This problem is relevant as an auxiliary component of many tasks involving intelligent systems.

Materials and Methods. For a point set in the plane, the principal-direction problem is solved explicitly for two polar angles specifying the directions of the extremal lines corresponding to the maximum and minimum of the error functional. If the denominator in the resulting formula degenerates because the points are arranged symmetrically, changing one coordinate of a single point by 10^{-14} is sufficient to break the symmetry, make the formula applicable, and obtain a double-precision result. For a point set in three-dimensional space, the problem is reduced to a numerical iterative algorithm based on gradient descent with respect to the orientation angles of a line passing through the centroid of the set. A minimal compact C++ implementation for determining the principal directions in the three-dimensional case is proposed.

Results. For a spatial point set, an explicit expression is obtained for the error functional and for the components of its gradient in the azimuthal and polar directions of a spherical coordinate system. An empirical formula for the gradient step is obtained as a function of the number of iterations, enabling a double-precision numerical solution with a relatively small number of iterations. It is shown that 100 iterations are sufficient to determine the extremal angles even when the initial orientation differs from the final orientation by $\pi/2$ for each angle.

Discussion. The principal-direction problems for point sets in the plane and in space are solved numerically with double precision. Two test examples confirm double-precision accuracy for cases with the largest deviation between the initial and final line orientations. Both the explicit formulas for the two-dimensional problem and the formulas of the three-dimensional iterative algorithm involve only coefficients represented by homogeneous second-order sums that depend on the point coordinates. There are three such coefficients in the two-dimensional case and six in the three-dimensional case.

Conclusions. Despite its algorithmic simplicity, the gradient descent method yields well-defined iterative formulas for any initial and intermediate orientation of a line passing through the centroid of a point set in space. The explicit formulas obtained for the principal directions of a point set in the plane and the gradient-descent formulas for the three-dimensional case can be used in analysis problems involving intelligent systems and neural networks.

Keywords: numerical methods, gradient descent, gradient, spherical coordinate system, artificial intelligence, neural networks

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Задача поиска главных направлений точечного множества на плоскости и в пространстве

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Аннотация

Введение. Численно решается задача поиска главных направлений точечного множества на плоскости или в пространстве, то есть поиска ориентации экстремальной прямой, проходящей через центр тяжести множества и доставляющей минимум или максимум функционалу суммы квадратов расстояний от каждой точки множества до прямой. Данная задача является актуальной как вспомогательная для многих задач с интеллектуальными системами.

Материалы и методы. Задача поиска главных направлений на плоскости решена в явном виде для двух полярных углов направления экстремальных прямых соответственно для максимума и минимума функционала ошибок. Для вырожденности знаменателя приведенной формулы с симметричным расположением точек множества достаточно координату одной из точек изменить на 10^{-14} для корректности использования формулы и получения ответа с двойной точностью (добавление малой асимметрии в геометрию множества). Задача поиска главных направлений точечного множества в пространстве сведена к численному итерационному алгоритму методом градиентного спуска изменением углов ориентации прямой, проходящей через центр тяжести множества. Предложен минимальный компактный код программы поиска главных направлений в трехмерном случае на языке C++.

Результаты исследования. Для пространственного расположения точек в явном виде получена формула функционала ошибок и компонентов его градиента в азимутальном и полярном направлениях сферической системы координат. Экспериментально получена формула градиентного шага в зависимости от числа итераций цикла для получения двойной точности численного решения задачи с небольшим числом итераций. Оказалось, что достаточно 100 итераций для поиска экстремальных углов, при котором начальная ориентация отличается на $\pi/2$ от конечной ориентации по каждому углу.

Обсуждение. С двойной точностью численно решены задачи поиска главных направлений точечного множества на плоскости и в пространстве. Приведено два тестирующих примера, подтверждающих двойную точность решения задачи с наибольшим отклонением начальных углов ориентаций прямой. Оказалось, что как в формулах явного вида в двумерной задаче, так и в формулах трехмерного итерационного алгоритма появляются только коэффициенты в виде однородных сумм второго порядка, зависящие от координат точек множества. В первом случае это три, а во втором случае шесть коэффициентов.

Заключение. Метод градиентного спуска, несмотря на простоту алгоритма, обеспечивает корректность итерационных формул при любой начальной и промежуточной ориентации прямой, проходящей через центр тяжести множества точек в пространстве. Полученные формулы в явном виде для главных направлений точечного множества на плоскости и формулы градиентного спуска в трехмерном случае могут быть использованы в задачах анализа с применением интеллектуальных систем и нейронных сетей.

Ключевые слова: численные методы, метод градиентного спуска, градиент, сферическая система координат, искусственный интеллект, нейронные сети

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Introduction. Neural-network methods are now used in an exceptionally wide range of scientific problems. In [1], the authors developed a neural-network model for storm forecasting on the basis of experimental data on physical fields during storms, using the Navier-Stokes equations, a large-eddy simulation (LES) turbulence model, ensemble probabilistic forecasting, and machine-learning methods. In [2–14], neural networks are applied to object-recognition problems, including personal identification from photographs, recognition of sea-depth values on pilot charts, control of mechanical-system balancing, recognition of railway-car numbers and pollen-grain classes, segmentation of cardiac arteries in coronary angiography images, and other applications. In a number of studies, including [4], A.V. Galaburdin

applied neural networks to nonlinear boundary-value problems in complex-shaped domains. The authors of [6] used several neural-network architectures to recognize protein conformational states from single-molecule diffraction data for bacteriorhodopsin. In [3, 10, 12, 13], neural networks are used to analyze the effect of electromagnetic radiation on equipment-corrosion rates, detect anomalies in cyber-physical systems, model the spread of a network virus, and recognize deceptive emotions. All of these problems are formulated and solved using neural-network methods.

At the same time, one can identify a class of specialized problems that admit exact mathematical treatment while using a neural network as a ready-made data-processing tool. This study considers two problems concerned with determining the principal directions of a point set in the plane and in three-dimensional space. The coordinates of the points may be obtained by a neural network from technical measurements, for example, by identifying the boundary of a wildfire area in a smoke-filled forest from aerial imagery or by determining the positions of fish from reflected ultrasonic waves during fishing operations. For the planar case, an explicit formula for the principal directions is derived. For the three-dimensional case, an algorithm in spherical coordinates is presented for determining the extremal directions of a line passing through the centroid of the point set.

Materials and Methods. Problem Statement

1. Determining the principal directions of a point set in the plane.

Consider a set of n points in the plane specified by their coordinates $(x_i, y_i, i = \overline{1, n})$. The coordinates of the centroid of the set are determined by the formulas

$$x_0 = \frac{1}{n} \sum_{i=1}^n x_i, y_0 = \frac{1}{n} \sum_{i=1}^n y_i. \quad (1)$$

The coordinates of each point relative to the centroid of the set are

$$\bar{x}_i = x_i - x_0 = x_i - \frac{1}{n} \sum_{j=1}^n x_j = \frac{1}{n} \sum_{j=1, j \neq i}^n (x_i - x_j), \bar{y}_i = y_i - y_0 = y_i - \frac{1}{n} \sum_{j=1}^n y_j = \frac{1}{n} \sum_{j=1, j \neq i}^n (y_i - y_j), i = \overline{1, n}. \quad (2)$$

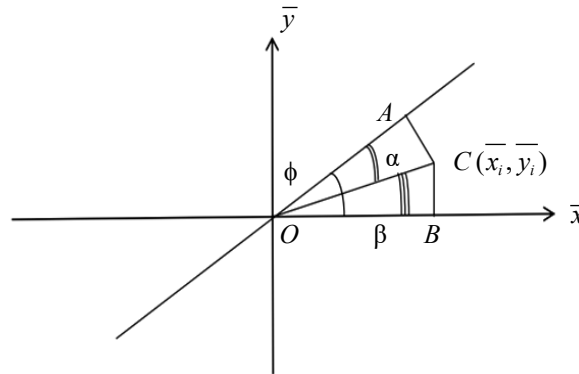


Fig. 1. Determination of the distance from a point to a line passing through the centroid O of the set at an angle ϕ to the axis

Consider a line passing through the centroid O at an angle ϕ to the axis $\bar{x}$. As can be seen from Fig. 1, the angle ϕ is equal to the sum of the angles $\phi = \alpha + \beta$. Then the distance $|OC|$ from the origin O to the point $(\bar{x}_i, \bar{y}_i)$ is $\rho_i = \sqrt{\bar{x}_i^2 + \bar{y}_i^2}$, and the square of the required distance from the point to the line $|CA|$ is found as

$$\begin{aligned} d_i^2 &= \rho_i^2 \sin^2 \alpha = \rho_i^2 \sin^2 (\phi - \beta) = \rho_i^2 (\sin(\phi) \cos(\beta) - \cos(\phi) \sin(\beta))^2 = \\ &= (\sin(\phi)(\rho_i \cos(\beta)) - \cos(\phi)(\rho_i \sin(\beta)))^2 = (\sin(\phi) \bar{x}_i - \cos(\phi) \bar{y}_i)^2. \end{aligned}$$

If all points of the set lie on a radial (optimal) line, the sum of the squared distances from the points to that line is zero. We therefore introduce an error functional for the planar problem; it depends on all point coordinates and on the angle ϕ as a parameter:

$$E(\bar{x}_i, \bar{y}_i, \phi) = \sum_{i=1}^n d_i^2 = \sum_{i=1}^n (\sin(\phi) \bar{x}_i - \cos(\phi) \bar{y}_i)^2 \rightarrow \text{extr}_{\phi \in [0, 2\pi]}. \quad (3)$$

Expanding the expression in parentheses and collecting terms, we obtain

$$E(\bar{x}_i, \bar{y}_i, \phi) = \sum_{i=1}^n d_i^2 = \sum_{i=1}^n (\sin(\phi) \bar{x}_i - \cos(\phi) \bar{y}_i)^2 = \sin^2(\phi) \sum_{i=1}^n \bar{x}_i^2 - 2 \sin(\phi) \cos(\phi) \sum_{i=1}^n \bar{x}_i \bar{y}_i + \cos^2(\phi) \sum_{i=1}^n \bar{y}_i^2 =$$

$$= \sin^2(\phi) A - 2 \sin(\phi) \cos(\phi) B + \cos^2(\phi) C \rightarrow \text{extr}_{\phi \in [0, 2\pi]}, A = \sum_{i=1}^n \bar{x}_i^2, B = \sum_{i=1}^n \bar{x}_i \bar{y}_i, C = \sum_{i=1}^n \bar{y}_i^2. \quad (4)$$

Differentiating the functional $E(\bar{x}_i, \bar{y}_i, \phi)$ in (4) with respect to the polar angle ϕ , we obtain the directions of the optimal (principal) lines passing through the centroid of the point set:

$$\partial E(\phi) / \partial \phi = 0 \Leftrightarrow 2 \sin(\phi) \cos(\phi) A + 2B(\sin^2(\phi) - \cos^2(\phi)) - 2 \cos(\phi) \sin(\phi) C = 0.$$

Dividing the last equation by $\cos^2(\phi)$:

$$\tan(\phi) A + B(\tan^2(\phi) - 1) - \tan(\phi) C = 0 \Leftrightarrow B \tan^2(\phi) + (A - C) \tan(\phi) - B = 0. \quad (5)$$

Solving the quadratic equation (5), we obtain the optimal directions

$$\tan(\phi) = \frac{C - A \pm \sqrt{(A - C)^2 + 4B^2}}{2B}, \phi_{1,2} = \arctan\left(\frac{C - A \pm \sqrt{(A - C)^2 + 4B^2}}{2B}\right), \quad (6)$$

where $A = \sum_{i=1}^n \bar{x}_i^2, B = \sum_{i=1}^n \bar{x}_i \bar{y}_i, C = \sum_{i=1}^n \bar{y}_i^2$.

Remark 1. The numerator of formula (6) is always defined, whereas its denominator may become zero, for example, when the points are arranged symmetrically with respect to the centroid. When $B = \sum_{i=1}^n \bar{x}_i \bar{y}_i = 0$ the symmetry of the point arrangement can be broken artificially by adding a small parameter to the coordinate $\bar{x}_i$ or $\bar{y}_i$ a small parameter $\varepsilon = 10^{-12} - 10^{-14}$.

Example 1. Consider a symmetric set of eight points located at the vertices of a rectangle or at the intersections of the rectangle with the coordinate axes:

$$x_1 = -2.0; y_1 = 1.0 + \varepsilon; \varepsilon = 0.000000000000001; x_2 = 0.0; y_2 = 1.0; x_3 = 2.0; y_3 = 1.0; x_4 = 2.0; y_4 = 0.0;$$

$$x_5 = 2.0; y_5 = -1.0; x_6 = 0.0; y_6 = -1.0; x_7 = -2.0; y_7 = -1.0; x_8 = -2.0; y_8 = 0.0.$$

C++ program in which all functions and arguments in formula (6) are represented in double precision yields the following values of the principal-direction angles (the angles $\alpha = -\pi/2$ and $\alpha = \pi/2$ define the same y-axis):

$$\alpha_1 = -1.5707963267948954,$$

$$\alpha_2 = 0.0000000000000000,$$

$$\pi/2 = 1.5707963267948966.$$

The principal directions coincide with the coordinate-axis directions to essentially double precision. The optimal directions can also be obtained analytically:

$$E(\bar{x}_i, \bar{y}_i, \phi = 0) = 2 \cdot 0^2 + 6 \cdot 1^2 = 6 = E_{\min}, E(\bar{x}_i, \bar{y}_i, \phi = \pi/2) = 2 \cdot 0^2 + 6 \cdot 2^2 = 24 = E_{\max}.$$

2. Determining the principal directions of a point set in space.

Consider a line passing through the centroid of a three-dimensional point set defined by the formulas

$$x_0 = \frac{1}{n} \sum_{i=1}^n x_i, y_0 = \frac{1}{n} \sum_{i=1}^n y_i, z_0 = \frac{1}{n} \sum_{i=1}^n z_i, \bar{x}_i = x_i - x_0, \bar{y}_i = y_i - y_0, \bar{z}_i = z_i - z_0, i = \overline{1, n}. \quad (7)$$

The coordinates of each point in the coordinate system associated with the centroid of the set (x_0, y_0, z_0) are denoted by $(\bar{x}_i, \bar{y}_i, \bar{z}_i), i = \overline{1, n}$. On the specified line, oriented by the angles (θ, ϕ) in the spherical coordinate system and passing through the centroid O , we place a vector $\mathbf{k}$ of unit length, as shown in Fig. 2.

The radius vector directed from the centroid O to the point $(x_i, y_i, z_i, i = \overline{1, n})$, and the unit vector $\mathbf{k}$ form a parallelogram whose area, on the one hand, can be determined in Cartesian coordinates by the vector product

$$\bar{S} = \mathbf{k} \times \bar{\rho}_i = \begin{vmatrix} i & j & k \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \bar{x}_i & \bar{y}_i & \bar{z}_i \end{vmatrix} = i(\sin \theta \sin \phi \bar{z}_i - \cos \theta \bar{y}_i) - j(\sin \theta \cos \phi \bar{z}_i - \cos \theta \bar{x}_i) +$$

$$+ k(\sin \theta \cos \phi \bar{y}_i - \sin \theta \sin \phi \bar{x}_i). \quad (8)$$

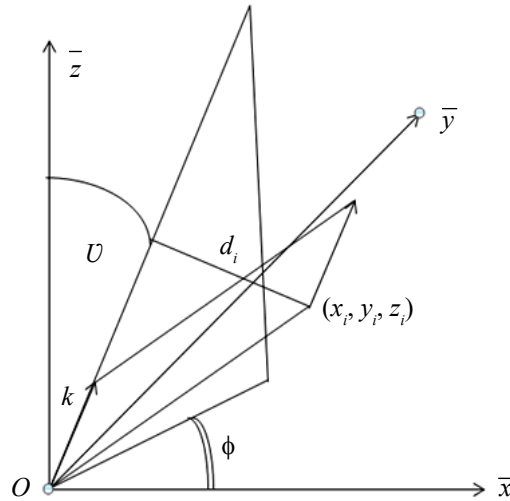


Fig. 2. Determination of the distance from point $(\bar{x}_i, \bar{y}_i, \bar{z}_i, i = \overline{1, n})$ to a line passing through the centroid of the set O and oriented by the angles (θ, ϕ) in a spherical coordinate system

On the other hand, the area of the parallelogram is $|\bar{S}| = |\bar{k}| d_i = 1 \cdot d_i = d_i$, where d_i is the distance from the points $(\bar{x}_i, \bar{y}_i, \bar{z}_i, i = \overline{1, n})$ to the selected line (θ, ϕ) in the spherical coordinate system. Without changing the error functional (equal to the sum of the squared distances from the points to the line), we obtain the extremum problem (9):

$$E(\bar{x}_i, \bar{y}_i, \bar{z}_i, \theta, \phi) = \sum_{i=1}^n d_i^2 = \sum_{i=1}^n (\sin \theta \sin \phi \bar{z}_i - \cos \theta \bar{y}_i)^2 + (\sin \theta \cos \phi \bar{z}_i - \cos \theta \bar{x}_i)^2 + (\sin \theta \cos \phi \bar{y}_i - \sin \theta \sin \phi \bar{x}_i)^2 \rightarrow \text{extr}_{\theta \in [0, \pi], \phi \in [0, 2\pi]}, \quad (9)$$

$$\begin{aligned} E(\theta, \phi) &= (\cos^2 \theta + \sin^2 \theta \sin^2 \phi) \sum_{i=1}^n \bar{x}_i^2 + (\cos^2 \theta + \sin^2 \theta \cos^2 \phi) \sum_{i=1}^n \bar{y}_i^2 + (\sin^2 \theta \sin^2 \phi + \sin^2 \theta \cos^2 \phi) \sum_{i=1}^n \bar{z}_i^2 + \\ &\quad - 2 \sin^2 \theta \sin \phi \cos \phi \sum_{i=1}^n \bar{x}_i \bar{y}_i - 2 \cos \phi \sin \theta \cos \theta \sum_{i=1}^n \bar{x}_i \bar{z}_i - 2 \sin \phi \sin \theta \cos \theta \sum_{i=1}^n \bar{y}_i \bar{z}_i = \\ &= (1 - \sin^2 \theta \cos^2 \phi) \sum_{i=1}^n \bar{x}_i^2 + (1 - \sin^2 \theta \sin^2 \phi) \sum_{i=1}^n \bar{y}_i^2 + \left(\frac{1 - \cos 2\theta}{2} \right) \sum_{i=1}^n \bar{z}_i^2 - \sin^2 \theta \sin 2\phi \sum_{i=1}^n \bar{x}_i \bar{y}_i - \\ &\quad - \sin 2\theta \left(\cos \phi \sum_{i=1}^n \bar{x}_i \bar{z}_i + \sin \phi \sum_{i=1}^n \bar{y}_i \bar{z}_i \right) \Leftrightarrow \\ E(\theta, \phi) &= (1 - \sin^2 \theta \cos^2 \phi) A + (1 - \sin^2 \theta \sin^2 \phi) B + \left(\frac{1 - \cos 2\theta}{2} \right) C - \sin^2 \theta \sin 2\phi D - \\ &\quad - \sin 2\theta (\cos \phi E + \sin \phi F) \rightarrow \text{extr}_{\theta \in [0, \pi], \phi \in [0, 2\pi]}, \end{aligned} \quad (10)$$

$$\text{where } A = \sum_{i=1}^n \bar{x}_i^2, B = \sum_{i=1}^n \bar{y}_i^2, C = \sum_{i=1}^n \bar{z}_i^2, D = \sum_{i=1}^n \bar{x}_i \bar{y}_i, E = \sum_{i=1}^n \bar{x}_i \bar{z}_i, F = \sum_{i=1}^n \bar{y}_i \bar{z}_i.$$

The domain of the variables (θ, ϕ) is finite and forms the rectangle $[0, \pi] \times [0, 2\pi]$. Therefore, the unconstrained extremum problem (10) can be solved numerically by the gradient descent method [15, 16]. The components of the gradient of the functional $E(\theta, \phi)$:

$$\begin{aligned} E_\theta(\theta, \phi) &= -2A \sin \theta \cos \theta \cos^2 \phi - 2B \sin \theta \cos \theta \sin^2 \phi + C \sin 2\theta - 2D \sin \theta \cos \theta \sin 2\phi - \\ &\quad - 2 \cos 2\theta (\cos \phi E + \sin \phi F) = \sin 2\theta (C - A \cos^2 \phi - B \sin^2 \phi) - D \sin 2\theta \sin 2\phi - 2 \cos 2\theta (\cos \phi E + \sin \phi F), \end{aligned} \quad (11)$$

$$\begin{aligned} E_\phi(\theta, \phi) &= A \sin 2\phi \sin^2 \theta - B \sin 2\phi \sin^2 \theta - 2 \sin^2 \theta \cos 2\phi D - \sin 2\theta (-\sin \phi E + \cos \phi F) = \\ &= (A - B) \sin 2\phi \sin^2 \theta - 2 \sin^2 \theta \cos 2\phi D + \sin 2\theta (\sin \phi E - \cos \phi F). \end{aligned} \quad (12)$$

The iterative gradient-descent formula for the minimization problem, using formulas (10), (11), and (12), takes the form

$$\begin{cases} \theta^{m+1} = \theta^m - hE_{\theta}(\theta^m, \phi^m), m = 0, 1, 2, \dots, N, \\ \phi^{m+1} = \phi^m - hE_{\phi}(\theta^m, \phi^m), \end{cases} \quad (13)$$

where $h = \pi / (5N)$ is the gradient-step formula; N is the number of iterations. The center of the rectangle $[0, \pi] \times [0, 2\pi]$ can be chosen as the initial point, i. e., as the initial orientation angles:

$$\theta^0 = \frac{\pi}{2}, \phi^0 = \pi. \quad (14)$$

Remark 2. The terms in formulas (11), (12), and (13) are defined (and all coefficients A, B, C, D, E, F of the point set in (10) are defined as well) and remain finite for different orientation angles of the line with vector $\mathbf{k}$. Therefore, algorithm (11)–(13) for minimizing functional (10) is well defined for all orientation angles.

The following program code is written in C++, with all functions and arguments represented in double precision. In the program, variables x and y correspond to θ and ϕ , respectively, in formulas (10), (11), (12), and (13):

```
#include<stdio.h>
#include<math.h>
double EE(double x, double y, double A, double B, double C, double D, double E, double F)
{return(1.0-cos(2.0*x))*C/2.0+(1.0-sin(x)*sin(x)*sin(y)*sin(y))*B+(1.0-sin(x)*sin(x)*cos(y)*cos(y))*A-
sin(2.0*x)*(F*sin(y)+E*cos(y))-sin(2.0*y)*sin(x)*sin(x)*D;}
double Ex(double x, double y, double A, double B, double C, double D, double E, double F)
{returnsin(2.0*x)*(C-B*sin(y)*sin(y)-A*cos(y)*cos(y))-2.0*cos(2.0*x)*(F*sin(y)+E*cos(y))-sin(2.0*y)*sin(2.0*x)*D;}
double Ey(double x, double y, double A, double B, double C, double D, double E, double F)
{return sin(2.0*y)*sin(x)*sin(x)*(A-B)-sin(2.0*x)*(F*cos(y)-E*sin(y))-2.0*cos(2.0*y)*sin(x)*sin(x)*D;}
int const n=10, m=100;
int main()
{int i; double x,y,h,x1[n+1],y1[n+1],z1[n+1],x0,y0,z0;
double x2[n+1],y2[n+1],z2[n+1],s1,s2,s3,s4,s5,s6;double A,B,C,D,E,F,pi;
pi=2.0*asin(1.0);h=pi/double(5*m); s1=0.0;s2=0.0;s3=0.0;
for(i=0;i<=n;i++)
{y1[i]=double(i);x1[i]=0.0;z1[i]=0.0; s1=s1+x1[i];s2=s2+y1[i];s3=s3+z1[i];}
x0=s1/double(n+1);y0=s2/double(n+1);z0=s3/double(n+1);
s1=0.0;s2=0.0;s3=0.0;s4=0.0;s5=0.0;s6=0.0;
for(i=0;i<=n;i++)
{x2[i]=x1[i]-x0;y2[i]=y1[i]-y0;z2[i]=z1[i]-z0;s1=s1+x2[i]*x2[i]; s2=s2+y2[i]*y2[i]; s3=s3+z2[i]*z2[i]; s4=s4+z2[i]
*y2[i];s5=s5+z2[i]*x2[i];s6=s6+x2[i]*y2[i]; }
A=s1;B=s2;C=s3;F=s4; E=s5;D=s6; x=pi*0.1;y=pi*0.9;
printf("inicial teta=%.16lf fi=%.16lf\n",x,y);
for(i=0;i<=m;i++)
{x=x-h*Ex(x,y,A,B,C,D,E,F); y=y-h*Ey(x,y,A,B,C,D,E,F);}
printf("end teta=%.16lf fi=%.16lf\n",x,y);
printf("exact teta=%.16lf fi=%.16lf\n",pi/2.0,pi/2.0); printf("E=%.16lf\n",EE(x,y,A,B,C,D,E,F));}
```

Consider **Example 2**. Eleven points are located on the y -axis at unit intervals, with the first point coinciding with the origin $x_i = 0, y_i = i, z_i = 0, i = 0, 10$. The initial values of the line-orientation angles, in radians, are $\theta^0 = x = \pi \cdot 0.1, \phi^0 = y = \pi \cdot 0.9$.

Starting from the prescribed initial angular orientation, the program returns the final orientation. The final orientation coincides with the exact solution to double precision, i. e., to approximately 15 significant digits. The number of iterations is $N(m \text{ in the program}) = 100$. During the iteration cycle according to formula (13), the line changes its initial orientation for each angle by almost $\pi/2$, which shows that algorithm (10)–(13) is applicable over a wide range of initial line orientations. Formula (10), i. e., the value of the functional, is used to verify the numerical solution. If the line passes through all points of the set, the functional $E(\theta^N, \phi^N) = 0$ (for any initial values θ^0, ϕ^0 in Example 2) is:

```
inicial teta=0.3141592653589793 fi=2.8274333882308138
end teta=1.5707963267948966 fi=1.5707963267948966
exact teta=1.5707963267948966 fi=1.5707963267948966
E=0.0000000000000000.
```

It should be noted that the gradient method (10)–(13) requires more iterations ($N = 100$) than, for example, the matrix form of Newton's tangent method. However, the matrix Newton method requires inversion of the Jacobian matrix of the

system of equations, and if the determinant of the Jacobian is zero, the Newton formula is not applicable. By contrast, the gradient-descent algorithm (10)–(13) remains well defined for all line-orientation angles, although the residual converges to zero more slowly.

Results. The problem of determining the principal directions of a set of points in the plane or in space has been solved. The coordinates of all points are assumed to be known. For a two-dimensional point set, the numerical solution for the optimal directions is given by formula (6). For a three-dimensional point set, the principal directions are determined by the algorithm (10)–(13) with double precision.

Discussion

1. For a point set in the plane, the orientations of the principal lines passing through the centroid are obtained from formula (6).
2. The coefficients in formula (6) are homogeneous second-order sums of the prescribed point coordinates.
3. For a point set in space, the gradient-descent method yields the algorithm (10)–(13) for determining the principal directions in a spherical coordinate system.
4. The six coefficients A , B , C , D , E , F , and F in algorithm (10)–(13) are homogeneous second-order sums of the known point coordinates.
5. The error functional in the extremum problem is equal to the sum of the squared distances from each point to the line.
6. In this problem, both directions corresponding to the minimum and maximum values of the error functional are relevant. For example, for a forest area affected by fire, the direction associated with the minimum of the functional can be interpreted as the direction for water discharge, whereas the direction associated with the maximum can define the shortest evacuation route for people and animals from the burning area.
7. In the two test examples, the extremal directions are determined numerically with double precision.

Conclusion. Despite its algorithmic simplicity, the gradient descent method yields well-defined iterative formulas for any initial and intermediate orientation of a line passing through the centroid of a point set in space. The explicit formulas obtained for the principal directions of a point set in the plane and the gradient-descent formulas for the three-dimensional case can be used in analysis problems involving intelligent systems and neural networks.

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Mathematical Modelling of the Black Sea Coastline Considering its Fractal Structure and Grid Generation

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Abstract

Introduction. The coastline of a reservoir is a complex natural object, which geometry has irregularity and signs of self-similarity, and the length depends on the scale of measurement. Despite the widespread study of its fractal properties, the transition to a digital representation of the coastline is a complex procedure and is a current topic of research. The purpose of the article is to develop a mathematical-algorithmic scheme for analyzing the coastline, including geometry validation, geodetic measurement, scale dependence analysis, fractal dimension assessment and selection of an adaptive quadrilateral grid using the example of the Black Sea.

Materials and Methods. Normalization and validation procedures are applied to the coastline represented as a finite sequence of geographic points to obtain an intermediate, geometrically plausible contour. The contour length is calculated on a sphere using the haversine formula. The “box-counting” method is used to address the coastline’s multi-scale nature and fractal characteristics. To transition from the line to a 2D representation of the water area, the *Lambert azimuthal equal-area (LAEA)* projection is used, followed by the construction of a grid with the required cell size. Subsequently, detailed and coarse full-quadrilateral grids are generated using the *Delaunay*, *Frontal-Delaunay for Quads*, and *Packing of Parallelograms* algorithms. A comparison of the grid generators is then conducted.

Results. A computational experiment was conducted using Black Sea coastline data. A comparison of two sets of grids demonstrated that *Delaunay* is the superior generator based on the specified multi-criteria metric. The *Delaunay* method with a range of 125–250 m should be used to preserve significant coastal features across the entire water area. A high-resolution *Delaunay* configuration with a range of 50–250 m should be selected when accurate reconstruction of the seabed topography is the priority.

Discussion. The practical significance lies in the ability to prepare verified contours and grids for seabed topography modelling, geoinformation analysis, and subsequent hydrodynamic calculations.

Conclusions. Future research prospects involve expanding the range of coastal systems analyzed, comparing various methods for estimating fractal dimension, investigating the impact of source geodata spatial resolution on the stability of calculated metrics, and adapting the approach for multi-scale monitoring of coastal dynamics.

Keywords: coastline, adaptive quadrilateral grid, mathematical modelling, fractal geometry, box-counting, bathymetry, scale dependence

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Математическое моделирование береговой линии Черного моря с учетом ее фрактальной структуры и построение сеток

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Аннотация

Введение. Береговая линия водоема представляет собой сложный природный объект, геометрия которого проявляет нерегулярность и признаки самоподобия, а длина зависит от масштаба измерения. Несмотря на широкое изучение её фрактальных свойств, переход к цифровому представлению береговой линии представляет собой комплексную процедуру и является актуальной темой исследования. В настоящей работе описана созданная математико-алгоритмическая схема анализа береговой линии, включающая валидацию геометрии, геодезическое измерение, анализ масштабной зависимости, оценку фрактальной размерности и выбор адаптивной четырёхугольной сетки на примере Чёрного моря.

Материалы и методы. Для береговой линии, представленной конечной последовательностью географических точек, проводятся процедуры нормализации и валидации, что позволяет получить промежуточный геометрически правдоподобный контур. Длина контура вычисляется на сфере с использованием формулы гаверсина. Для работы с многомасштабностью и фрактальными характеристиками береговой линии применяется метод «подсчета ящиков». Для перехода от линии к 2D-представлению акватории применяется равноплощадная проекция *Lambert azimuthal equal-area (LAEA)*, а затем строится поле требуемого размера ячейки. Далее строятся подробные и укрупненные полноквадратные сетки с помощью генераторов *Delaunay*, *Frontal-Delaunay for Quads* и *Packing of Parallelograms*. Производится сравнение генераторов сеток.

Результаты исследования. Проведен вычислительный эксперимент на данных береговой линии Чёрного моря. Сравнение двух наборов сеток показало, что лучшим генератором по заданному многокритериальному показателю является *Delaunay*. Для сохранения существенных береговых особенностей на масштабе всей акватории следует использовать *Delaunay* с диапазоном 125–250 м. Подробный *Delaunay* с диапазоном 50–250 м следует выбирать, когда в приоритете точность восстановления рельефа дна.

Обсуждение. Практическая значимость работы состоит в возможности подготовки верифицированных контуров и сеток для моделирования рельефа дна, геоинформационного анализа и последующих гидродинамических расчётов.

Заключение. Перспективы дальнейших исследований связаны с расширением класса анализируемых береговых систем, сопоставлением различных методов оценки фрактальной размерности, исследованием влияния пространственного разрешения исходных геоданных на устойчивость вычисляемых метрик, а также с адаптацией подхода к задачам многомасштабного мониторинга береговой динамики.

Ключевые слова: береговая линия, адаптивная четырёхугольная сетка, математическое моделирование, фрактальная геометрия, подсчёт покрывающих ячеек, батиметрия, масштабная зависимость

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Introduction. Coastlines are natural objects for which classical geometric concepts are insufficient. Unlike smooth Euclidean curves, coastline length depends substantially on the measurement scale, while the coastline shape itself exhibits geometric irregularity and self-similarity. This is why coastlines became one of the best-known examples that contributed to the development of the concept of the fractal nature of natural structures [1, 2].

In contemporary studies of coastal systems, digital geodata play an essential role by representing a contour as a finite sequence of coordinates and thereby making it amenable to computational analysis. However, the transition from a real natural object to its digital representation is not a neutral procedure. It involves discretization, scale selection, filtering, geometric simplification, and metric selection, i. e., a set of assumptions that directly affect the final result [3–5].

Accordingly, coastline analysis is not only an applied problem but also a problem of general scientific significance. On the one hand, a mathematically sound and reproducible procedure for measuring and analyzing contour geometry must be developed. On the other hand, the applicability limits of the digital model must be established, together with an understanding of which properties of the natural object are preserved in its discrete representation.

The aim of this study is to develop and substantiate a mathematical and algorithmic framework for coastline analysis that includes:

- 1) validation and normalization of discrete geometry;
- 2) geodetic length measurement;
- 3) analysis of scale dependence;
- 4) estimation of the empirical fractal dimension;
- 5) construction and quantitative selection of an adaptive quadrilateral grid for preserving significant coastal features.

The scientific novelty of this work lies in treating coastline analysis not as a set of isolated computational procedures, but as a unified reproducible mathematical and algorithmic framework that combines geometric validation, geodetic measurement, fractal analysis, and grid generation. In contrast to studies in which fractal dimension is estimated without explicit quality control of the source geometry or a generator is selected solely by visual inspection, the proposed computational formulation includes polyline normalization, self-intersection checks, analysis of the robustness of the dimension estimate, and a formalized criterion for preserving coastal features.

The practical significance of the study lies in the applicability of the proposed approach to the analysis of digital coastline contours, improvement of the reliability of morphometric calculations, comparison of data with different spatial resolutions, and preparation of verified geometric data for further modelling in geoinformatics, geomorphology, and coastal-system monitoring [5–24]. A separate applied result is a rule for selecting the generator and the cell-size range that does not depend on a subjective visual assessment of the grid on a map.

Materials and Methods. Problem formulation and mathematical representation of the object. Let the coastline be represented by a finite sequence of geographic points [1]:

$$C = p_i'', \quad (1.1)$$

where $p_i = (\varphi_i - \lambda_i)$, $i = \overline{1, n}$, φ_i — latitude, λ_i — longitude of the i -th point. The source Black Sea contour was obtained from *Marine Regions* [6].

The study requires the development of a computational framework that, for a given sequence of points, provides:

- verification of the geometric validity of the contour;
- calculation of its length in a geodetic metric;
- analysis of the dependence of length on the representation scale;
- estimation of the empirical fractal dimension;
- selection of the generator and the cell-size range of the adaptive quadrilateral grid.

It is essential that the object under consideration not be identified with its digital record. The coordinate sequence is interpreted as an operational model of the coastline suitable for mathematical analysis but inevitably dependent on the resolution of the source data, the point-selection procedure, and the chosen measurement method. Thus, the focus of the study shifts from describing a particular software implementation to constructing a mathematically interpretable and reproducible analytical framework suitable for investigating scale-dependent properties of coastline contours.

Geometry normalization and validation. Normalization removes duplicates and checks traversal order, self-intersections, and abnormally long segments. For nonadjacent segments, an orientation test is applied [7–9]:

$$\text{orient}(A, B, C) = (b_x - a_x)(c_y - a_y) - (b_y - a_y)(c_x - a_x).$$

The original polyline remains the sole source for quantitative calculations; simplification is allowed only when rendering *SVG* graphics so that visual simplification is not mixed with measurement. Thus, validation establishes a geometrically plausible contour before the length, fractal characteristic, and grid are calculated.

Geodetic length measurement. Since the coastline points are specified in geographic coordinates, the contour length is calculated not in a Euclidean plane but on a sphere using the haversine formula [3, 4]. For two consecutive points $p_i = (\varphi_i - \lambda_i)$ and $p_{i+1} = (\varphi_{i+1} - \lambda_{i+1})$ the distance is defined as:

$$d_i = 2R \arctan \frac{\sqrt{h_i}}{\sqrt{1 - h_i}}, \quad (2.1)$$

where $h_i = \sin\left(\frac{2\Delta\varphi_i}{2}\right) + \cos\varphi_i \cos\varphi_{i+1} \sin\frac{2\Delta\lambda_i}{2}$, $\Delta\varphi_i = \varphi_{i+1} - \varphi_i$, $\Delta\lambda_i = \lambda_{i+1} - \lambda_i$, and R — the mean radius of the Earth.

The total polyline length is calculated by summing the distances between adjacent points:

$$L(C) = \sum_{i=1}^{n-1} d_i. \quad (2.2)$$

The implementation uses $R = 6371$ km (the mean radius of the Earth). When projecting coordinates for the box-counting method (*box-counting*) and for geometry simplification, the following local approximation is used: $metresPerDegLat = 111\,194.9$ m/degree, $metresPerDegLon = 87\,300.0$ m/degree (relative to the reference point 43.5°N , 35.0°E for the Black Sea).

This approach makes it possible to obtain a physically interpretable coastline length in kilometers and to avoid systematic distortions that arise when a planar metric is applied directly to geographic data.

Scale dependence and fractal characteristic. Coastline length depends on the level of detail of its digital contour [1, 10]; therefore, the fractal characteristic is calculated directly from the observed line without replacing it by synthetic iterations of the Koch curve. The box-counting method is applied for 13 scales and four relative grid offsets [1–3]:

$$\ln N_\varepsilon = D \ln \left(\frac{1}{\varepsilon} \right) + C. \quad (2.3)$$

The coefficient D is interpreted only within the adopted range of scales. The working window requires at least four scales, $R^2 \geq 0.98$ and a spread of local slopes not exceeding 0.18. This excludes unstable regression from further interpretation.

Adaptive quadrilateral grid and generator-selection criterion. To move from a line to a 2D representation of the water area, the the *Lambert azimuthal equal-area (LAEA)* projection is applied, after which the required cell-size field is constructed $h(x)$. It depends on the distance to the coast, local contour curvature, and the depth gradient; the cell-size growth rate is limited across neighboring edges. This choice refines the grid near geometrically complex coastlines and in areas of sharp topographic change, without treating the grid itself as a hydrodynamic model.

The following generators were compared *Delaunay*, *Frontal-Delaunay for Quads* and *Packing of Parallelograms*, implemented in *Gmsh* 4.15.2 [11]. For each algorithm, all-quadrilateral grids of two levels were constructed from identical input data: detailed (h from 200 to 1000 m) and coarse (h from 500 to 1000 m). The same coastline boundary, GEBCO 2024 [12] bathymetry and the field $h(x)$, were used as input; therefore, the comparison does not confound the generator effect with changes in the source geometry.

The quality of coastal-feature preservation is defined by the total relative deviation of their areas:

$$\delta_A = 100 \frac{\sum_{j=7}^m |A_j^{mesh} - A_j^{ref}|}{\sum_{j=1}^m A_j^{ref}}, \quad (2.4)$$

where A_j^{ref} and A_j^{mesh} — the areas of the j -th spit, bay, island, or another selected coastal feature for the source boundary and the grid, respectively. Together with the *RMS*-deviation of the boundary e_b this quantity is converted into a coastline score:

$$C = \frac{1}{t + \delta_A + e_b / h_{\min}}. \quad (2.5)$$

The quantity $S = 0.15C + 0.35B + 0.2G + 0.2T + 0.1E$, where B — bathymetry-preservation score (*RMSE* of depth, volume, areas of depth zones, slopes, and the fraction of nearest replacements), G — quadrilateral quality, T — fraction of edges within the range $0.5 \leq \frac{1}{h} \leq 1.5$, E — efficiency in terms of runtime and peak memory. The approach to accounting for cell shape and degeneracy is consistent with algebraic mesh-quality metrics [13]. All weights are fixed in advance, ranking is performed by S , rather than manually.

For successful *MSH* files, additional checks were performed for the absence of triangles and degenerate cells, connectivity of the water area, correctness of physical boundary tags, and coincidence with the input contour. Equality of C for two successful generators at the same level is interpreted not as a lack of control, but as confirmation that both generators obtained the same discretized coastline boundary. Therefore, the difference in their final ranking is explained by the internal cell structure, bathymetry, compliance with $h(x)$ and computational resources.

Results.

Normalized contour and scale characteristic. The calculation included 9,635 points of the complete Black Sea contour. After normalization, its geodetic length was 6,398 km. The box-counting method over 13 scales yielded $D = 1.10840$, $R^2 = 0.9999$ and a local-slope spread of 0.1659. The stability criterion is satisfied. These values describe only the given resolution of the source geometry and are not interpreted as evidence of coincidence with the idealized Koch curve (Figs. 1–2).

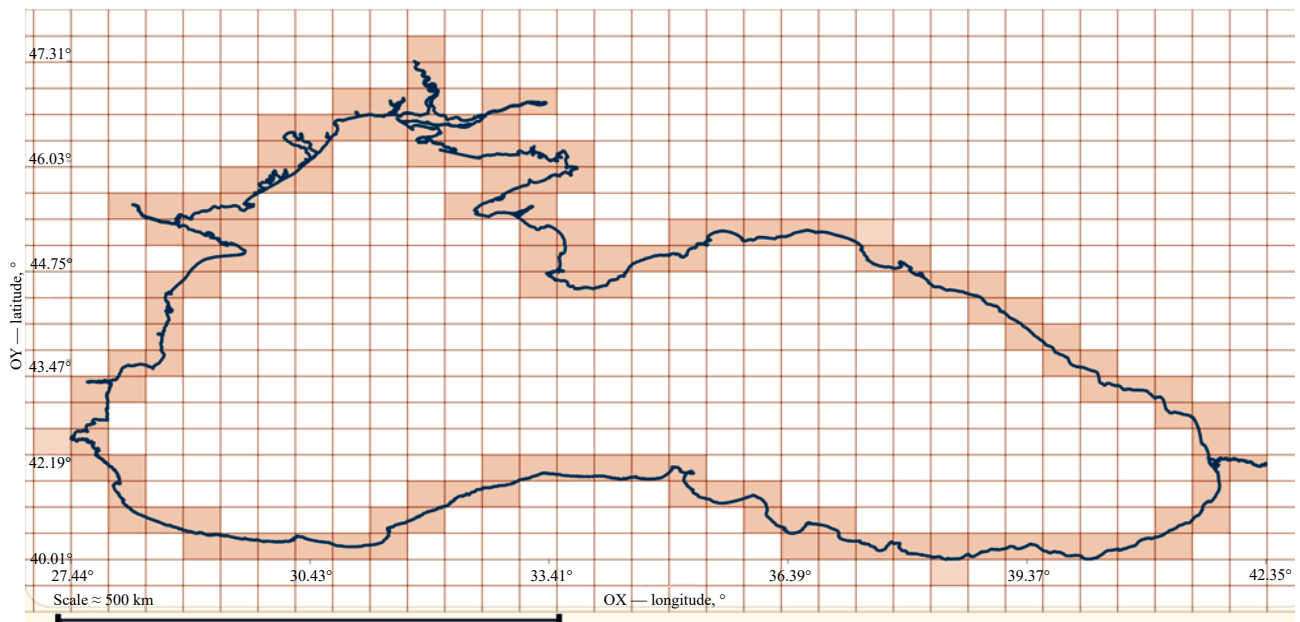


Fig. 1. Normalized Black Sea contour and box-counting coverage

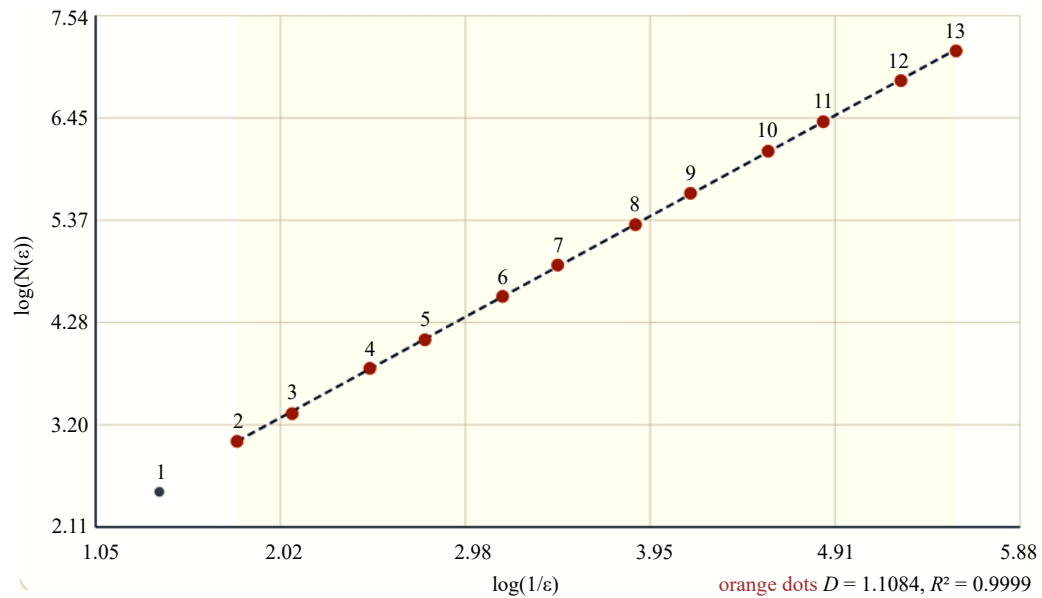


Fig. 2. Logarithmic regression of the box-counting method for the observed coastline

Comparison of adaptive quadrilateral grid generators. The bathymetric map provides the spatial context of the experiment: depths, isobaths, and coastline-shape complexity. It is not a navigational product but serves as the source model for evaluating topography preservation (Figs. 3–5).

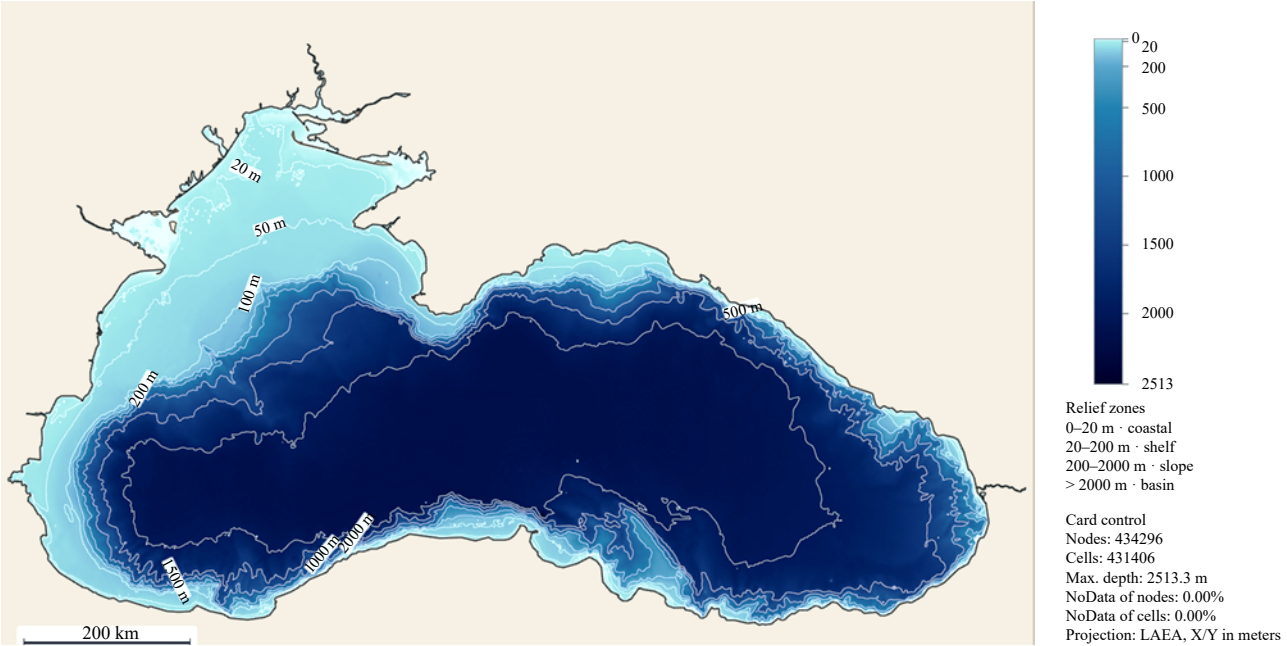


Fig. 3. Overview bathymetric map of the Black Sea and reference isobaths

The field provides refinement near a complex coastline and in zones of increased depth gradient; the final cells are generated by the selected generator.

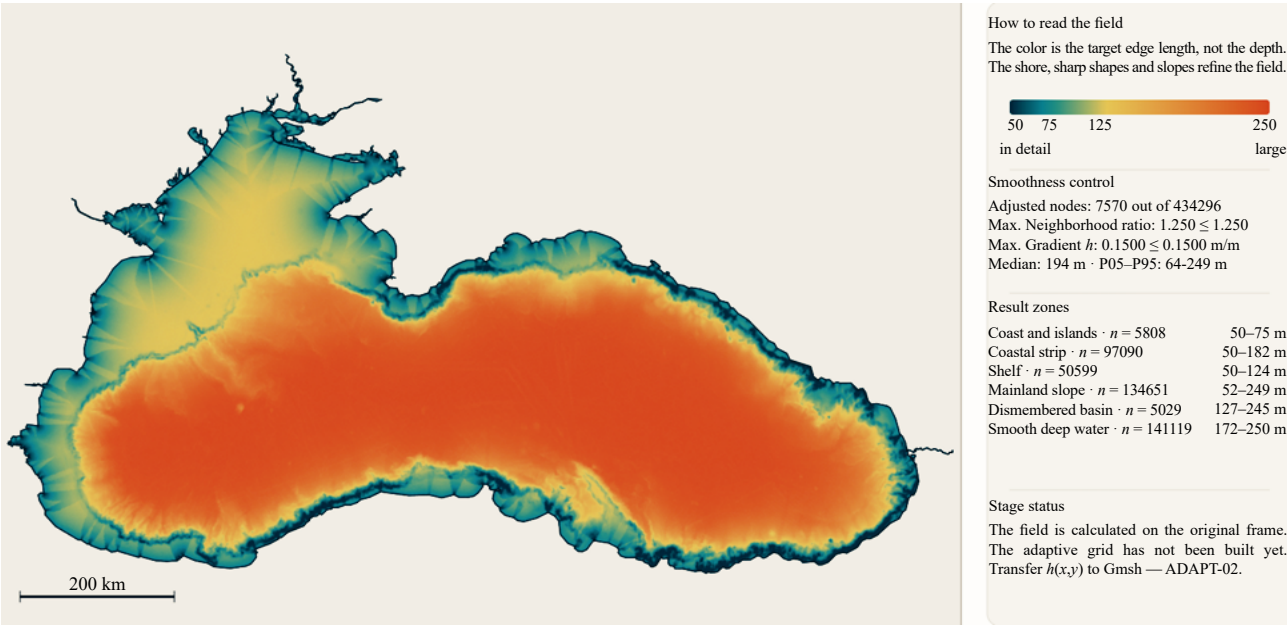


Fig. 4. Required cell-size field for the Black Sea

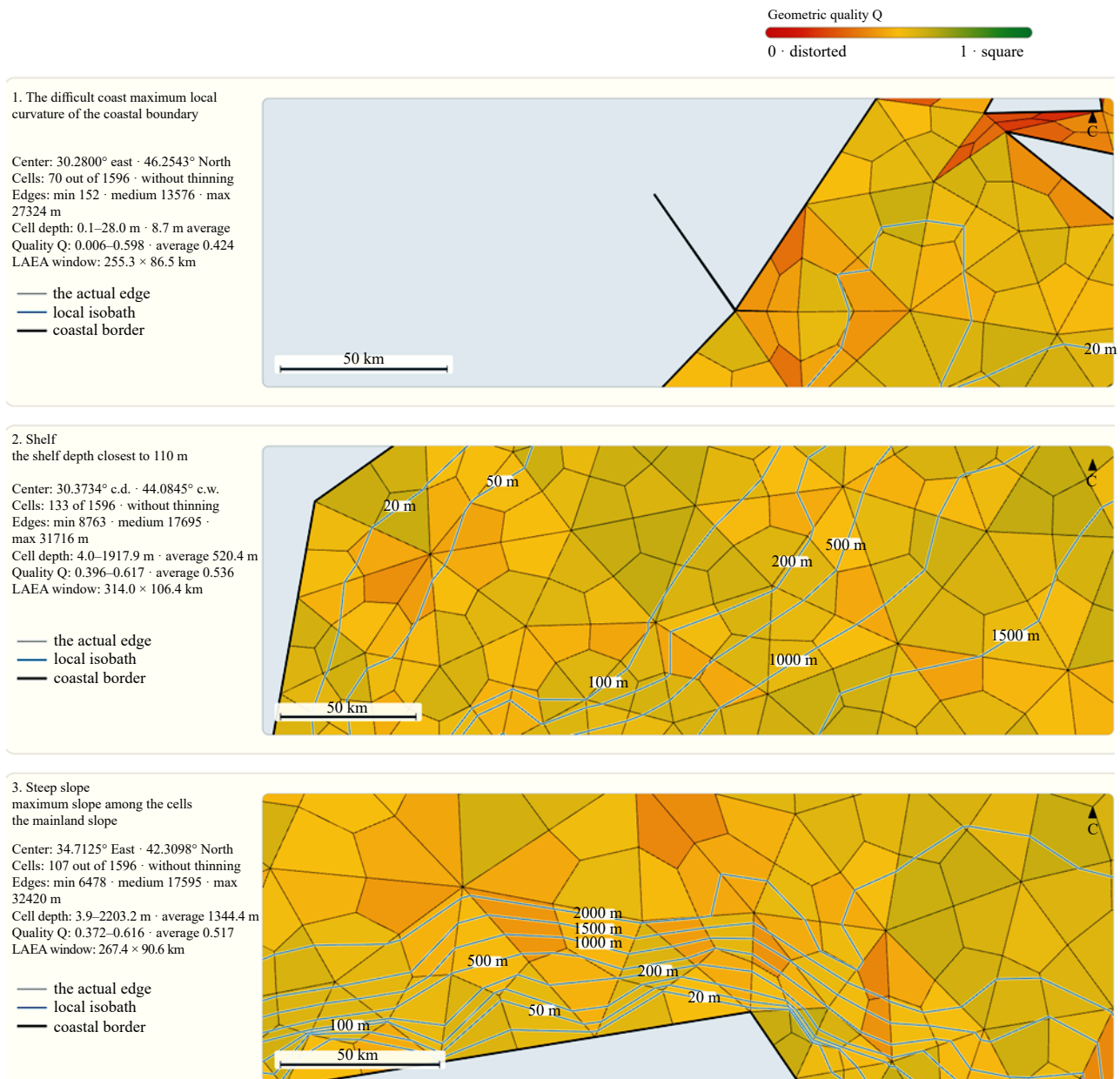


Fig. 5. Actual grid fragments and isobaths: complex coastline, shelf, and steep slope

The table contains the results of two independent reproducible runs. All successful variants contain 100 % quadrilaterals, the accepted topology, and machine-level coincidence of the boundary $e_b \approx 3 \cdot 10^{-7}$ m. Therefore, within a given level, the generators preserve the specified input coastline equally well. Difference in between levels characterizes the influence of the detail level and discretization of the source boundary (Table 1).

Table 1

Quantitative comparison of generators at two levels of the adaptive grid

Level	Generator	Cells	δ_A , %	Depth RMSE, m	Q_{cp}	Edges within tolerance, %	Time, s	S
50–250	<i>Delaunay</i>	3 041 643	195.46	0.333	0.537	93.97	47.4	71.50
50–250	<i>Frontal-Delaunay for Quads</i>	2 452 929	195.46	0.376	0.548	92.06	41.2	70.54
125–250	<i>Delaunay</i>	1 191 504	78.18	0.638	0.536	94.94	18.1	72.51
125–250	<i>Frontal-Delaunay for Quads</i>	967 734	78.18	0.727	0.547	93.26	16.6	71.33

At the detailed level, *Delaunay* outperforms *Frontal-Delaunay for Quads* by 0.96 points. Its advantage is due to a lower depth RMSE (0.333 versus 0.376 m), a larger fraction of edges within tolerance, and lower peak RSS (381.6 versus 551.9 MiB); *Frontal-Delaunay* is slightly faster and has a higher mean cell-quality value. At the coarse level, *Delaunay* retains first place with a margin of 1.18 points: the depth RMSE is lower by 0.090 m, and the fraction of edges within tolerance is higher by 1.68 percentage points; *Frontal-Delaunay* is faster by 1.5 s and has a slightly higher mean cell-quality value.

For a clear visual comparison, the two successful generators are shown for the same Kızılırmak Spit at the recommended 125–250 m level (Figs. 6 and 7).

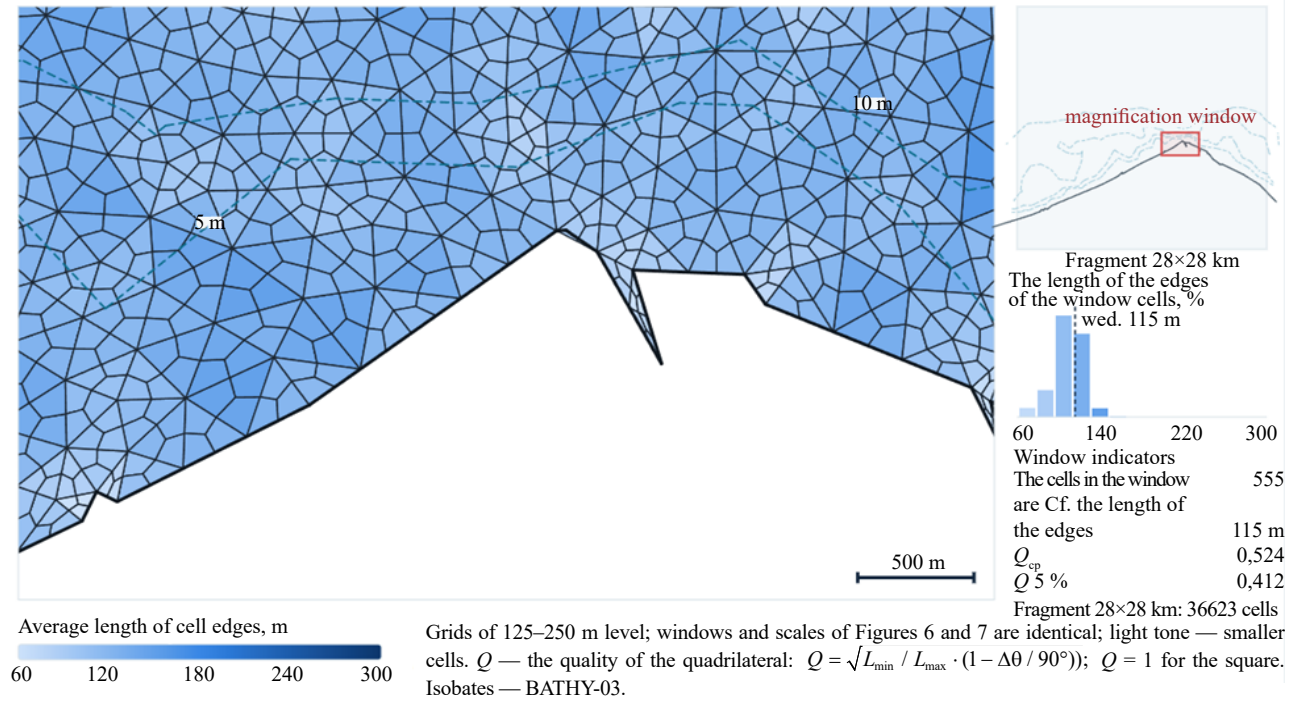


Fig. 6. Kızılırmak Spit: Delaunay, recommended 125–250 m level

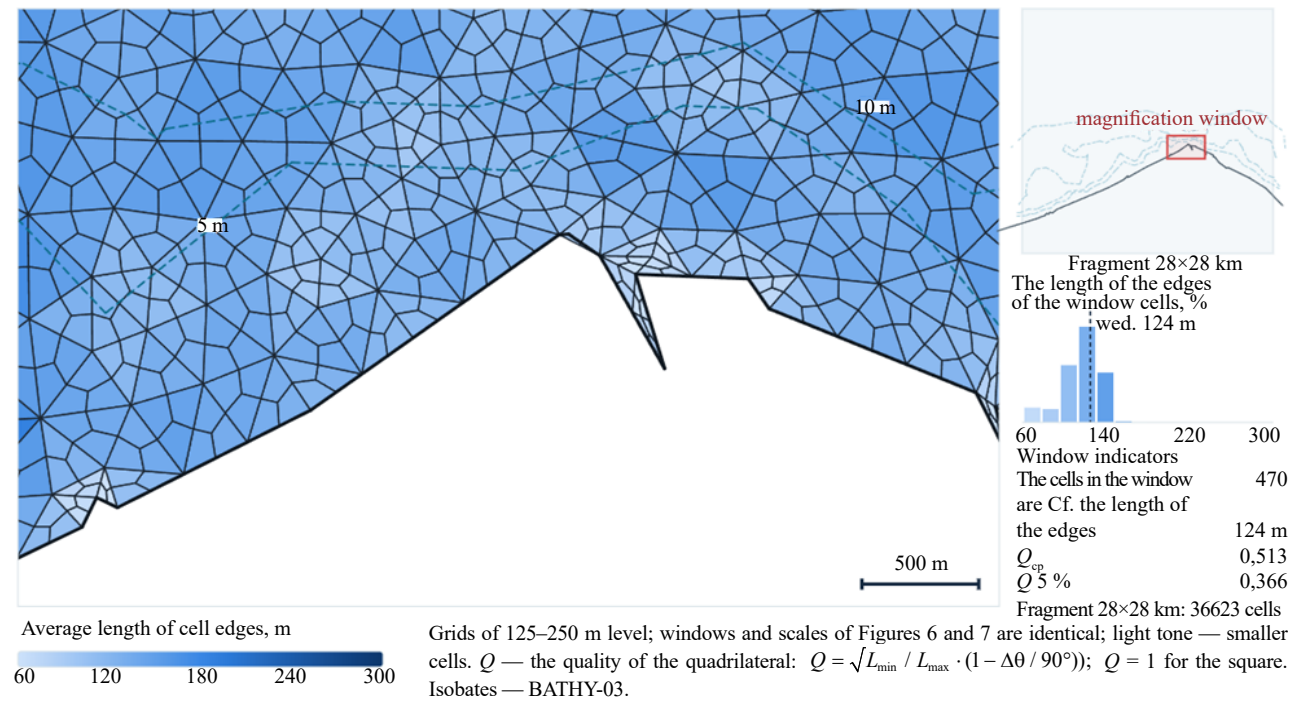


Fig. 7. Kızılırmak Spit: Frontal-Delaunay for Quads, 125–250 m level

Packing of Parallelograms is not included in the table of successful grids: in the adopted validation run, it exceeded the 10 min limit at both levels and did not produce an *MSH* file suitable for topological and bathymetric verification. Therefore, this algorithm cannot be recommended for the formulation considered here; the absence of a result is not replaced by a zero or calculated score.

Discussion. Based on the computational experiment, it can be concluded, that the choice of a grid generator should depend on the intended purpose of the model. To preserve significant coastal forms at the scale of the entire water area according to the specified criterion δ_A it is recommended to use *Delaunay* with a range of 125–250 m: it provides the smallest total area deviation (78.18%), the highest final score (72.51), and almost 2.6 times fewer cells than the detailed *Delaunay*. If local refinement and minimum depth-reconstruction error are required, *Delaunay* 50–250 m should be used, accepting an increase in the number of cells to 3.04 million and a poorer value of this area-based criterion. This is not a contradiction but an explicitly observed trade-off between the morphological criterion, bathymetry, and computational cost. The scientific novelty of the work lies in a unified reproducible workflow: geometry normalization, geodetic and fractal characterization, construction of an adaptive size field, and quantitative generator selection based on the preservation of coastal features. The practical result is a grid-selection rule for preparing seabed-topography problems and subsequent applied calculations without subjective ranking by visual appearance. This conclusion applies to the fixed Black Sea contour, bathymetry, cell-size ranges, and criterion weights. The fractal dimension characterizes only the working scale range, while the final generator score cannot be transferred to another water area without recalculation. Coincidence of the input boundary for successful generators means that, within one level, they represent the specified coastline equally well; the advantage of *Delaunay* is due to the combined bathymetric, size, and resource metrics.

Conclusion. A mathematical and algorithmic framework has been proposed and substantiated for analyzing a coastline as a natural geometrically irregular object and for selecting an adaptive quadrilateral grid for the Black Sea water area. It is shown that a correct study of such an object requires not only numerical computations but also preliminary geometry normalization, selection of an appropriate metric, consideration of scale dependence, and an explicit grid-quality criterion.

The computational experiment using Black Sea coastline data yielded a geodetic length of 6.398 km and a robust box-counting estimate of $D = 1.10840$ within the adopted range of scales. The result reflects the complexity of the observed digital contour specifically and is not identified with the idealized Koch curve.

Comparison of the two grid sets showed that the best generator according to the specified multicriteria metric is *Delaunay*: 71.50 points for the 50–250 m range and 72.51 points for the 125–250 m range. To preserve significant coastal features at the scale of the entire water area, *Delaunay* 125–250 m should be used: the total area deviation is 78.18% compared with 195.46% at the detailed level, while the number of cells is 2.6 times smaller. The detailed *Delaunay* 50–250 m configuration should be selected when accurate seabed-topography reconstruction is the primary objective.

The scientific novelty of the study lies in combining geometric validation, geodetic measurement, fractal analysis, and quantitative generator selection within a unified reproducible framework. The practical significance lies in the possibility of preparing verified contours and grids for seabed-topography modelling, geoinformation analysis, and subsequent applied calculations without replacing quantitative selection by subjective visual assessment.

Further research will focus on expanding the class of coastal systems under analysis, comparing different methods for estimating fractal dimension, investigating the influence of the spatial resolution of source geodata on the robustness of calculated metrics, and adapting the approach to multiscale monitoring of coastal dynamics.

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Comparative Analysis of the Effectiveness of Petroleum-Product Contamination Detection in Aquatic Systems Using Pre-Trained *YOLO*-Family Neural Network Models and the *SMDNetV4* and *PSPNet* Architectures

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Abstract

Introduction. The PSPNet, SMD-Net, and YOLO26-Seg semantic segmentation architectures are considered for the task of detecting oil spills in satellite imagery. Particular attention is paid to comparing segmentation accuracy under identical training conditions.

Materials and Methods. PSPNet uses a ResNet101 backbone with a pyramid pooling module for semantic segmentation. SMD-Net is a lightweight shared-backbone architecture with two decoders and CTGM, CoordAtt, and ASPP modules. YOLO26-Seg combines a YOLO26 encoder backbone with fixed weights and a lightweight decoder. The following metrics are compared: *IoU* (Intersection over Union), defined as the ratio of the intersection of the predicted and ground-truth regions to their union; *Dice* (Dice coefficient), a measure of segment overlap based on the intersection area; accuracy, the proportion of correct predictions among all predictions; inference time, the time required by the model to process one input; and the number of trainable parameters.

Results. After 50 epochs of training on the existing dataset masks without manual annotation, PSPNet significantly outperformed SMD-Net in binary oil-spill segmentation (*IoU* = 0.704, *Dice* = 0.770, *Acc* = 0.896 versus *IoU* = 0.467, *Dice* = 0.575, *Acc* = 0.671 for SMD-Net). The additionally fine-tuned YOLO26l-seg model demonstrated moderate pixel-level metrics (*IoU* = 0.320) at a reduced detection threshold, which limits its applicability to this task.

Discussion. The choice of architecture for oil-spill segmentation is governed by a trade-off between accuracy and speed: PSPNet provides the highest segmentation quality, SMD-Net performs worse across all metrics, and YOLO26-SemSeg cannot reliably detect oil spills without task-specific training.

Conclusion. PSPNet significantly outperforms SMD-Net and YOLO26-Seg in binary oil-spill segmentation under identical training conditions (*IoU* = 0.704 versus 0.467 for SMD-Net), confirming the effectiveness of the ResNet101 backbone combined with pyramid pooling.

Keywords: oil-spill segmentation, semantic segmentation, deep learning, computer vision, comparison of neural network architectures

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Сравнительный анализ эффективности обнаружения загрязнений нефтепродуктами водных систем на основе предобученных нейросетевых моделей семейства YOLO и архитектур SMDNetV4 и PSPNet

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Аннотация

Введение. Рассматриваются архитектуры семантической сегментации *PSPNet*, *SMD-Net* и *YOLO26-Seg* для задачи обнаружения нефтяных разливов на спутниковых снимках. Особое внимание уделено сравнению точности сегментации при идентичных условиях обучения.

Материалы и методы. *PSPNet* использует архитектуру *ResNet101* с модулем пирамидального пулинга для семантической сегментации. *SMD-Net* — лёгкая архитектура общей магистрали с двумя декодировщиками и модулями *CTGM*, *CoordAtt*, *ASPP*. *YOLO26-Seg* сочетает кодирующую магистраль *YOLO26* с фиксированными весами и лёгкий декодировщик. Проведён сравнительный анализ метрик: *IoU* (пересечение над объединением) — отношение пересечения предсказанной и истинной областей к их объединению; *Dice* (коэффициент Дайса) — мера перекрытия сегментов, вычисляется на основе площади пересечения; *accuracy* (точность) — доля верных предсказаний среди общего их числа; время вывода — время, потраченное моделью на обработку одного входа.

Результаты исследования. *PSPNet* после обучения на пятидесяти эпохах по существующим маскам набора данных (без ручной разметки) значительно превзошёл *SMD-Net* по бинарной сегментации нефти (*IoU* = 0,704, *Dice* = 0,770, *Acc* = 0,896) против *SMD-Net* (*IoU* = 0,467, *Dice* = 0,575, *Acc* = 0,671). Дополнительно обученная *YOLO26l-seg* показала умеренные пиксельные метрики (*IoU* = 0,320) при сниженном пороге детекции, что делает её ограниченно применимой для данной задачи.

Обсуждение. Выбор архитектуры для сегментации нефтяных разливов определяется компромиссом между точностью и скоростью: *PSPNet* обеспечивает наилучшее качество, *SMD-Net* проигрывает по всем метрикам, а *YOLO26-SemSeg* не способен определять разливы нефтяных пятен без обучения.

Заключение. *PSPNet* значительно превосходит *SMD-Net* и *YOLO26-Seg* по бинарной сегментации нефтяных разливов при одинаковых условиях обучения (*IoU* = 0,704 против 0,467), что подтверждает эффективность архитектуры *ResNet101* с пирамидальным пулингом.

Ключевые слова: сегментирование нефтяных пятен, сегментирование, глубокое обучение, компьютерное зрение, сравнение архитектур нейросетей

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Introduction. Semantic segmentation, which involves pixel-wise image classification, is one of the fundamental tasks of computer vision and is widely used in various fields, including medical diagnostics and the analysis of Earth remote sensing data [1–3]. This task is particularly important for detecting oil spills on water surfaces, because accurate localization of slick boundaries is essential for quantitatively assessing the extent of contamination [4, 5].

Recent studies have proposed a range of semantic segmentation architectures that differ in their approaches to contextual information extraction and feature aggregation, among which *PSPNet* and *SMD-Net* are of particular interest. Comparative analysis of different neural-network approaches remains relevant because the choice of architecture is determined by a trade-off between accuracy and computational complexity [6, 7]. *PSPNet* is based on a deep *ResNet101* backbone with a pyramid pooling module that aggregates global context at multiple scales. *SMD-Net* has lower computational complexity and includes a shared backbone with two decoders as well as cross-task interaction modules. In addition, this study considers the *YOLO26l-seg* architecture, which performs polygon-based segmentation using a *YOLO* detector and is fine-tuned on the same dataset [8–12]. Hybrid approaches combining local binary patterns and neural networks are also used to detect objects on the sea surface.

The aim of this study is to comparatively evaluate *PSPNet*, *SMD-Net*, and *YOLO26l-seg* in terms of oil-spill segmentation accuracy, computational performance, and generalization capability. The relevance of the study is determined by the need to improve the reliability of contamination-extent assessment for environmental modeling of marine systems. To achieve this aim, the architectural features of the models are analyzed, an experimental evaluation is performed on a satellite-image dataset of oil spills, the results are quantitatively compared using *IoU*, *Dice*, accuracy, inference time, and number of parameters, and the most effective architecture for binary oil-spill segmentation is identified.

Materials and Methods. An open dataset of satellite images of oil spills was used, containing 2,074 images with corresponding segmentation masks. The masks were provided in grayscale format; for binary classification, they were converted to “oil” (pixel value >127) and “background” (all other values) without additional manual correction. The original image resolution was 512×512 pixels; before being fed into the models, all images were resized to 256×256 pixels. The dataset was divided into a training set (2,000 images) and a test set (74 images). Data augmentation was not applied in order to ensure identical conditions for all models.

PSPNet is a semantic segmentation architecture based on *ResNet101* with a *Pyramid Pooling Module (PPM)*. The *PPM* performs adaptive average pooling at four levels (1×1, 2×2, 3×3, and 6×6), followed by bilinear interpolation and concatenation, thereby aggregating global context at different scales. The output layer is a 1×1 convolution with the number of channels equal to the number of classes (one class in this study for binary oil-spill segmentation). The model was trained for 50 epochs on 2,000 images using the existing dataset masks converted to binary form, without additional manual annotation. The input resolution was 256×256 pixels. The model architecture is shown in detail in Figs. 1 and 2.

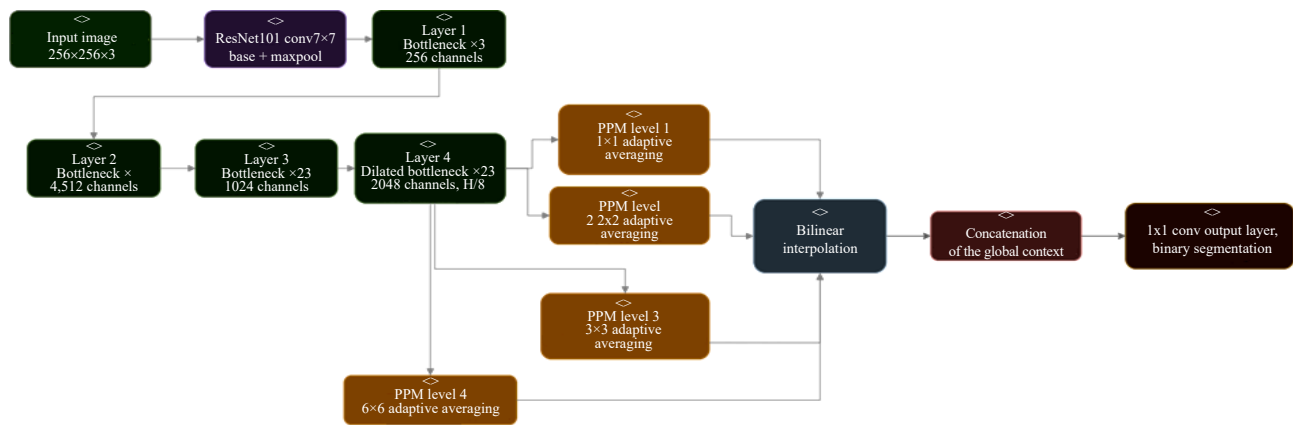


Fig. 1. *PSPNet* scheme

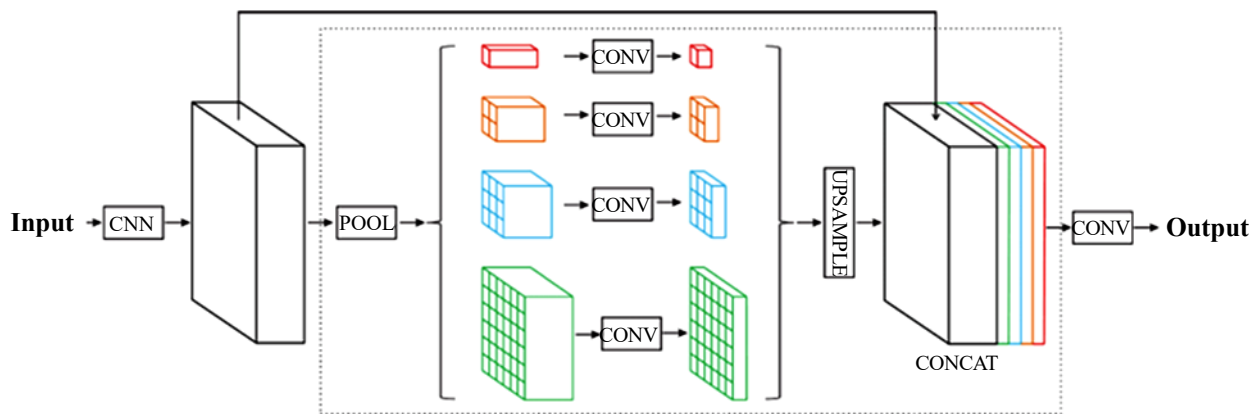


Fig. 2. *PSPNet* architecture

SMD-Net is a semantic segmentation architecture based on a shared backbone with two decoders [13]. It contains *CTGM* modules for cross-task feature exchange between decoders, *CoordAtt* for coordinate attention along the vertical and horizontal directions, *ECA* for efficient channel attention, and *ASPP* using convolutions with different dilation rates to capture multi-scale context. The model was trained for binary oil-spill segmentation (one class) for 50 epochs on 2,000 images using *AMP* (automatic mixed precision). The input resolution was 256×256 pixels.

The following metrics were used to quantitatively evaluate segmentation quality:

- *IoU* (Intersection over Union): the proportion of overlap between the predicted region and the ground-truth region:

$$IoU = TP / (TP + FP + FN); \quad (1)$$

- *Dice* (Sørensen–Dice coefficient): twice the overlap relative to the sum of the sizes of the predicted and ground-truth regions; equivalent to the *F1* score:

$$Dice = 2*TP / (2*TP + FP + FN); \quad (2)$$

- *Accuracy* (pixel accuracy): the proportion of pixels correctly classified as background or object:

$$Acc = (TP + TN) / (TP + TN + FP + FN). \quad (3)$$

The following hyperparameters were used for *PSPNet* and *SMD-Net*: *AdamW* optimizer; initial learning rate of $1E-3$ with cosine decay to $1E-5$; a loss function combining binary cross-entropy and the *Dice* coefficient (*BCE+Dice*); batch size of 16; and 50 training epochs. Automatic mixed precision (AMP) was used to accelerate computations. All experiments were performed on an *NVIDIA A100 GPU* (40 GB) using the *PyTorch 2.0* framework. For *YOLO26l-seg*, the standard settings from the Ultralytics repository were used, with fine-tuning for 50 epochs on the same dataset.

PSPNet and *SMD-Net* were trained at a resolution of 256×256 pixels using the *BCE+Dice* loss, the *AdamW* optimizer, cosine learning-rate decay, and AMP.

Inference time was measured over 100 runs on a single image after warm-up. The *SMD-Net* model architecture is illustrated in Fig. 3.

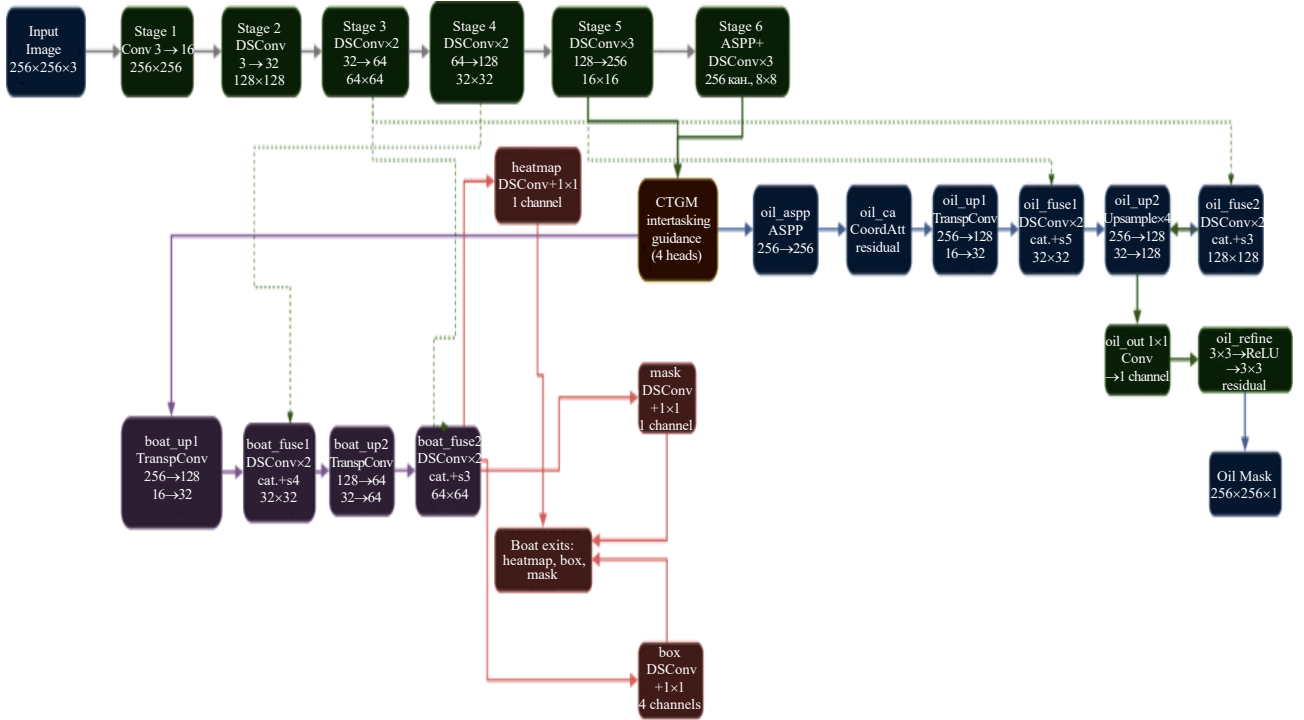


Fig. 3. *SMD-Net* architecture

YOLO26l-seg is an instance-segmentation architecture based on the *YOLO26* detector. Unlike full semantic segmentation models (*PSPNet* and *SMD-Net*), *YOLO26l-seg* identifies the contours of individual objects using polygons rather than pixel-wise masks.

The *YOLO26l-seg* architecture comprises three main components:

1. Backbone — a modified *CSPDarknet* with *C2f* blocks that provides multi-level feature extraction (*P3*, *P4*, and *P5*) at resolutions of 80×80 , 40×40 , and 20×20 , respectively. The backbone was pre-trained on the COCO dataset.
2. Neck — *PAN-FPN* (Path Aggregation Network with Feature Pyramid Network), which aggregates features from different backbone levels to improve the detection of objects at different scales.
3. Head — a multi-task head with three branches: classification (cls), bounding-box regression (reg), and segmentation (seg). The segmentation branch generates mask prototypes and coefficients for their combination, forming a polygonal representation of the contours of detected objects.

At the standard confidence threshold ($conf = 0.25$), the model detects oil in only 23% of the test images, resulting in low average metrics. When the threshold is reduced to $conf = 0.001$, detections are obtained in 86% of the images and the pixel-level metrics increase to $IoU = 0.320$, $Dice = 0.414$, and $Accuracy = 0.541$. This indicates that *YOLO* can identify oil slicks but performs worse than full semantic segmentation architectures because of the polygonal mask representation and unstable detection across heterogeneous scenes [14]. The *YOLO26-Seg* architecture is shown in Fig. 4.

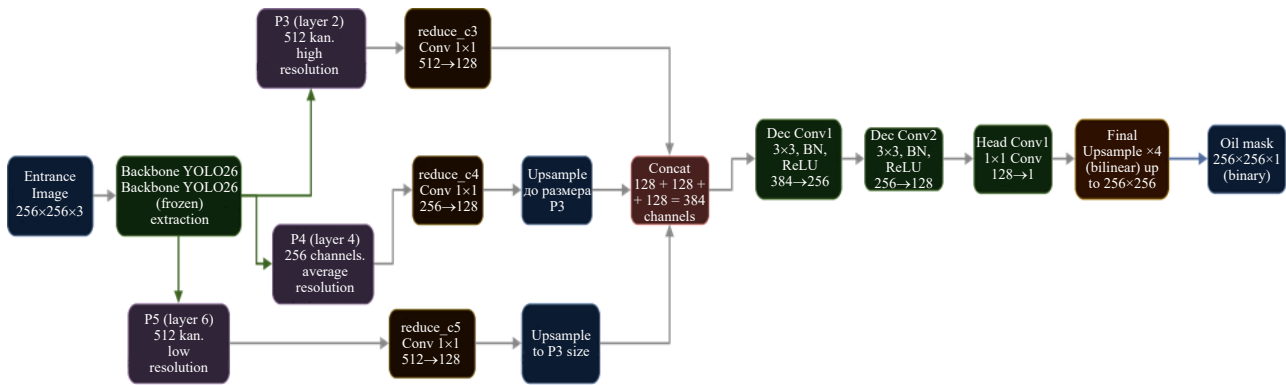


Fig. 4. *YOLO26-Seg* model architecture

Results. The quantitative results of the comparative analysis are presented in Table 1.

Table 1

Comparative metrics of semantic segmentation architectures

Model	Parameters	Time, ms	IoU	$Dice$	$Accuracy$
PSPNet	57.4M	25.4	0.704	0.770	0.896
SMD-Net	5.8M	16.9	0.467	0.575	0.671
YOLO26l-seg ft	31.4M	35.0	0.320	0.414	0.541

All models were trained on the same dataset using binary oil/background labels derived from the existing dataset masks, without manual annotation [15]. *PSPNet* and *SMD-Net* were trained under identical conditions (50 epochs, 2,000 images, 256×256 resolution). *PSPNet* substantially outperformed *SMD-Net*: $IoU = 0.704$ versus 0.467 (+50.7%), $Dice = 0.770$ versus 0.575 (+33.9%), and $Accuracy = 0.896$ versus 0.671 (+33.5%). *YOLO26l-seg* achieved an IoU of 0.320 , lower than *PSPNet*, even with the reduced confidence threshold [5].

Segmentation Results. Representative outputs of the models are shown in Figs. 5–7.

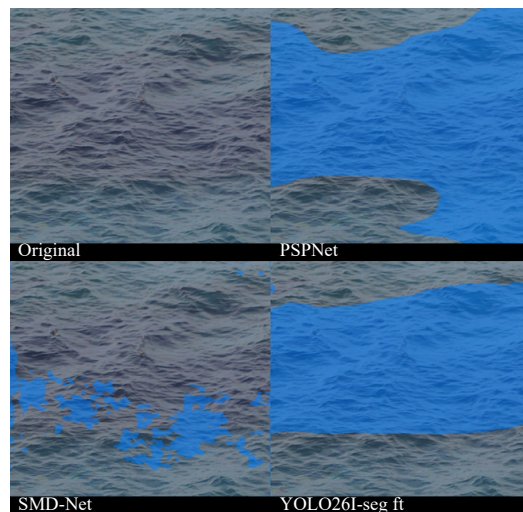


Fig. 5. Model output 1

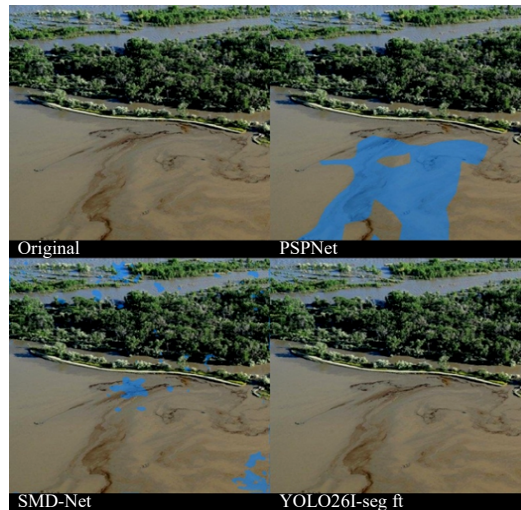


Fig. 6. Model output 2

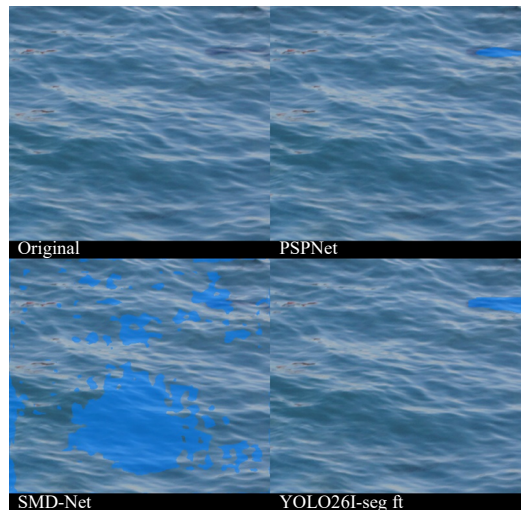


Fig. 7. Model output 3

The substantial advantage of *PSPNet* ($IoU = 0.704$ versus 0.467 for *SMD-Net*) can be attributed to the use of the deep *ResNet101* backbone and the Pyramid Pooling Module (PPM). PPM aggregates contextual information at four scales, which is particularly important for segmenting semi-transparent objects with diffuse boundaries, such as oil films on the water surface. Despite incorporating attention modules (*CoordAtt* and *ECA*) and *ASPP*, *SMD-Net* performs worse due to the lower capacity of its backbone, which leads to a higher rate of false positives in areas affected by glare and surface waves.

The higher inference speed (16.9 ms versus 25.4 ms for *PSPNet*) and smaller number of parameters (5.8 million versus 57.4 million) make *SMD-Net* attractive for mobile systems. However, in environmental monitoring applications, where misclassification can have substantial consequences, the 33.9% loss in *Dice* score is not offset by the gain in computational performance.

The *YOLO26l-seg* model was originally designed for detecting and polygon-based instance segmentation of individual objects. For the continuous binary “oil/background” mask task, its polygonal representation is redundant and results in loss of fine details. Reducing the confidence threshold to 0.001 is effectively a tuning artifact that increases recall but sharply increases the number of false positives, as reflected by the low Accuracy value of 0.541 . In addition, *YOLO* inference time (35 ms) is longer than that of *PSPNet*, further reducing its competitiveness for this task.

Discussion. The choice of architecture for oil-spill segmentation is determined by a trade-off between accuracy and speed: *PSPNet* provides the highest segmentation quality, *SMD-Net* performs worse across all metrics, and *YOLO26l-SemSeg* cannot reliably detect oil spills without task-specific training.

Conclusion. A comparative experimental analysis of three semantic segmentation architectures (*PSPNet*, *SMD-Net*, and *YOLO26l-seg*) was performed for oil-spill detection in satellite imagery. All models were trained under identical conditions on the same dataset using automatically generated binary masks [16–17].

The results show that *PSPNet*, based on *ResNet101* and pyramid pooling, provides the highest binary segmentation accuracy and significantly outperforms *SMD-Net* and *YOLO26l-seg* across all pixel-level metrics. Despite its high processing speed, *SMD-Net* exhibits a false-positive rate that is unacceptable for environmental applications. *YOLO26l-seg*, even after fine-tuning, is poorly suited to pixel-wise classification due to its polygon-based representation.

Thus, the choice of architecture depends on the required segmentation accuracy and inference speed. For binary oil-spill segmentation, *PSPNet* provides the strongest performance and substantially outperforms *SMD-Net* across all evaluated metrics.

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Original Empirical Research

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An Index-Based Algorithm for Detecting Water Soil Erosion Indicators from *Sentinel-2* Multispectral Data and Its Verification on Field Data of Rostov Oblast and Krasnodar Krai

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Abstract

Introduction. The paper presents the development and testing of an index-based algorithm for detecting indicators of soil erosion by water from *Sentinel-2* remote sensing data, considered as the first practical stage in constructing a hybrid physics-based mass-transport model with neural-network parameter identification.

Materials and Methods. The algorithm relies on joint analysis of time series of the Normalized Difference Vegetation Index (NDVI) and the Bare Soil Index (BSI), followed by spatial change detection (ΔBSI) with threshold filtering at the 90th percentile. The algorithm was first validated on synthetic data and then applied to real *Sentinel-2* imagery for two test sites – the Kamensky District of Rostov Oblast and the vicinity of Moldovanskoye village, Krymsky District, Krasnodar Krai – covering 10 dates within 2024.

Results. The algorithm is shown to consistently identify spatially localised zones of increasing bare-soil exposure.

Discussion. A key limitation – the inability to distinguish erosion-related exposure from ordinary agrotechnical bare soil – is discussed, along with directions for resolving it (digital elevation model overlay, multi-year analysis).

Conclusion. An index-based algorithm for detecting indicators of water-induced soil erosion from *Sentinel-2* multispectral time series was developed and validated. Verification on synthetic data and application to two real test areas in southern Russia confirmed its operability while demonstrating the need to separate erosion-related changes from agricultural soil exposure in subsequent model development.

Keywords: remote sensing, *Sentinel-2*, water soil erosion, *NDVI* vegetation index, *BSI* bare soil index, change detection, mathematical modelling, neural network parameter identification

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Оригинальное эмпирическое исследование

Индексный алгоритм детекции признаков водной эрозии почв по мультиспектральным данным *Sentinel-2* и его верификация на натурных данных территорий Ростовской области и Краснодарского края

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Аннотация

Введение. Работа посвящена разработке и валидации индексного алгоритма выявления признаков водной эрозии почв по данным дистанционного зондирования Земли (ДЗЗ) *Sentinel-2*, рассматриваемого как первый

практический этап построения гибридной физико-математической модели переноса массы с нейросетевой параметрической идентификацией.

Материалы и методы. Алгоритм основан на совместном анализе временных рядов вегетационного индекса *NDVI* и индекса оголенности почвы *BSI* с последующей детекцией пространственных изменений (ΔBSI) с пороговой фильтрацией по 90-му процентилю. Работоспособность алгоритма проверена на модельных (синтетических) данных, после чего проведена верификация на натурных данных — реальных снимках *Sentinel-2* для двух тестовых территорий — Каменского района Ростовской области и окрестностей с. Молдаванское Крымского района Краснодарского края — с временным охватом 10 дат за 2024 год.

Результаты исследования. Авторами установлено, что алгоритм устойчиво выделяет пространственно-локализованные зоны роста оголенности почвы.

Обсуждение. Определено ограничение метода, связанное с неразличимостью эрозионного и агротехнического происхождения оголения, а также намечены пути его устранения (привязка к цифровой модели рельефа, многолетний анализ).

Заключение. Разработан и валидирован индексный алгоритм детекции признаков водной эрозии почв по временным рядам мультиспектральных снимков *Sentinel-2*, основанный на совместном анализе индексов *NDVI* и *BSI* с пространственной детекцией изменений. Алгоритм верифицирован на модельных данных и применен к реальным снимкам для двух тестовых территорий юга России, что подтвердило его работоспособность при одновременном выявлении принципиального ограничения — неразличимости эрозионного и агротехнического происхождения оголения почвы в пределах одного сезона наблюдений.

Ключевые слова: дистанционное зондирование Земли, *Sentinel-2*, водная эрозия почв, вегетационный индекс *NDVI*, индекс оголенности почвы *BSI*, детекция изменений, математическое моделирование, нейросетевая идентификация параметров

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Introduction. Water erosion of soils (sheet and gully erosion) remains one of the most significant threats to the agricultural potential of chernozem areas in southern Russia. Traditional methods of soil-erosion mapping (large-scale field surveys and generalized-scale map schemes) do not provide the timeliness required to monitor process dynamics, whereas *Sentinel-2* Earth remote sensing data enable regular (every 5 days) acquisition of multispectral imagery at a spatial resolution of 10 m/pixel free of charge.

The objective of this study is formulated as the development of a hybrid model combining a physics-based mathematical description of mass transport with neural-network identification of the model parameters from remote sensing data. The present paper reports the results of the first stage: the development and validation of an index-based algorithm for extracting erosion indicators from *Sentinel-2* imagery, which will subsequently serve as a source of input data for a neural-network identifier of the parameters of the physical model.

An analysis of publications in computer science has shown that no previous studies have simultaneously combined a physics-based mass-transport model, neural-network parameter identification, and remote sensing data for water-induced soil erosion. Methodologically related studies are being pursued in three non-overlapping directions:

- sediment-transport models for coastal systems based on splitting schemes and computationally efficient parallel algorithms [1–3];
- neural-network methods for processing satellite imagery to monitor water bodies and coastal systems [4];
- the use of *RUSLE/USLE* and their modifications in combination with remote sensing and machine learning for erosion-risk assessment [5–8].

Spectral indices (primarily *NDVI* and *BSI*) are standard input layers for *RUSLE* + remote-sensing models [5, 6] and for hybrid physics-informed models in which a neural network identifies physical parameters from preprocessed, physically interpretable observation fields [9–12]. Therefore, constructing and verifying *NDVI/BSI* time series with a change-detection algorithm is a necessary preliminary stage before integrating a neural network into a physics-based mass-transport model. The corresponding model, provisionally based on a diffusion-convection-reaction equation for the suspended-matter concentration $c(x, y, z, t)$ and a solid-phase continuity equation for surface evolution (similar to [1, 2], adapted to hillslope runoff through the shallow-water equations), is considered the subject of subsequent stages of the study.

Materials and Methods.

1. Data Used. The study uses Sentinel-2 multispectral images at the *L2A* processing level obtained through the Copernicus Data Space Ecosystem. Five spectral bands with a spatial resolution of 10 m/pixel are used to calculate the indices:

Table 1

Sentinel-2 spectral bands used to calculate the indices

Band	Name	Wavelength
<i>B02</i>	Blue	~490 nm
<i>B03</i>	Green	~560 nm
<i>B04</i>	Red	~665 nm
<i>B08</i>	Near-infrared (<i>NIR</i>)	~842 nm
<i>B11</i>	Short-wave infrared (<i>SWIR</i>)	~1610 nm

2. Spectral Indices. The Normalized Difference Vegetation Index (*NDVI*) is calculated as

$$NDVI = (B08 - B04) / (B08 + B04). \quad (1)$$

NDVI ranges from -1 to $+1$; high values correspond to healthy vegetation because reflectance in the near-infrared range increases sharply relative to the red range owing to the cellular structure of leaves.

The *Bare Soil Index* (*BSI*) is calculated as

$$BSI = ((B11 + B04) - (B08 + B02)) / (B11 + B04) + (B08 + B02). \quad (2)$$

Positive *BSI* values are characteristic of bare soil (higher reflectance in the *SWIR* and red bands), whereas negative values are characteristic of vegetation and water bodies.

3. Temporal Analysis and Change Detection. To distinguish a persistent erosion signal from seasonal agricultural dynamics (fallow, stubble, tillage), indices (1)–(2) are calculated for a time series of N dates within a single growing season (in this study, $N = 10$, with an interval of approximately 5 weeks). A probable erosion indicator is not a single low *NDVI* value but its persistence throughout the season against the background of seasonal dynamics in the surrounding areas.

The spatial change in bare-soil exposure is estimated from the ΔBSI difference between the final and initial observation dates:

$$\Delta BSI = BSI(t_N) - BSI(t_1). \quad (3)$$

Before difference (3) is calculated, each *BSI* map is spatially smoothed using a Gaussian filter to suppress sensor pixel noise unrelated to actual spatial surface structures. Pixels for which the smoothed ΔBSI exceeds the 90th percentile of the distribution over the entire scene are classified as zones of probable active erosion expansion.

4. Generalized Algorithm.

1. Build a time series of N *Sentinel-2* images of the same site over the growing season.
2. Calculate *NDVI* (1) and *BSI* (2) for each date over all pixels in the scene.
3. Analyze the seasonal *NDVI* dynamics to distinguish persistently low values (potential erosion) from temporary decreases caused by agricultural practices.
4. Calculate the ΔBSI difference (3) between the final and initial dates.
5. Spatially smooth the ΔBSI map using a Gaussian filter.
6. Apply threshold classification: pixels with ΔBSI above the 90th percentile are identified as zones of probable erosion.

Results.

1. Algorithm Verification on Synthetic Data. Before application to real imagery, the algorithm was verified using a synthetically generated scene reproducing a field crossed by a winding ravine. Seasonal vegetation dynamics in the field and controlled expansion of a bare-soil zone along the ravine margins over the course of a year were prescribed. The results (Fig. 1) show that the final detection mask (step 6 of the algorithm) is correctly localized along the ravine margins, precisely where expansion of the bare-soil zone was specified, confirming the correctness of the algorithm implementation independently of the quality of the input data.

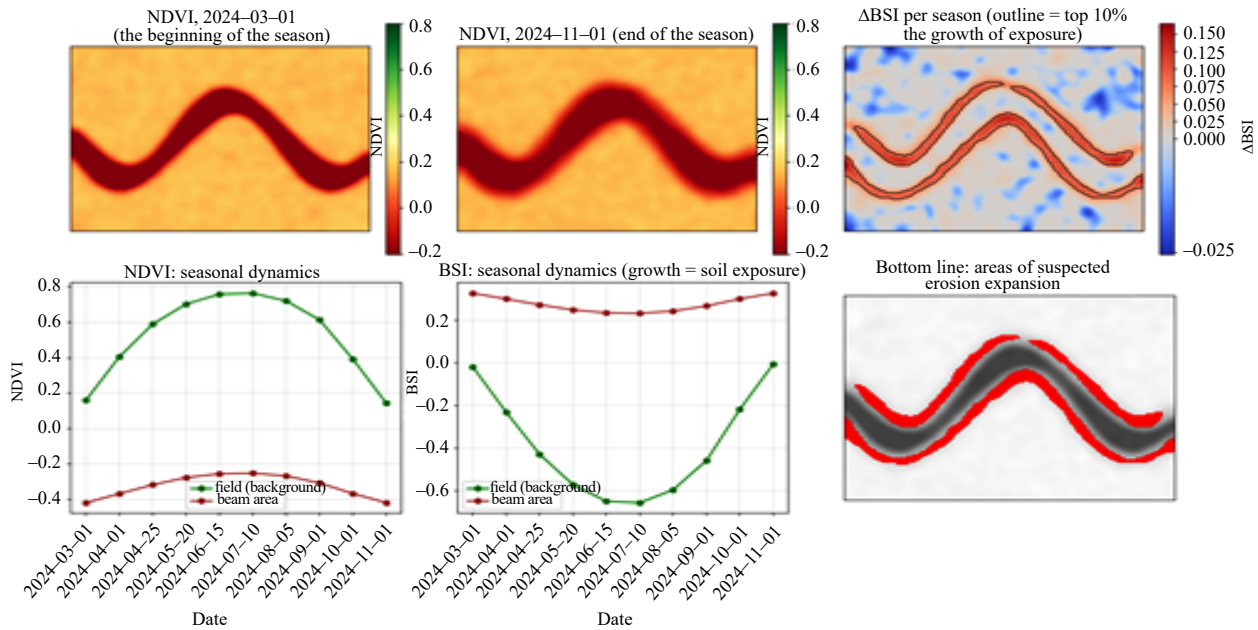


Fig. 1. Verification of the algorithm using synthetic data:
 $NDVI$ at the beginning and end of the season, ΔBSI map with detected changes, $NDVI/BSI$ time series for the ravine zone and background, and the final mask of erosion-growth zones

2. Verification on Field Data: Kamensky District, Rostov Oblast. The test site, approximately 10×8 km in area, is located in the Donetsk Ridge zone (48.28° N, 40.20° E), a typical erosion-prone area with a pronounced ravine-and-gully network. According to the calculations (Fig. 2), the scene-averaged $NDVI$ exhibits a profile typical of winter crops: an increase from 0.10 in March to a peak of 0.59 in late April, followed by a decrease to 0.13 in November (harvesting and subsequent soil cultivation). The ΔBSI map identifies compact clusters of increasing bare-soil exposure that spatially coincide with individual agricultural fields.

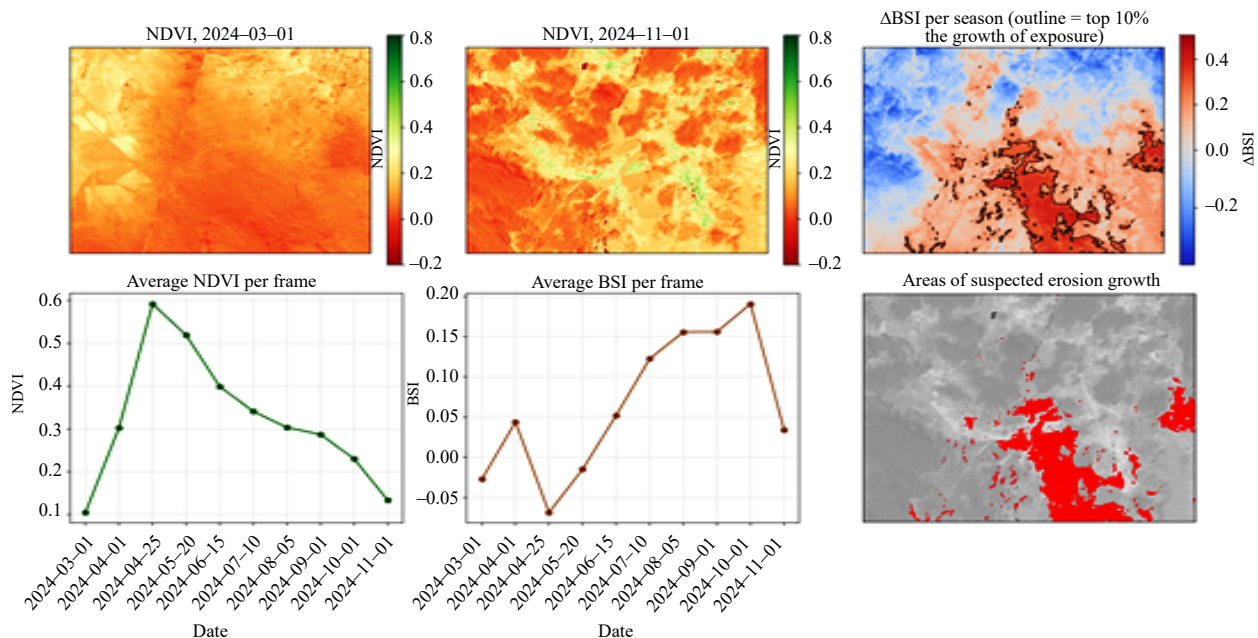


Fig. 2. Results of applying the algorithm to *Sentinel-2* data for Kamensky District, Rostov Oblast (2024)

The shape of the detected zones (compact patches within field boundaries rather than elongated structures aligned with terrain elements) indicates that most of the identified changes are likely related to agricultural operations (harvesting and tillage), with a possible contribution from active erosion; distinguishing these two causes from single-season data is limited.

3. Verification on Field Data: Moldovanskoye Village, Krymsky District, Krasnodar Krai. The second test site (44.9458° N, 37.8703° E; area approximately 10×8 km) was selected to compare algorithm performance in an area with a different land-use structure. The mean *NDVI* is higher and more stable throughout the season (peak 0.70, decreasing only to 0.47 by November), indicating the presence of perennial plantings alongside arable land. The ΔBSI map (Fig. 3) shows several large rectangular zones of increasing bare-soil exposure whose shapes precisely coincide with individual field boundaries; this even more clearly than at the first site indicates that the changes are predominantly agricultural in origin.

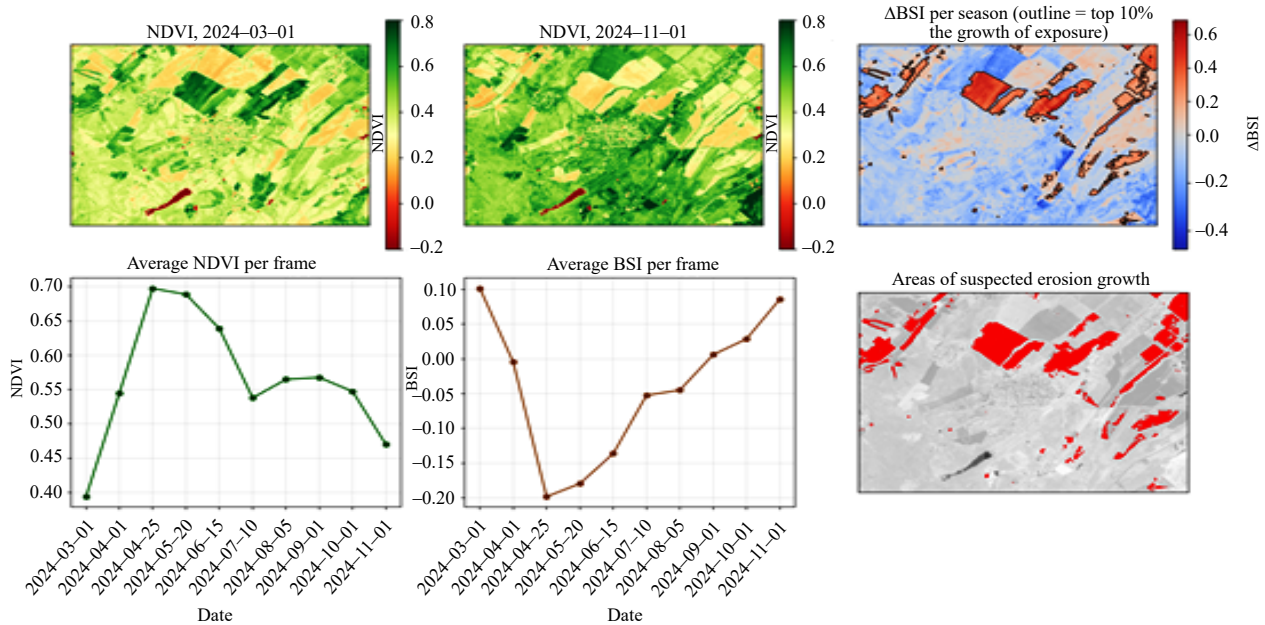
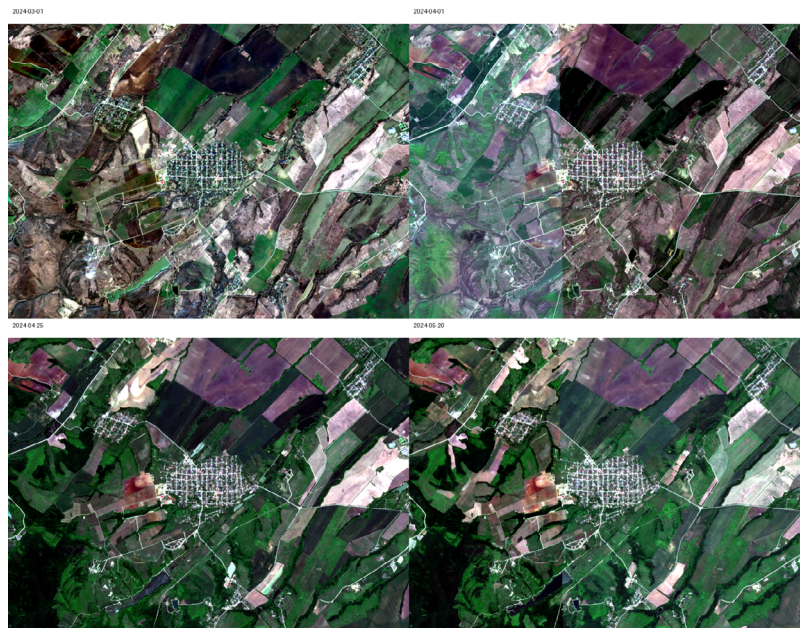


Fig. 3. Results of applying the algorithm to *Sentinel-2* data for the vicinity of Moldovanskoye village (2024)

4. Remote Sensing Data Illustrations. Figures 4 and 5 present the complete series of 10 *Sentinel-2* images used for 2024 for the Kamensky District test site in two color representations: true color ($R = B04$, $G = B03$, $B = B02$) and false-color near-infrared (*False Color NIR*, $R = B08$, $G = B04$, $B = B03$), in which vegetation is displayed in red and bare soil in dark or bluish tones. An analogous dataset was obtained for the test site near Moldovanskoye village and was used to construct Fig. 3.



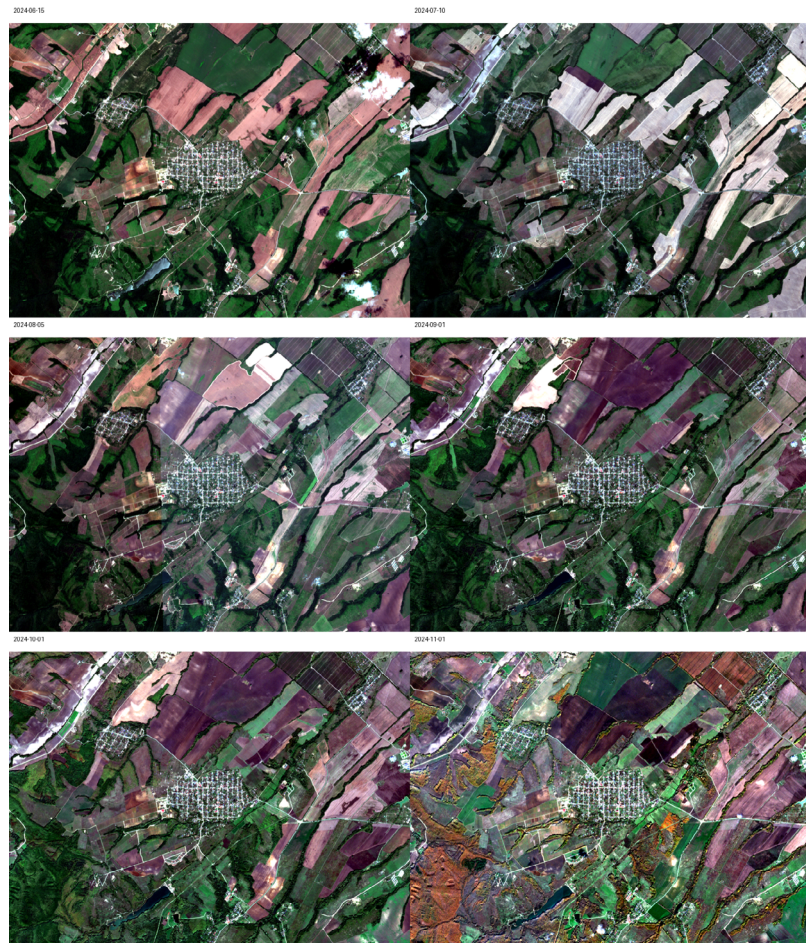
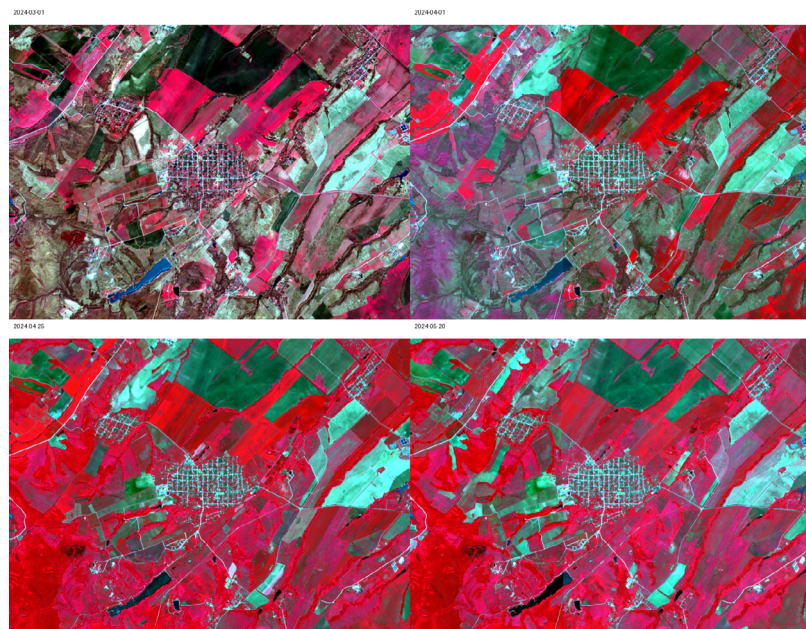


Fig. 4. *Sentinel-2* image time series for Kamensky District, 2024 (true color, 10 dates)



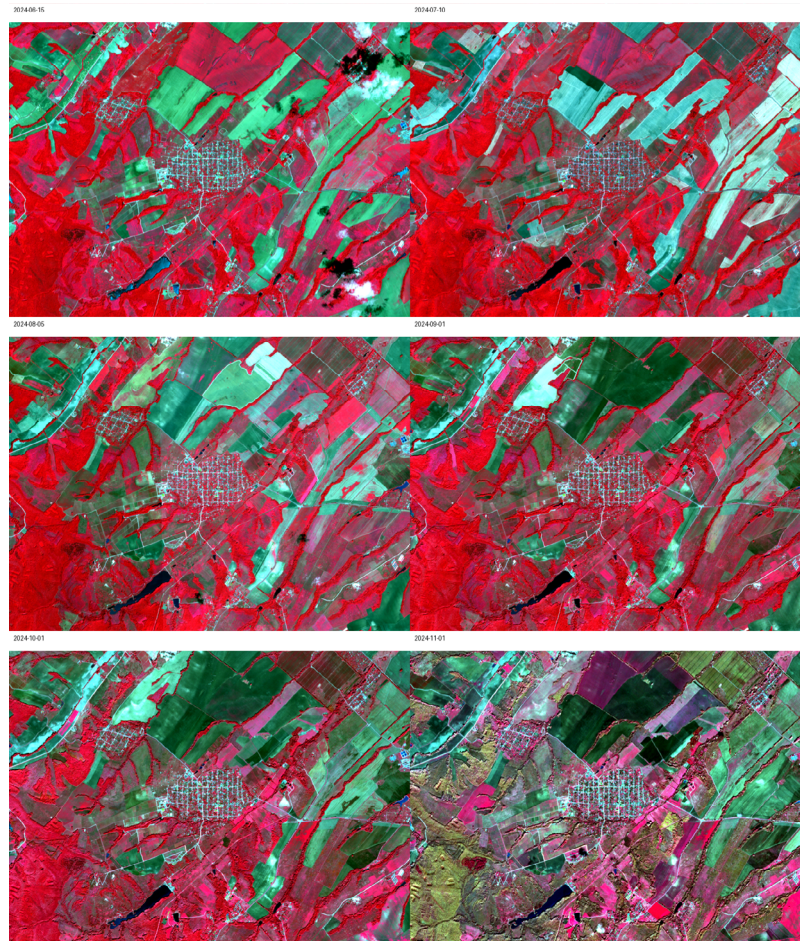


Fig. 5. *Sentinel-2* image time series for Kamensky District, 2024 (false-color infrared composite, 10 dates)

Discussion. Verification on field data from the two study areas confirmed the operability of the algorithm: in all cases, the detection procedure produces spatially localized, interpretable structures rather than random noise. At the same time, a fundamental limitation of the index-based approach was identified: within a single growing season, it is not possible to distinguish soil exposure caused by erosion from exposure caused by ordinary agricultural practices (harvesting, tillage, fallow), because both cases produce a similar increase in *BSI*. This limitation is expected to be addressed by:

- moving from a single season to a multi-year observation series (a persistent interannual increase in *BSI* is a stronger indicator of erosion than a one-time seasonal change);
- joint analysis with a digital elevation model (erosion is geographically associated with slopes and ravine bottoms, unlike regular agricultural operations);
- overlaying agricultural field boundaries to exclude planned field operations from the detection results.

Conclusion. An index-based algorithm for detecting indicators of water-induced soil erosion from time series of *Sentinel-2* multispectral imagery was developed and validated. The method is based on the joint analysis of *NDVI* and *BSI* with spatial change detection. The algorithm was verified using synthetic data and applied to real imagery for two test areas in southern Russia, confirming its operability while revealing a fundamental limitation: within a single observation season, it cannot distinguish erosion-induced soil exposure from exposure caused by agricultural practices.

The results of the present stage are regarded as a source of input data for subsequent identification of the parameters of a physics-based mass-transport model (in particular, the soil erodibility coefficient or the source term of the diffusion-convection-reaction equation) using a neural-network physics-informed component, which is the subject of further research.

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